Journal of Economic Studies



GDP AND POPULATION GROWTH. EVIDENCE OF FRACTIONAL COINTEGRATION WITH HISTORICAL DATA FROM 1820 ONWARDS

Journal:	Journal of Economic Studies
Manuscript ID	JES-06-2020-0307.R2
Manuscript Type:	Research Paper
Keywords:	GDP, GDP per capita, population, cointegration, persistence, long range dependence

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Abstract

This paper deals with the analysis of the relationship between GDP and population using historical data from 1820 onwards in a group of seven countries, namely, Australia, Chile, Denmark, France, the UK, Italy and the USA. We investigate if there is a long run equilibrium relationship between the two variables, using fractional integration and cointegration methods. Our results show first that the two series are highly persistent, presenting orders of integration close to or above 1 in practically all cases. Testing cointegration between the two variables, the results are quite variable depending on the methodology and the bandwidth numbers used, but if cointegration takes places, it only occurs in the cases of France, Italy and the UK.

Strole

Keywords: GDP; GPD per capita; population; cointegration; persistence

JEL Classification: C22

Comments from the Editor and two anonymous reviewers are gratefully acknowledged.

1. Introduction

The relationship between population and economic growth constitutes a key topic in economics. It is indeed as old as the discipline itself and has been studied extensively from many different perspectives. Yet, conclusions about the casual relationship between population and income growth remains complex and controversial, and the debate is still ongoing. While some empirical works suggest that economic growth and development are positively related to population growth, others argue that population growth puts a strain on the limited stock of resources available per person, thus reducing countries' potential growth in the long run, an argument that echoes the classical Malthusian theory.

Among the positive views on this topic, research has pointed out that population growth increases a country's supply of labor force and enlarges the size of markets which, in turn, offers opportunities to exploit scale and scope economies. In contrast, some other works have argued that population growth is detrimental to economic growth. Despite extensive research on the relationship between population and economic issues, there is no universal agreement as to whether population growth is positive, damaging or neutral to economic growth.

The variety of research methods employed to investigate this intriguing relationship between economic and population growth might explain the diversity of results. Different time frames, countries, control variables or statistical methods employed are likely potential and powerful factors causing diversity in results (Heady & Hodge, 2009). Despite the wide range of methods, no works have comprehensively explored fractional integration and cointegration and its potential for unpacking the long run relationship between GDP and population. This is the main contribution of this work, the use of updated time series techniques in the analysis of long run comovements between population and growth and using historical data of seven countries dating back from 1820. Unlike other econometric techniques

used in the analysis of the population – income nexus, fractional integration and cointegration allow for mean reversion and long-lasting effects of shocks in both the individual variables and in the long run relationship between the variables. Our results show that the two individual series, log of GDP and log of population, display large degrees of persistence and lack of mean reversion, implying thus permanency of shocks. Looking at the possibility of long run relationships between the two variables, plausible cointegration is only found in the cases of France, Italy and the UK.

2. Literature review

There are many papers dealing with economic growth in time series (Narayan et al., 2007; Payne, 2010; Wong, 2013; Sa Cardoso and Ravishankar, 2015), while others focus on population (Azam et al., 2020; Adeosun and Popogbe, 2020), and others on the relationship between them (Yezdani, 2013; Rehman, 2019). In general, there is agreement on the existence of a relationship between population and GDP per capita and its importance to understand income per capita distributions. Yet, there is no agreement on the direction of this relationship. Peterson (2017) argued that for the world as a whole, the correlation between demographic growth and income per capita growth was negative. However, aggregate world data tells us nothing about the actual relationship between such determinant variables. Indeed, recent works suggest that for some countries and for some periods, demographic growth and economic growth can be positively correlated, and that some scenarios (i.e. being a high or a low-income country) might also influence the results (Peterson, 2017).

The first economic interpretations of population growth and its impact on the economy date back to classical economists such as Malthus and Ricardo. The underlying theory of the Malthusian Trap concerned a stagnant agriculture with a limited supply of land and capital in which the workforce suffered from diminishing returns. The implication of the Malthus model is that income growth will be offset by population growth and brought back to a basic subsistence level. While this understanding provided an accurate explanation of the preindustrial societies, it missed the boat entirely for future societies, especially for the modern economic growth period, when technological change pushed incomes well above subsistence levels. Indeed, between 1820 and 2001, world population has multiplied 6-fold at the same time as there has been a 9-fold increase in income per capita.

