Testing Fractional Integration in Time Series with Artificial Neural Network Nonlinearity

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Abstract

This paper proposes a non-linear fractional integration time-series model based on the autoregressive neural network process. ……………………………………………………………
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Keywords: Autoregressive Neural Network; Fractional integration; Unit root test; Non-linearity
JEL Classification: C15; C22; C45

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1. Introduction

Testing unit-roots has become a standard practice in time series econometrics, since the processes of modelling and forecasting rely on stationary time structure (Box and Jenkins, 1976). Starting with the unit root tests of Dickey and Fuller (ADF, 1979), many other unit root tests have become available in the last twenty years including those of Phillips and Perron (1988), Kwiatkowski et al. (KPSS, 1992), Elliot et al. (ERS, 1996), Ng and Perron (NP, 2001), etc. Nevertheless, it is nowadays a well-known stylized fact that most unit root testing frameworks have very low power if the alternatives include breaks (Perron, 1989; Campbell and Perron, 1991), regime-switching (Nelson, Piger and Zivot, 2001) or more general non-linear structures (Enders and Granger, 1998; etc.). One way to improve the issue is to incorporate nonlinearities in the deterministic components of the auxiliary regressions of the unit root tests, as proxy for structural breaks.

Franses and van Dijk (2000) emphasized the appropriateness of the non-linear time series models as tools to explain and predict economic time series, and the utilization of unit root tests on models based on macroeconomic and financial constraints i.e. exchange rates, unemployment and industrial manufacturing, among others. Several other studies that have attempted to incorporate non-linear dynamics into the unit root testing framework include Caner and Hansen (2001), Shin and Lee (2001), Kapetanios et al. (2003); among many others. In another study, Allen et al. (2016) extended the unit root tests on non-linear models based on Euro, British pound, Chinese yuan, and Japanese yen over US dollar for the analysis of exchange rate movements.

Further, Trapletti et.al. (2000) presented significant findings on the stationarity in AutoRegressive Neural Network (ARNN) framework. According to their research, the current linear unit root stationarity examinations are satisfactory if the activation function is bounded. Steurer (1996) had earlier demonstrated by empirically that the neural networks only worked best for those stationary data. However, nonlinearity tests are useful additions to the common situation of time series modelling. Examining a time series with unknown functional form of nonlinearity before an ARNN adjusted model is essential for two reasons. First, the need to justify a non-linear model with a linear variant; and second, an ARNN model executes equally to a linear model if the examined time series is considered by a linear process.

The Artificial Neural Network (ANN) is a parametric model approximator for other non-linear time series models such as the Threshold Autoregressive (TAR), Smooth Transition AutoRegressive (STAR), Markov Switching (MS) and Bilinear models (Franses and van Dijk, 2000). Thus, it is uncommon to imagine an ANN model as a Data Generating Process (DGP) of any time-dependent system and test for neural network-type non-linearity, but more as a universal nonlinear approximator (Lee, White and Granger, 1993).

So far, unit root tests based on ANN non-linearity are scarce in the literature. A recent development in the literature on non-linear unit root tests is the proposed Autoregressive Neural Network – Augmented Dickey Fuller [ARNN-ADF] test by Yaya, Ogbonna, Furuoka and Gil-Alana (2021). The testing procedure is well suited for unknown functional forms of non-linearity in time series as it relies on the linear, quadratic and cubic components of the neural network process, to accounts for the nonlinearity in the ARNN-ADF testing framework. The authors have applied the simplest form of the Artificial Neural Network (ANN) model in the ARNN-ADF test regression. In this present paper, we extend the unit root testing to allow for fractional order of integration, relying on the notion of Hassler and Wolters (1994), Lee and Schmidt (1996) and others, who found that conventional classical unit root tests tends to have very low power against fractional alternatives. Thus, the new test is based on fractional integration using ANN nonlinearity, and it will serve as a more general test to the one proposed in Yaya, Ogbonna, Furuoka and Gil-Alana (2021).