That historical evidence does not indicate, however, that the question of how population influences GDP has been resolved. The relation between population and GDP seems to be much more complex than that shown by the Malthusian model. Indeed, growth in per capita income might have been faster if population growth rates had been somewhat lower (Peterson, 2017). Moreover, population growth can represent much more than a food problem (Savaş, 2008), and could also create problems on the development of savings, the evolution of foreign exchange or human resources (Meier, 1995). Neoclassical models have extended this sort of argument on the negative effects of demographic expansion. These models, often referred to as exogenous growth models, assume that smaller amounts of capital per worker are the result of fast population growth and therefore decelerate economic growth (Bucci, 2015). In addition, it is also considered that the combination of an increasing population and a relatively static growth in capital stock bring diminishing returns enter into play. Coale and Hoover (1958) suggested that a shift toward a younger population may induce governments and households to divert resources from directly productive areas to expenditures which are expected to be productive only in the long run such us health and education. Later studies have also highlighted the damaging consequences of a growing population on social and economic well-being, including rapid and disordered urbanization, resource depletion, environmental degradation, domestic conflicts, or less effective social services.

Yet, neoclassical explanations proved insufficient to explain modern economic growth so by the mid-1980s, a revisionist wave emerged, arguing that concerns about population growth had been excessive, and suggesting that modern growth is not only a question of population but is related with the accumulation of human and physical capital. Some works suggested that population growth would stimulate individuals' creativity, resourcefulness and technological innovations therefore finding new solutions to resource problems and allowing food production to keep up with the growing population. In contrast to the neoclassical predictions, endogenous growth models (see Strulik, 2005 for a complete review) have mostly supported the assumption of a positive relationship between population growth and per capita economic growth when the hypothesis about diminishing returns to capital as labor supply increases (Todaro and Smith, 2012). Yet, some others have found the opposite correlation between the two variables. Strulik (2015), based on empirical findings working with human capital accumulation, found that the correlation could be positive, negative or, in some cases, economic growth could be independent of population, so, if economic growth is not only explained by population growth could be sustainable in the long run. Prettner and Prskawetz (2010) supported Strulik's conclusions working with population ageing and R&D subsidies.

Regardless of the contradictory nature of the relationship between population growth and economic growth, proving their points of views is a big challenge for both sets of scholars. Empirical studies are still scarce while research methods vary extensively. Cross-section regression has been a common method to analyze the relationship between population growth and income growth. However, the results are not consistent: some found no statistically significant relationship between the two variables while others did not arrive at conclusive results. Dawson and Tiffin (1998) used annual time series data over the period 1950-93 to analyze the long-run relationship between population and economic growth in India. The study employed cointegration and Granger causality methods and reported that there is no long-run relationship between the two variables. Thornton (2001) studied the relationship between population and economic growth in the long run in seven Latin American countries: Argentina, Brazil, Chile, Colombia, Mexico, Peru, and Venezuela. Using the same methods as Dawson and Tiffin (1998) this study employed annual time series data over the period 1900-94, the results showed the inexistence of a relationship between the two variables in all of the seven countries in the long run. Several analysts, due to these contradictory results, consider the possibility that the impact of population growth on per capita output growth may not be uniform but, rather, variable. For example, Becker, Glaeser, and Murphy (1999) founded in a theoretical model, based on population to cities, investment in human capital and economic growth, both negative and positive impacts on productivity due to a large increase in population. They demonstrated that, due to diminishing returns to the growing labor force making more intensive use of a fixed resource base, population growth in low income, agricultural societies delay growth in per capita income. On the other hand, a growing population with high income in urban economies, as a result of increasing returns from greater specialization and growth in investments in human capital, may give rise to greater income growth. Therefore, to enable a net relationship between greater population and economic growth it is necessary to have incentives to human capital and expansion of knowledge that develop new technologies, which outweigh diminishing returns to natural resources. A positive relationship between population growth and productivity is found in Bucci (2015), also through specialization, but the author suggests that more complex production processes might offset this positive effect. Bucci's theory complements Kelley and Schmidt (2001) and Mireau and Turnovsky (2014) explanations which suggest that results vary depending on whether population growth comes from lowering mortality rates or increasing fertility. Headey and Hodge (2009) found a positive correlation in the case of high-income countries and a negative correlation in case of low-income countries. Using unit root tests however,

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Hosen (2019) arrived at the opposite result. As many authors propose, these findings should be taken into account by policymakers, especially in developing countries, in order to design a resource allocation that can efficiently boost human capital development.

This review through the most relevant literature shows the wide variety of approaches and methods used to study the complex relationship between population and economic growth, while highlighting the lack of conclusive results. This fact allows us to conclude that, despite there is no agreement on the direction of the relation between population and economic growth, much of the literature agree to highlight the complexity of that relationship and suggest the need to deepen the study by incorporating other variables that allow to narrow the behavior of the population, such as for example fertility rates, levels of education or health and of course institutions. In addition, recent literature also agrees to suggest that human capital acts a fundamental driver to generate economic growth in the long-run and seems to be the key needed to counteract diminishing returns on natural resources.

Following this path, the work of Bucci, Prettner and Prskawetz (2019) delves into the study of relevant variables that affect population, mainly health, education and demographic change, and concludes how the process of capital accumulation is a fundamental determinant of long-run economic growth. This analysis is based on Romer's (1986) work, which established how long-run growth is mainly driven by the accumulation of knowledge and has a long tradition in economic research (Uzawa, 1965; Ben-Porath, 1967; Haley, 1976). Recently, Pelloni et al. (2019) confirmed non-linearities in the relation between human capital and growth, highlighting the role that institutions and gender have on this relation. Bucci et al. (2019) paid attention to the positive relation between the level of per capita income and population's health. Mariani et al. (2019) analyzed in a dynamic perspective the relation between the endogenous forces of demographic change and the environmental conditions.

Finally, it is also important to mention the role of government on these issues and the implications of its action, mentioned by Agenor (2019) and Cipriani and Fioroni (2019).

Although all these works represent a significant breakthrough, much remains to be done. Based on this evidence, we introduce a new methodological approach to the analysis of these two variables and its relationship that is based on the concepts of fractional integration and cointegration which are explained in the following section.

3. Methodology

The standard approach of looking at long run equilibrium relationships between nonstationary variables is through the framework of cointegration. According to this theory, given two (or more) time series which are nonstationary and move apart one each other, it should be expected that any linear combination of the two should also be nonstationary and move apart; however, there might exist a linear combination of the series which is stationary or at least presents a lower order of integration than the individual series, implying that the series tend to converge to one another in the long run.

Following Engle and Granger (1987), two series x_t and y_t are cointegrated if:

i) both series are individually I(d), and

ii) there exists a linear combination of the two which is integrated of order d - b, with b > 0.

Though this definition was originally presented in Engle and Granger (1987) for any real values d and b, most empirical applications (and theoretical works such as those by Johansen, 1988, 1995, 1996) assume integer degrees of differentiation, mainly, d = b = 1. In other words, x_t and y_t are supposed to be I(1) and the linear combination of the two must be I(0).

In this paper we depart from this assumption and allow d and b to be potentially fractional values. In this context, if a series is I(d), it can be expressed as:

$$(1-L)^{d} x_{t} = u_{t}, \qquad t = 1, 2, ...,$$
(1)

where L is the lag operator $(Lx_t = x_{t-1})$ and u_t is I(0) defined for the purpose of the present work as a covariance stationary process with a spectral density function that is positive and bounded at the zero frequency. Note that the polynomial in the left-hand side of equation (1) can be expressed for all real d as:

$$(1-L)^{d} = \sum_{j=0}^{\infty} {d \choose j} (-1)^{j} L^{j} = 1 - dL + \frac{d(d-1)}{2} L^{2} - \dots,$$

and thus, (1) can be expressed as:

$$x_t = dx_{t-1} - \frac{d(d-1)}{2}x_{t-2} + \dots + u_t.$$

Thus, if d is a fractional value, x_t depends on all its past history and the higher the value of d is, the higher the level of dependence between the observations is. Moreover, it allows for a much higher degree of flexibility in the dynamic specification of the data than the classical methods based on integer degrees of differentiation, i.e., 0 in case of stationarity and 1 for nonstationarity, and permitting d to be a fractional value, we can consider alternatives such as anti-persistence (d < 0); long memory stationarity (0 < d < 0.5); nonstationarity mean reverting processes ($0.5 \le d < 1$) or explosive patterns (d > 1).

Fractional cointegration is the natural extension of the concept of fractional integration to the multivariate case. Preliminary papers using this concept include Cheung and Lai (1993) and Gil-Alana (2003) and it was Peter Robinson and his coauthors who were the first to investigate this issue theoretically (Marinucci and Robinson, 2001; Robinson and Yajima, 2002; Robinson and Hualde, 2003; Robinson and Marinucci, 2003; Hualde and Robinson, 2007; etc.). In recent years, Soren Johansen and his coauthors have also introduced the concept of fractional CVAR, extending the classical CVAR to the fractional case (Johansen, 2008; Johansen and Nielsen, 2010, 2012, 2015; etc.).

4. Data

We use data from the Maddison database¹ hosted by the Maddison project. The Maddison project was launched in 2010 in an effort to support cooperation among scholars to continue the work of Angus Maddison, who had compiled an extraordinary set of data. It provides historical statistics on population and GDP over the very long run for a wide range of countries. The last version, updated in 2018, includes 169 countries and introduces a new measure of real GDP per capita expressed in 2011 US dollars. Within the economic history approach, Maddison data is well known and has been used in many previous works with different purposes and results. Being aware of the difficulties of estimating the GDP of previous times, Maddison's work is really important and the corrections applied to the initial estimations of the Maddison Project give us a certain degree of objectivity.

Among the wide range of countries in the sample, we select seven countries, namely, Australia, Chile, Denmark, France, Italy, the UK and the USA, that have continuous data between 1820 and 2016 for both population and economic growth. There are countries with different characteristics in terms of growth, size and economic development within this group. This variety is quite interesting for the study since it allows us to compare European economies that addressed the changes of industrialization in the first moment with European and non-European countries that later came to this process.