Our contribution to the literature is twofold. First, we propose a fractional integration based ANN nonlinearity testing framework and provide the theoretical background for the testing framework. Second, we evaluate the size and power of the testing framework in comparison with extant unit root tests, as well as the recently proposed test by Yaya et al. (2021), in a Monte Carlo simulation exercise. Third, we apply the new test to empirically ascertain the stationarity stance of unemployment rates in selected countries.

The rest of the paper is structured as follows: the theoretical properties of the model are presented in Section 2. Section 3 displays the simulation analysis, while Section 4 contains an empirical application based on unemployment rates of five European countries**;** finally, Section 5 concludes the paper.

2. Theoretical properties

We consider the model,

$$y_t = f\left(\gamma_j, w_t\right) + x_t, \quad t = 1, 2, ..., T \tag{1}$$

where y_t is the time series under investigation, $f\left(\gamma_j, w_t\right)$ is the expression for the ANN non-linear function in time *t*, where γ_j and w_t are defined later, and x_t is the fractionally integrated process, given by:

$$\left(1-L\right)^d x_t = u_t, \quad t = 1, 2, ..., T \tag{2}$$

where L is the usual lag-operator, i.e., $L^k x_t = L x_{t-k}$ for every k lag integer, d is the fractional integration parameter, defined in the interval $-0.5 < d < 2$ for moving average invertibility range of time series (Sowell, 1992), and u_t is the covariance stationary I(0) process, assumed to be independently and identically distributed with mean 0 and variance σ_u^2. For the fractional integration parameter, at $d = 0$ from (2), $x_t = u_t$, and at $d = 1, 2$, we have the respective series differenced transformations $x_t - x_{t-1} = u_t$ and $x_t - 2x_{t-1} + x_{t-2} = u_t$. The fractional difference operator $\left(1-L\right)^d$ is expressed by the Maclaurin series as:

$$\left(1-L\right)^d = \sum_{k=o}^{\infty} \frac{\Gamma\left(-d+k\right)}{\Gamma\left(-d\right)\Gamma\left(k+1\right)} L^k, \tag{3}$$

where $\Gamma(.)$ is a Gamma function. By putting (3) in (2), u_t in (2) can be expressed as:

$$u_t = \sum_{k=0}^{\infty} \frac{\Gamma(-d+k)}{\Gamma(-d)\Gamma(k+1)} x_{t-k} . \tag{4}$$

The function $f(\gamma_j, w_t)$ in (1) is known as the hidden unit of the ANN. This is a bounded logistic function between 0 and 1, such that:

$$f(\gamma_j^T w_t) = \frac{1}{1+\exp(-\gamma_j^T w_t)} = \frac{1}{1+\exp\left[\left(c_1 - \gamma_{11} y_{t-1} - \gamma_{12} y_{t-2} - \ldots - \gamma_{1j} y_{t-j}\right)\right]} \tag{5}$$

where $\gamma_j = (-c, \gamma_{11}, \ldots, \gamma_{1j})^T$ and $(j+1)\times 1$ vector of parameters of j hidden units. These hidden units are then approximated using the third-order Taylor series expansion on the logistic function as:[1]

$$F(\gamma_j^T w_t) = F(\gamma_j^T w_t^0) + \frac{\delta(\gamma_j^T w_t^0)}{\delta w_t^T}(w_t - w_t^0)_{|w_t = w_t^0} + \frac{\delta(w_t - w_t^0)}{\delta w_t^T} \frac{\delta^2(\gamma_j^T w_t^0)}{\delta w_t^T \delta w_t}(w_t - w_t^0)_{|w_t = w_t^0}$$

$$+ \ldots + R_h(\gamma_j, w_t, w_t^0), \tag{6}$$

and $R_h(\gamma_j, w_t, w_t^0)$ is the remainder of the h^{th} order expansion in the Taylors series expansion (Rech, 2002; Medeiros, Terasvirta and Rech, 2006; Yaya, Ogbonna, Furuoka and Gil-Alana, 2021). By merging terms of the same orders in (6) gives,[2]

$$f(\gamma_j{}^T w_t) = \lambda_0 + \sum_{t=1}^{q} \lambda_i w_{ti} + \sum_{i=0}^{q}\sum_{j=1}^{q} \lambda_{ij} w_{ti} w_{tj} + \sum_{i=0}^{q}\sum_{j=i}^{q}\sum_{l=j}^{q} \lambda_{ijl} w_{ti} w_{tj} w_{tl} + R(\gamma_j, w_t, w_t^0) \tag{7}$$