5. **Empirical results**

We start by considering the following model,

$$y_t = \beta_0 + \beta_1 t + x_t;$$
 $(1 - L)^d x_t = u_t,$ $t = 0, 1, ...,$ (2)

¹ Maddison Project Database, version 2018. Bolt, Jutta, Robert Inklaar, Herman de Jong and Jan Luiten van Zanden (2018), "Rebasing 'Maddison': new income comparisons and the shape of long-run economic development", Maddison Project Working paper 10.

where y_t refers to each of the observed time series (log of real GDP, log of population and log of real GDP per capita); β_0 and β_1 are unknown coefficients referring, respectively, to an intercept and a linear time trend, while we suppose x_t to be I(d), where d can be any real value; finally, u_t is I(0), expressed in terms of both uncorrelated and autocorrelated (Bloomfield, 1973) errors. The latter is a non-parametric approach of modelling the I(0) error term. It is non-parametric in the sense that the model is only implicitly determined in terms of its spectral density function, producing autocorrelations that decay exponentially fast, as in the AR case.

In all cases, we estimate the value of d under three different assumptions, corresponding to the cases of i) no deterministic terms (i.e., $\beta_0 = \beta_1 = 0$ in equation (2)), ii) with an intercept ($\beta_1 = 0$ in (2)), and iii) with an intercept and a linear time trend (β_0 and β_1 unknown), and we select for each series the specification that produces significant coefficients for these deterministic terms.

We start in Table 1 by presenting the results for the log of GDP. Panel i) displays the results for white noise errors, while Panel ii) focuses on the case of autocorrelation. We see that a time trend is required in all cases except for Italy with no autocorrelation. If we focus now on the estimated values of d we see that the differencing parameter is significantly below 1 in the case of Australia with no autocorrelation, and for Chile and the USA with autocorrelation. For the remaining cases (Denmark, France, the UK and Italy), d is found to be equal to or higher than 1. In fact, the I(1) hypothesis cannot be rejected in the majority of the cases, and evidence of I(d, d > 1) is found in the cases of France and Italy with no autocorrelation. In conclusion, evidence of mean reversion is only found for Australia (under the assumption of white noise errors) and for Chile and the USA (with autocorrelated disturbances). In all the other cases, our results indicate high levels of persistence, lack of mean reversion and thus permanency of shocks.

[Insert Tables 1 and 2 about here]

If we focus now on the time series for population, in Table 2, we see first that the time trend is required in all cases except for France with no autocorrelation. The values of d are now larger than in the previous table, being higher than 1 in most cases. Only for Australia are the results slightly different, finding evidence for this country of mean reversion (i.e., d < 1) under the two assumptions of white noise and autocorrelation. Thus, any random shock in the population series will have a permanent effect on its trend, in all except the Australian case where the shock will disappear by itself in the long run.

If we look now at the difference between the two series, i.e., the log of the real GDP per capita, the results are reported in Table 3. Assuming white noise errors, the I(1) hypothesis cannot be rejected in any series except for Italy, where the estimated value of d is found to be higher than 1. With autocorrelation, mean reversion (d < 1) is found in the cases of Chile and the USA, while the unit root cannot be rejected in the remaining cases.

[Insert Tables 3 and 4 about here]

Table 4 summarizes the results from the above three tables in terms of the estimated values of d for the two cases of uncorrelated and autocorrelated errors. We see in this table that only for Chile and USA is there a slight reduction in the degree of integration of the GDP per capita in relation to GDP. Moreover, the difference is clearly insignificant, suggesting that there is no evidence of cointegration of any degree in the series examined, at least when using the observed data on the three variables.

We next examine the possibility of cointegration by looking at the residuals from a linear regression of log GDP on log Population, that is, we consider the regression model,

$$Log GDP_{t} = \alpha + \beta Log POP_{t} + x_{t}, \qquad (3)$$

assuming once more that the errors might be fractionally integrated, i.e., as in equation (1) and estimating the value of d in this context. Thus, if that value is significantly smaller than the one of the individual parent series (log GDP and log POP), these two series would be

cointegrated showing a long run equilibrium relationship. This is, in fact, the approach proposed in Engel and Granger (1987), though examined in the fractional case in Cheung and Lai (1993) and Gil-Alana (2003). However, a necessary condition for cointegration in the bivariate context, is that the two individual series must display the same integration order. Thus, a preliminary step here is to test if the two series (log GDP and log POP) statistically share the same value of the differencing parameter, d. We use the statistics proposed in Robinson and Yajima (2002) and Robinson and Hualde (2003), testing the null of H_o: $d_x = d_y$, where d_x and d_y are the orders of integration for log GDP and log Population respectively. The results using the Robinson and Yajima's (2002) approach are displayed across Tables 5 and 6. In Table 5 we use for the orders of integration of the individual series, the values reported in Table 4, while in Table 6 we employ the local Whittle semiparametric approach proposed in Robinson (1995).

[Insert Tables 5 and 6 about here]

The results are very consistent in the two tables and we find evidence of homogeneous orders of integration in the two variables for only four of the variables, Australia, France, Italy and the UK. For the remaining three: Chile, Denmark and the US, our results reject the null of equal orders of integration for all the bandwidth numbers examined. Almost identical results were obtained when using the method proposed in Hualde (2003).

[Insert Table 7 about here]

Based on the above results, in Table 7, we estimate d on the errors of the regression model (3) in the four countries that show homogeneous degrees of integration, and the first thing that we observe is that the estimated differencing parameter is close to 1 in almost all cases, rejecting thus the hypothesis of cointegration of any degree between the two variables. In fact, the only evidence of mean reversion (d < 1) and potential cointegration between the two variables is found for France under no autocorrelation. This approach, however, may be

biased due to several facts. First, the estimates of d are obtained on the estimated errors and not on the observed data; moreover, the results can substantially change depending on the estimation method used in the calculation of α and β in (3), and they can be even inconsistent if some conditions are not satisfied. Because of this, as a final step, we conduct a Hausman test as proposed in Marinucci and Robinson (2001), testing the null hypothesis of no cointegration versus the alternative of fractional cointegration.

We present first in Table 8 the estimates of d* (which is the restricted estimate of d obtained in the bivariate representation of the two series under the assumption that both individual orders are the same) for a range of bandwidth numbers from 10 to 15. We observe here that for Italy and the UK and in some cases for France, the values are significantly smaller than the values observed for the individual series.

[Insert Tables 8 and 9 about here]

Table 9 displays the test results. We observe evidence of cointegration for all bandwidth numbers in the cases of Italy and the UK, and also for m = 12, 13, 14 and 15 for France. Thus, according to these results, some evidence of cointegration between GPD and population is found in these three countries, France, Italy and the UK, while no long run equilibrium relationship is found for the remaining four countries (Australia, Chile, Denmark and the USA).

6. **Concluding comments**

The aim of this study has been to analyze the relationship between GDP and population by drawing on the long-term historical data of seven countries, between 1820 and 2016. The seven countries covered in this study (France, Italy, the UK, Denmark, Australia, Chile and the USA) are high-income countries, according to the classification of the World Bank. Yet, not all of them were high-income countries at the beginning of the period of study: by 1820,

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notable differences in income and population existed among them. UK ranked the first in 1820 in terms of income per capita, closely followed by the USA. Australia, Denmark and France also ranked high, exceeding \$ 2,000, whereas Italy and Chile appeared at the bottom. These differences of income per capita results have several causes including relatively lower demographic pressure, in countries such as Australia, the USA or Denmark, or being an early (UK and France) or a late comer to Industrialization (Italy). However, during our period of study (1820-2016) all countries in the group, even including those in the lower positions, became high-income countries. Regarding population, Australia was the least inhabited in 1820, yet the one in which population grew the most during our period of study followed by the USA and Chile, both also with a high rate of demographic growth. In contrast, France, Italy and the UK are the countries in which population grew the least between 1820 and 2016.

In this study, we have investigated the possible existence of a relationship between the two variables, population and GDP, using fractional integration and cointegration methods. Our results show first that both series are highly persistent, presenting orders of integration close to or above 1 in practically all countries. This high level of persistence indicates that shocks in the series will have permanent effects and evidence of reversion to the mean (or transitory shocks) is only found in very few cases (Australia, for the log of real GDP and log population, and Chile and USA in case of log of real GPD and GDP per capita). In the rest of the cases, the estimates of the differencing parameter are equal to or higher than 1. When testing cointegration between the two variables, the results are quite variable depending on the methodology and the bandwidth numbers used, but if it does take place, it only occurs in the cases of France, Italy and the UK. Thus, only for these three countries we obtain some evidence of a long run commovement between the two variables.

According to Hosen (2019), correlation between GDP and population depends on income levels. He made four income groups, and showed that most of the countries (eight out

of the ten, Spain and USA being the exceptions) that belong to the high-income group explored a long running significant relationship between the growth of GDP and the growth of population. Our work shows that with a greater temporal perspective this is not clear, at least for all the rich countries examined in this work. In fact, Australia, the USA and Denmark did not confirm this relation. Evidence of correlation between GDP and population is only found in France, Italy and the UK. Although these three share a common feature, they are the ones in our group in which population grew the least between 1820 and 2016, this explanation cannot be generalized, nor considered as a conclusive moderator variable of the populationincome nexus. From a theoretical perspective, our results suggest that a lower steady-rate of population growth could enhance its capacity to transform itself into human capital and keep up investment in capital to avoid diminishing returns which in turn support economic growth. In contrast, a relatively high rate of population growth may negatively affect economic growth because investment in capital and expansion of knowledge find it more difficult to keep up population growth therefore leading to smaller amounts of capital per worker, diminishing returns, and therefore lower economic results.