Then, by expanding (7) around $\gamma_{j0} = 0$ gives,

$$f(\gamma_j{}^T w_t) = m_0 + \sum_{t=1}^{q} m_i w_{ti} + \sum_{i=0}^{q}\sum_{j=1}^{q} m_{ij} w_{ti} w_{tj} + \sum_{i=0}^{q}\sum_{j=i}^{q}\sum_{l=j}^{q} m_{ijl} w_{ti} w_{tj} w_{tl} + R(\gamma_j, w_t, 0), \tag{8}$$

[1] This is necessary because the distribution of parameters of ARNN does not exist, whereas that of the Taylor polynomial exists.

[2] This expansion is similar to Volterra functional series or the Kolmogorov-Gabor polynomial (see Priestley, 1988; Madala. and Ivakhnenko, 1994; Ivakhnenko and Müller, 1995). The Kolmogorov-Gabor polynomial can approximate any random series and its estimation follows adaptive methods or a system of Gaussian normal equations (Allen et al., 2015).

where $R_h(\gamma_j, w_t, 0)$ is another remainder of the h^{th} order expansion. By using equation (8) in (1), one obtains,

$$(1-L)^d y_t = m_0 + \sum_{t=1}^{q} m_i w_{ti} + \sum_{i=0}^{q}\sum_{j=1}^{q} m_{ij} w_{ti} w_{tj} + \sum_{i=0}^{q}\sum_{j=i}^{q}\sum_{l=j}^{q} m_{ijl} w_{ti} w_{tj} w_{tl} + \tilde{\varepsilon}_t \,, \tag{9}$$

where q is the order of the hidden unit (neuron) and $\tilde{\varepsilon}_t$ is the remainder term of the Taylor series expansion. Note, $w_t = (1, y_{t-1}, y_{t-2}, \ldots, y_{t-k})'$ from (5) in the case of an AR(p) process. Then, a simple form of the ARNN logistic function in (8) uses $q = 1$, $p = 1$, and $k = 1$ such that $w_t = (1, y_{t-1})'$ and q is the order of the hidden unit and p is the order of the autoregressive part. Based on this, (9) is re-specified as,

$$(1-L)^d y_t = m_0 + \sum_{i=1}^{q} m_i y_{t-i} + \sum_{i=0}^{q}\sum_{j=1}^{q} m_{ij} y_{t-i} y_{t-j} + \sum_{i=0}^{q}\sum_{j=i}^{q}\sum_{l=j}^{q} m_{ijl} y_{t-i} y_{t-j} y_{t-l} + \tilde{\varepsilon}_t \,, \tag{10}$$

where m_0 is the model intercept; m_i is the parameter for the linear logistic component where y_{t-i} is the linear component; m_{ij} is the parameter for the quadratic component where $y_{t-i}y_{t-j}$ is the quadratic component, and m_{ijl} is the parameter for the cubic component and $y_{t-i}y_{t-j}y_{t-j}$ is the cubic component. The remainder term $\tilde{\varepsilon}_t$ is now the error process which is a white noise process or a weakly autocorrelated processes.

About the linearity of the process, acceptance of the null hypothesis:

$$H_0 : \begin{cases} m_{ij} = 0; \ i = 1, \ldots, q \\ m_{ij} = 0, \quad i = 0, \ldots, q; \ j = i, \ldots, q \\ m_{ij} = 0, \quad i = 0, \ldots, q; \ j = i, \ldots, q; \ l = j \end{cases} \tag{11}$$

implies linearity of the time structure. A suitable F test for linearity against nonlinearity, therefore, is conducted.

To estimate the parameters in the model in (10), one needs to minimize the errors $\tilde{\varepsilon}_t$ having re-written the nonlinear model in linear parameter form,

$$\tilde{\varepsilon}_t = y_t^* - \hat{m}_0 1_t^* + \sum_{i=1}^{q} \hat{m}_i z_t^* - \sum_{i=0}^{q}\sum_{j=1}^{q} \hat{m}_{ij} zz_t^* - \sum_{i=0}^{q}\sum_{j=i}^{q}\sum_{l=j}^{q} \hat{m}_{ijl} zzz_t^* \qquad (12)$$

where $y_t^* = (1-L)^{d_o} y_t$: $1_t^* = (1-L)^{d_o} 1_t$: $z_t^* = (1-L)^{d_0} y_{t-i}$; $zz_t^* = (1-L)^{d_0} y_{t-i} y_{t-j}$ and $zzz_t^* = (1-L)^{d_0} y_{t-i} y_{t-j} y_{t-l}$ with the hypothesized value $d = d_0$. The error process $\tilde{\varepsilon}_t$ is assumed to be an I(0).

In the absence of nonlinearity, that is the acceptance of nested null hypotheses, the testing ARNN-FI framework in (10) reduces to the linear specification of Robinson (1994) fractional integration test for $q = 1$, and $y_{t-i} = t$, that is a time trend with coefficient β and intercept $m_0 = \alpha$.

$$(1-L)^d y_t = \alpha + \beta t + \tilde{\varepsilon}_t \qquad (13)$$

With the error process $\tilde{\varepsilon}_t$, one can easily estimate the coefficients α and β by standard OLS methods such that,

$$\tilde{\varepsilon}_t = y_t^* - \hat{\alpha}_0 1_t^* + \hat{\beta} t_t^* \qquad (14)$$

and $y_t^* = (1-L)^{d_o} y_t$: $1_t^* = (1-L)^{d_o} 1_t$: $t_t^* = (1-L)^{d_o} t_t$, and with the aid of the complex form of the test statistic,

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a}, \qquad (15)$$

where T is the sample size, and

$$\hat{a} = \frac{-2\pi}{T} \sum_j^{*} \psi(\lambda_j) g_u(\lambda_j;\hat{\tau})^{-1} I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g_u(\lambda_j;\hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} = \frac{2}{T}\left\{ \sum_j^{*} \psi(\lambda_j)\psi(\lambda_j)' - \sum_j^{*} \psi(\lambda_j)\hat{\xi}(\lambda_j)' \left[\sum_j^{*} \hat{\xi}(\lambda_j)\hat{\xi}(\lambda_j)' \right]^{-1} \sum_j^{*} \hat{\xi}(\lambda_j)\psi(\lambda_j)' \right\};$$

$$\psi(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|; \qquad \hat{\xi}(\lambda_j) = \frac{\partial}{\partial\tau} \log g_\varepsilon(\lambda_j;\hat{\tau})$$

where $\lambda_j = 2\pi j/T$, and * indicates that the sums are taken over all frequencies bounded in the spectrum, with periodogram $I(\lambda_j)$ for $\tilde{\varepsilon}_t$ and $\hat{\tau} = \arg\min_{\tau \in T^*} \sigma^2(\tau)$, ($T^*$ is a subset of the R^q Euclidean space).