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i) White noise errors						
Series	No terms	An intercept	A linear trend			
AUS	0.62 (0.57, 0.68)	12.4005 (71.02)	0.0437 (17.56)			
CHL	0.92 (0.80, 1.08)	13.0344 (194.24)	0.0340 (10.34)			
DK	0.97 (0.89, 1.07)	14.4683 (399.99)	0.0249 (11.19)			
FRA	1.11 (1.01, 1.24)	17.5998 (278.86)	0.0206 (2.67)			
UK	1.11 (0.98, 1.29)	17.6312 (563.86)	0.0205 (5.35)			
ITA	1.32 (1.20, 1.50)	17.2066 (417.05)				
USA	1.04 (0.91, 1.20)	16.8129 (393.75)	0.0332 (9.26)			
	ii) Autocorrelated	(Bloomfield) errors				
Series	No terms	An intercept	A linear trend			
AUS	0.91 (0.82, 1.05)	12.2902 (63.20)	0.0432 (4.80)			
CHL	0.67 (0.55, 0.85)	13.0304 (170.21)	0.0333 (25.89)			
DK	0.94 (0.82, 1.11)	14.4674 (396.63)	0.0249 (12.84)			
FRA	0.97 (0.82, 1.17)	17.6033 (278.17)	0.0206 (5.30)			
UK	0.80 (0.68, 1.02)	17.6273 (552.37)	0.0204 (22.71)			
ITA	1.05 (0.95, 1.19)	17.1887 (406.40)	0.0214 (5.58)			
USA	0.78 (0.63, 0.99)	16.8320 (374.88)	0.0346 (29.83)			

 Table 1: Estimated coefficients for the log of real GDP

The values in parenthesis in the second column refers to the 95% confidence band of the non-rejection values of d. Those in parenthesis in the third and fourth columns are the corresponding t-values for the intercept and the time trend respectively.

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3 4	Table 2: Estimated co
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7	Series
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21 22	Series
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25	CHL
26 27	DK
28	FRA
29 30	UK
31 32	ITA
33	USA
34 35	The values in parenthesis i of d. Those in parenthesis i
36 37	the time trend respectively.
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oefficients for the log of population

White noise errors i) No terms A linear trend An intercept 0.50 (0.45, 0.57) 5.8100 (42.27) 0.0230 (15.89) 1.98 (1.90, 2.07) 6.6246 (7771.18) 0.0230 (19.31) 1.79 (1.69, 1.92) 7.0.414 (4552.14) 0.0103 (5.49) 1.58 (1.45, 1.75) 10.3464 (2416.51) _____ 1.29 (1.22, 1.39) 9.9538 (1803.79) 0.0068 (4.25) 1.92 (1.66, 2.30) 9.9058 (4926.97) 0.0064 (2.35)1.74 (1.65, 1.87) 9.1780 (4749.27) 0.0295 (15.33) ii) Autocorrelated (Bloomfield) errors A linear trend No terms An intercept 0.73 (0.64, 0.86) 5.7739 (32.01) 0.0225 (5.93) 1.90 (1.79, 2.04) 6.6247 (4943.56) 0.0228 (12.79) 1.71 (1.77, 2.01) 7.0.415 (4335.83) 0.0101 (5.67) 1.25 (1.10, 1.48) 10.3446 (2190.86) 0.0042 (3.67) 1.42 (1.27, 1.63) 9.9527 (1844.22) 0.0083 (2.92) 1.03 (0.94, 1.16) 9.9065 (5979.25) 0.0056 (41.05) 1.63 (1.53, 1.80) 9.1792 (3766.15) 0.0274 (11.77)

in the second column refers to the 95% confidence band of the non-rejection values in the third and fourth columns are the corresponding t-values for the intercept and

45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60

i) White noise errors					
Series	No terms	An intercept	A linear trend		
AUS	0.94 (0.88, 1.04)	6.5035 (107.80)	0.0212 (6.59)		
CHL	0.92 (0.80, 1.08)	6.4006 (94.35)	0.0179 (5.43)		
DK	0.99 (0.91, 1.08)	7.4249 (205.22)	0.0167 (6.81)		
FRA	1.04 (0.95, 1.17)	7.2563 (119.59)	0.0167 (3.18)		
UK	1.11 (0.99, 1.28)	9.6755 (252.51)	0.0142 (3.90)		
ITA	1.32 (1.21, 1.49)	7.2973 (176.80)			
USA	1.02 (0.89, 1.19)	7.6823 (179.33)	0.0295 (15.33)		
	ii) Autocorrelated	(Bloomfield) errors			
Series	No terms	An intercept	A linear trend		
AUS	0.96 (0.85, 1.11)	6.5017 (107.84)	0.0212 (6.00)		
CHL	0.65 (0.53, 0.84)	6.3778 (98.93)	0.0171 (16.96)		
DK	0.99 (0.88, 1.14)	7.4249 (204.95)	0.0167 (6.80)		
FRA	0.91 (0.76, 1.12)	7.2567 (120.36)	0.0168 (6.02)		
UK	0.87 (0.76, 1.03)	9.6661 (255.08)	0.0147 (12.75)		
ITA	1.06 (0.96, 1.19)	7.2827 (172.94)	0.0158 (3.99)		
USA	0.74 (0.60, 0.96)	7.6102 (181.34)	0.0167 (18.16)		

Table 3: Estimated coefficients for the log of real GDP per capita

The values in parenthesis in the second column refers to the 95% confidence band of the non-rejection values of d. Those in parenthesis in the third and fourth columns are the corresponding t-values for the intercept and the time trend respectively.

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	No autocorrelation			No autocorrelation Model of Blomfield (autocorrelation)		
Series	Log GDP	Log POP	Log CAP	Log GDP	Log POP	Log CAP
AUS	0.62	0.50	0.94	0.91	0.73	0.96
CHL	0.92	1.98	0.92	0.67	1.90	0.65
DK	0.97	1.79	0.99	0.94	1.71	0.99
FRA	1.11	1.58	1.04	0.97	1.25	0.91
UK	1.11	1.29	1.11	0.80	1.42	0.87
ITA	1.32	1.92	1.32	1.05	1.03	1.06
USA	1.04	1.74	1.02	0.78	1.63	0.74

Table 4: Summary results across Tables 1 – 3

Table 5: Testing homogeneity in the integration order I (Robinson and Yajima, 2002)

Parametric	netric No autocorrelation		Bloomf	ield (autocorr	relation)	
method	$m = (T)^{0.25}$	$m = (T)^{0.35}$	$m = (T)^{0.45}$	$m = (T)^{0.25}$	$m = (T)^{0.35}$	$m = (T)^{0.45}$
AUS	0.449	0.762	1.293	0.674	1.143	1.939
CHL	-3.971	-6.735	-11.423	-4.608	-7.815	-13.256
DK	-3.072	-5.210	-8.837	-2.884	-4.892	-8.298
FRA	-1.760	-2.986	-5.06	-1.048	-1.779	-3.017
UK	-0.674	-1.143	-1.939	-2.322	-3.939	-6.681
ITA	-2.247	-3.812	-6.466	0.074	0.127	0.215
USA	-2.622	-4.447	-7.544	-3.184	-5.401	-9.160

In bold, evidence of homogeneity in the order of integration at the 5% level. m is a bandwidth number.

Table 6: Testing	homogeneity i	in the integ	ation order	II (Rohinso	n and Vaiima	2002)
Table 0. Testing	nomogeneity	in the mitgi	anon or ucr		n anu rajima	, 2002)

Whittle s.	10	11	12	13	14	15
AUS	1.340	0.605	1.211	1.417	2.576	1.695
CHL	-9.289	-10.813	-13.284	-14.532	-20.648	-22.770
DK	-4.390	-4.949	-5.760	-5.512	-6.089	-7.425
FRA	-1.319	-1.793	-1.787	-3.873	-3.415	-5.370
UK	-2.160	-0.219	-0.359	-5.031	-5.768	-7.709
ITA	-0.041	-1.102	-0.624	-1.729	-2.394	-3.390
USA	-9.469	-9.734	-14.629	-13.793	-14.629	-16.110

In bold, evidence of homogeneity in the order of integration at the 5% level.

	No autocorrelation			Model of Blomfield (autocorrelation)		
Series	d	Intercept	Time trend	d	Intercept	Time trend
ALIC	0.97	6.979	-0.021	1.04	6.966	-0.026
AUS	(0.88, 1.07)	(53.29)	(-1.11)	(0.79, 1.22)	(58.92)	(-1.50)
FRA	0.86	-34.891	4.098	0.87	-34.989	4.108
ГКА	(0.78, 0.96)	(-9.65)	(11.75)	(0.75, 1.06)	(-9.39)	(11.42)
ITA	1.28	-8.183	1.635	1.02	-10.585	1.878
IIA	(1.17, 1.46)	(-0.76)	(1.51)	(0.93, 1.16)	(-2.14)	(3.77)
LIV	1.10	0.124	0.799	0.96	-4.081	1.220
UK	(1.01, 1.24)	(0.04)	(2.63)	(0.89, 1.06)	(-1.87)	(5.59)

 Table 7: Testing fractional cointegration with Robinson and Yajima (2001)

In bold, evidence of fractional cointegration and mean reversion at the 5% level.