The fractional Flexible Fourier fractional (FFFF) integration test, recently proposed in Gil-Alana and Yaya (2020) uses the Fourier function as nonlinearity source instead of the linear time trend function as in Robinson (1994) case.

$$(1-L)^d y_t = \alpha + \beta t + \sum_{k=1}^{n} \lambda_k \sin\left(2\pi j_k t/T\right) + \sum_{k=1}^{n} \gamma_k \cos\left(2\pi j_k t/T\right) + \tilde{\varepsilon}_t \quad (16)$$

where $j_k = 1, 2, \ldots, n$ are the fractional Fourier frequency and $j_k = k$ (k =1, 2) in the case of integer frequency (Omay, 2015; Omay, Gupta and Bonaccolto, 2017; Gil-Alana and Yaya, 2020). Parameters λ_k and γ_k measure the amplitude and displacement of the sine/cosine sinusoidal component of the Fourier function which acts as the deterministic term; $\pi = 3.142$. Empirically, (15) can easily be re-written as,

$$y_t^* = \alpha 1_t^* + \beta t_t^* + \sum_{k=1}^{n} \lambda_k \sin_{k,t}^* + \sum_{k=1}^{n} \gamma_k \cos_{k,t}^* + \hat{\varepsilon}_t \quad (17)$$

where $\sin_{k,t}^* = (1-L)^{d_0} \sin\left(2\pi j_k t/T\right)$, $\cos_{k,t}^* = (1-L)^{d_0} \cos\left(2\pi j_k t/T\right)$ and y_t^*, 1_t^* and t_t^* as in (14) above. The process in (17) is actually linear in parameters, thus it is easier to estimate its parameters using OLS method as well.

3. Simulation analysis

The Monte Carlo simulations are conducted to examine the finite-sample behaviour of three different fractional integration tests: the Robinson test (Robinson, 1994), the Alana-Yaya (AY) test (Gil-Alana and Yaya, 2020) and the proposed artificial neural network (ANN)-fractional integration (ANN-FI) test. Firstly, the Robinson test (Robinson, 1994) is based on the following equation:

$$y_t = \mu + \beta t + x_t; \quad (1-L)^d x_t = \varepsilon_t \quad (18)$$

where L is the lag operator, d is the fractional integration parameter, μ is the intercept, t is the trend, β is the slope parameter and ε is the white noise. Secondly, the Alana-Yaya (AY) test Gil-Alana and Yaya, 2020) is based on these equations:

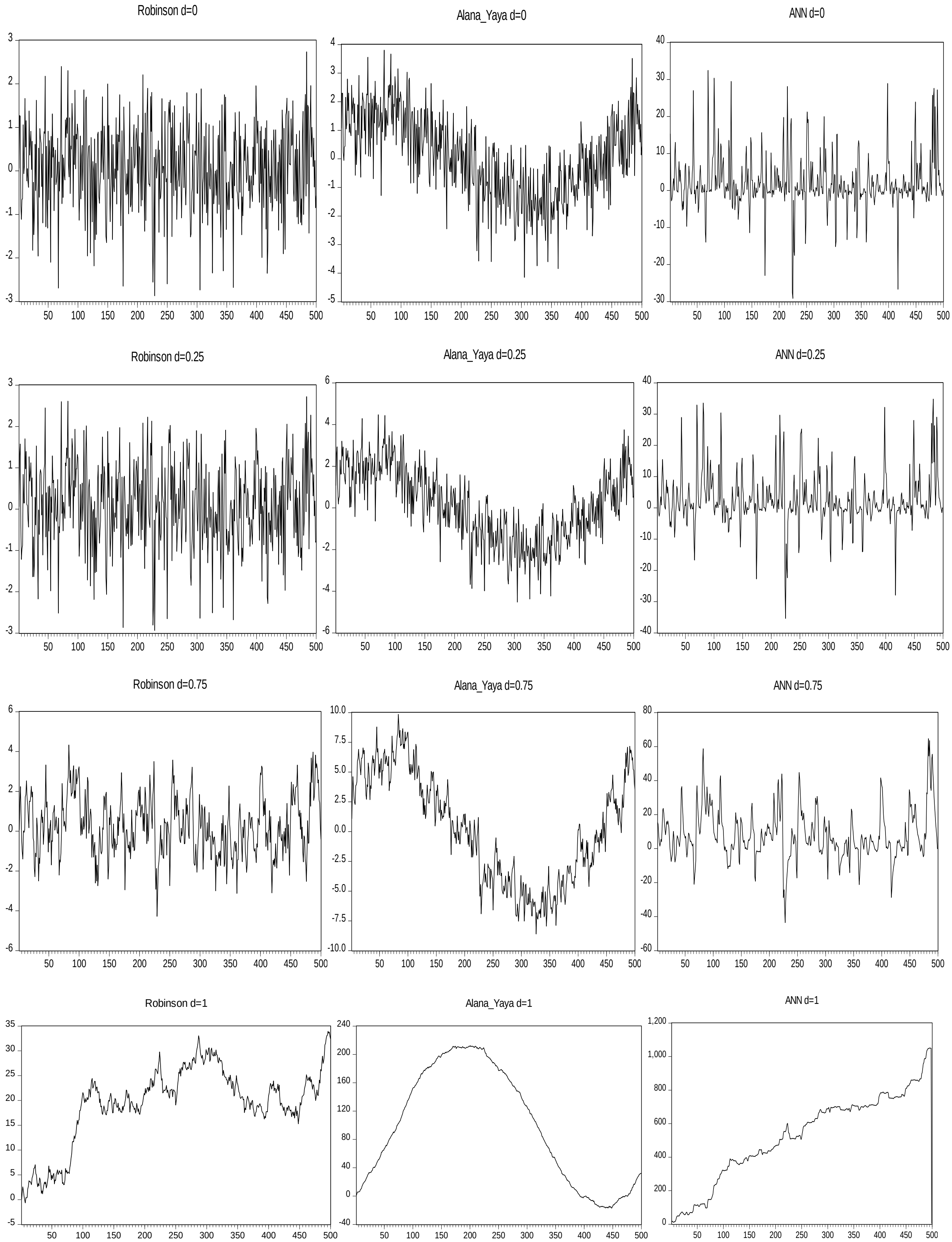
$$y_t = \mu + \beta t + \gamma_1 \sin\left(\frac{2\pi k t}{T}\right) + \gamma_2 \cos\left(\frac{2\pi k t}{T}\right) + x_t; \quad (1-L)^d x_t = \varepsilon_t \tag{19}$$

where *sin* is a sine function, *cos* is a cosine function and γ is the slope parameter. The ANN-FI test is based on these equations:

$$y_t = \mu + \beta t + \sum_{i=1}^{q} m_i y_{t-i} + \sum_{i=0}^{q}\sum_{j=i}^{q} m_{ij} y_{t-i}\, y_{t-j} + \sum_{i=0}^{q}\sum_{j=i}^{q}\sum_{l=j}^{q} m_{ijl} y_{t-i}\, y_{t-j} y_{t-l} + x_t;\ (1-L)^d x_t = \varepsilon_t \tag{20}$$

Figure 1 shows the graphs of simple realization of *I(d)* process for the Robinson test (Equation 18), the Alana-Yaya test (Equation 19) and the ANN-FI test (Equation 20), with $d = 0, 0.25, 0.75, 1$. The figure shows that, if $d = 0, 0.25$, the generated series tend to be stationary. By contrast, if $d = 0.75, 1$, the generated series tend to be nonstationary.

Figure 1: Examples of simple realization of *I(d)* process



In this simulation analysis, the intercept and trend are set to zero. The null hypothesis in this Monte Carlo simulation is:

$$d = d_0 \qquad (21)$$

In this simulation analysis, two scenarios would be examined, namely the short-memory process ($d_0 = 0$) and the unit root process ($d_0 = 1$), each scenario tested against the alternative values, $d = -0.5, (0.25), 1.75$. The simulation study uses the 1,000 replications with foure different sample sizes, $T = 50, T = 100, T = 250, T = 500$. Table 1 reports the simulation results from the first scenario (i.e. short memory process) while Table 2 reports those from the second scenario (unit root process).