Table 8: Estimates of d* in the bivariate representation of the series

S	10	11	12	13	14	15
AUS	0.840	0.974	0.928	1.000	0.861	0.940
FRA	0.896	0.850	0.586	0.426	0.562	0.701
IT	-0.090	-0.105	-0.012	0.059	0.173	0.303
UK	0.119	0.062	0.146	0.407	0.549	0.482

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s / Country	AUS	FRA	IT	UK
10	$H_{10} = 8.554$	$H_{10} = 0.199$	$H_{10} = 186.29$	$H_{10} = 92.27$
	$H_{20} = 2.797$	$H_{20} = 6.362$	$H_{20} = 187.27$	$H_{20} = 152.73$
	$\hat{d}_* = 0.840$	$\hat{d}_* = 0.896$	$\hat{d}_* = -0.090$	$\hat{d}_* = 0.119$
11	$H_{10} = 0.666$	$H_{10} = 0.970$	$H_{10} = 196.68$	$H_{10} = 128.2$
	$H_{20} = 0.091$	$H_{20} = 11.152$	$H_{20} = 229.80$	$H_{20} = 140.5$
	$\hat{d}_* = 0.974$	$\hat{d}_* = 0.850$	$\hat{d}_* = -0.105$	$\hat{d}_* = 0.062$
12	$H_{10} = 2.807$	$H_{10} = 17.503$	$H_{10} = 190.31$	$H_{10} = 126.9$
	$H_{20} = 0.470$	$H_{20} = 43.869$	$H_{20} = 204.63$	$H_{20} = 138.2$
	$\hat{d}_* = 0.928$	$\hat{d}_* = 0.586$	$\hat{d}_* = -0.012$	$\hat{d}_* = 0.140$
13	$H_{10} = 0.585$	$H_{10} = 37.440$	$H_{10} = 169.06$	$H_{10} = 31.91$
	$H_{20} = 0.120$	$H_{20} = 83.866$	$H_{20} = 206.17$	$H_{20} = 92.09$
	$\hat{d}_* = 1.000$	$\hat{d}_* = 0.426$	$\hat{d}_* = 0.059$	$\hat{d}_* = 0.407$
14	$H_{10} = 7.513$	$H_{10} = 19.569$	$H_{10} = 135.02$	$H_{10} = 16.94$
	$H_{20} = 0.629$	$H_{20} = 52.093$	$H_{20} = 180.36$	$H_{20} = 71.85$
	$\hat{d}_* = 0.861$	$\hat{d}_* = 0.562$	$\hat{d}_* = 0.173$	$\hat{d}_* = 0.549$
15	$H_{10} = 2.628$	$H_{10} = 3100$	$H_{10} = 92.928$	$H_{10} = 20.56$
	$H_{20} = 0.027$	$H_{20} = 32.323$	$H_{20} = 146.78$	$H_{20} = 103.3$
	$\hat{d}_{*} = 0.940$	$\hat{d}_{*} = 0.701$	$\hat{d}_* = 0.303$	$\hat{d}_* = 0.482$

Table 9: Testing fractional cointegration with Robinson and Marinucci (2001)

 $\chi_1^2(5\%) = 3.84$. * indicates rejection of the null hypothesis of no cointegration at the 5% level.

Editor Journal of Economic Studies		Prof. Luis A. Gil-Alana University of Navarra E-31080 Pamplona Spain		
20 February	2021			
Ref.:	JES-06-2020-0307			
Title:	GDP AND POPULATION GROWTH. EVIDENCE OF FRACTIONAL COINTEGRATION WITH HISTORICAL DATA FROM 1820 ONWARDS			
Journal:	Journal of Economic Studies			
Dear Editor,				
of fractioana	s for accepting our paper entitled "GDP and l cointegration with historical data from 182 n the Journal of Economic Studies			
Following your recommendations, we have included some papers related with the topic of the paper that have recently appeared in JES and other Emeral Insight journals Also, we have reduced the length of the paper, from the original 8398 words to 7129 in the revised version.				
These are the new references incorporated in the new version of the paper:				
Adeosun, O.T. and O.O. Popogbe (2020), Population growth and human resource utilization nexus in Nigeria, Journal of Humanities and Applied Social Sciences				
Azam M., H.N. Khan and F. Khan (2020), Testing Malthusian's and Kremer's population theories in developing economy, International Journal of Social Economics 47, 4, 523-538				
Narayan, P.K., S. Narayan, B. Prasan and A. Prasad (2007), Survey of the International Evidence on the Causal Relationship Between Energy Consumption and Growth, Journal of Economic Studies 34, 4, 341-351.				
Payne, J.E. (2010), Survey of the International Evidence on the Causal Relationship Between Energy Consumption and Growth, Journal of Economic Studies 37, 53-95.				

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Sincerely yours,

Prof. Luis A. Gil-Alana