As the simulation for the first case ($d_0 = 0$ **)** in the Table 1 indicated, there would be serious power distortion in all methods when the number of observation is small. The number of observation is set to 250, there are a minor power distortion in the Robinson test and the AY test. However, at the higher number of observation, the ANN-FI test still suffered from the power distortion when *d* is set to -0.25 when the power of the test is only 0.54. Also, the size of the test for the ANN-FI is 0.21 when the number of observation is set to 500 which is worse than the test size for the Robinson test and the AY test.

Table 1: The rejection frequencies of the null hypothesis with two-sided tests at the 95% significant level ($d_0 = 0$)

		Fractional integration parameters									
		-0.5	*-0.25*	*0*	*0.25*	*0.5*	*0.75*	*1*	*1.25*	*1.5*	*1.75*
Robinson	T=50	0.75	0.27	**0.18**	0.72	0.97	0.99	1.00	1.00	1.00	1.00
	T=100	0.99	0.66	**0.15**	0.91	1.00	1.00	1.00	1.00	1.00	1.00
	T=250	1.00	0.98	**0.14**	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	T=500	1.00	1.00	**0.10**	1.00	1.00	1.00	1.00	1.00	1.00	1.00
AY	T=50	0.29	0.12	**0.39**	0.83	0.98	0.99	1.00	1.00	1.00	1.00
	T=100	0.92	0.30	**0.26**	0.93	1.00	1.00	1.00	1.00	1.00	1.00
	T=250	1.00	0.94	**0.21**	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	T=500	1.00	1.0	**0.16**	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ANN-FI	T=50	0.12	0.11	**0.28**	0.44	0.64	0.79	0.90	0.95	0.98	0.99
	T=100	0.16	0.10	**0.21**	0.52	0.81	0.95	0.99	0.99	1.00	1.00
	T=250	0.26	0.24	**0.18**	0.61	0.97	0.99	0.99	0.99	1.00	1.00
	T=500	0.45	0.54	**0.21**	0.37	0.99	0.99	0.99	1.00	1.00	1.00

Note: The size of test are in bold

Similarly, as the simulation results for the second case ($d_0 = 1$) in the Table 2 showed, there would be serious power distortion in all three methods when the number of observation is small. The number of observation is set to 250, there is a little power distortion in the Robinson test and the AY test. However, at the higher number of observation (i.e. 500), the ANN-FI test still suffered from a serious power distortion when d is set to 0.5 and 0.7 when the power of the test is less than 0.20. By contrast, the size of the test for the ANN-FI is 0.05 when the number of observation is set to 500 that the better than test size for the Robinson test and the AY test.

Table 2: The rejection frequencies of the null hypothesis with two-sided tests at the 95% significant level ($d_0 = 1$)

		Fractional integration parameters									
		-0.5	*-0.25*	*0*	*0.25*	*0.5*	*0.75*	*1*	*1.25*	*1.5*	*1.75*
Robinson	T=50	1.00	1.00	0.90	0.97	0.85	0.40	**0.14**	0.66	0.97	1.00
	T=100	1.00	1.00	1.00	1.00	0.96	0.74	**0.13**	0.89	0.99	1.00
	T=250	1.00	1.00	1.00	1.00	1.00	0.99	**0.11**	0.99	1.00	1.00
	T=500	1.00	1.00	1.00	1.00	1.00	1.00	**0.11**	1.00	1.00	1.00
AY	T=50	1.00	0.99	0.93	0.75	0.45	0.13	**0.33**	0.81	0.98	1.00
	T=100	1.00	1.00	1.00	0.99	0.95	0.40	**0.21**	0.93	0.99	1.00
	T=250	1.00	1.00	1.00	1.00	1.00	0.96	**0.18**	1.00	1.00	1.00
	T=500	1.00	1.00	1.00	1.00	1.00	1.00	**0.12**	1.00	1.00	1.00
ANN-FI	T=50	0.30	0.17	0.09	0.09	0.09	0.14	**0.21**	0.21	0.52	0.74
	T=100	0.95	0.78	0.40	0.08	0.03	0.03	**0.09**	0.34	0.69	0.92
	T=250	1.00	1.00	0.93	0.37	0.04	0.01	**0.07**	0.53	0.95	0.99
	T=500	1.00	1.00	1.00	0.77	0.13	0.01	**0.05**	0.79	1.00	1.00

Note: The size of test are in bold

4. Empirical Application

This paper examined the mean-reversion of monthly unemployment rates (1990M1–2019M5) in five European countries—France (FR), Ireland (IR), Luxembourg (LX), Portugal (PT) and the United Kingdom (UK). The total number of observations was 353. The source of data was the Thomson Datastream (Thomson, 2021). Three fractional integration tests, namely the Robinson test (Robinson, 1994), the Alana-Yaya (AY) test (Gil-Alana and Yaya, 2020) and the artificial neural network (ANN)-fractional integration (ANN-FI, Yaya et al., 2021) test, are used for the empirical analysis. Firstly, the Robinson test (Robinson, 1994) is based on the Equation 12. In this fractional integration test, the 95% confidence interval (CI) could be estimated by discarding the significant parameters (Gil-Alana 2004). The estimated 95% CI can be used to separate time-series into two types: (1) the CI in the range (0, 1.0) would indicate that the time-series data would be mean-reverting; (2) the CI in the range of [1.0, ∞] would show that the time-series data would be not mean-reverting. Table 3 shows that unemployment rates in all five countries are not mean-reverting.

Table 3: Findings from Robinson test

	Fractional integration parameters												*95%CI*
	0.7	*0.8*	*0.9*	*1.0*	*1.1*	*1.2*	*1.3*	*1.4*	*1.5*	*1.6*	*1.7*	*1.8*	
FR	11.7*	7.8*	6.7*	5.0*	3.6*	2.4*	1.4	0.4	-0.4	-1.2	-1.9*	-2.6*	(1.28-1.64)
IR	23.0*	16.9*	11.6*	7.9*	5.3*	3.3*	1.8*	0.5	-0.5	-1.5	-2.4*	-3.2*	(1.32-1.60)
LX	6.3*	4.3*	2.5*	1.0	-0.2	-1.4	-2.5*	-3.4*	-4.2*	-4.9*	-5.5*	-6.1*	(0.96-1.20)
PT	31.4*	25.0*	18.5*	12.8*	8.3*	4.9*	2.5*	0.6	-0.7	-1.9*	-2.9*	-3.7*	(1.36-1.56)
UK	12.3*	8.4*	5.7*	3.8*	2.5*	1.5	0.7	0.1	-0.5	-1.0	-1.5	-2.0*	(1.20-1.70)

Note: * indicates significance at the 5 percent level

Secondly, the Alana-Yaya (AY) test (Gil-Alana and Yaya, 2020) is based on the Equation 13. This test would be able to take account of unknown nonlinearity by using the Fourier approximation function. As Table 4 shows, the AY test confirmed the findings from the Robinson test that unemployment rates in all these five countries are not mean-reverting.

Table 4: Findings from the Alana–Yaya test

	Fractional integration parameters												*95%CI*
	0.7	*0.8*	*0.9*	*1.0*	*1.1*	*1.2*	*1.3*	*1.4*	*1.5*	*1.6*	*1.7*	*1.8*	
FR	8.6* [1.6]	7.0* [1.6]	5.6* [1.6]	4.0* [1.6]	3.2* [1.6]	2.2* [1.6]	1.3 [1.6]	0.4 [1.6]	-0.4 [1.6]	-1.2 [1.6]	-1.9* [1.6]	-2.6* [1.6]	(1.28-1.64)
IR	12.2* [1.6]	9.4* [1.6]	7.2* [1.6]	5.4* [1.6]	3.9* [1.6]	2.6* [1.8]	1.4 [1.8]	0.3 [1.8]	-0.6 [2.0]	-1.6 [2.0]	-2.4* [2.0]	-3.3* [1.8]	(1.28-1.60)
LX	5.8* [1.0]	4.0* [1.0]	2.4* [1.0]	0.9 [1.0]	-0.3 [1.0]	-1.4 [1.0]	-2.5* [1.0]	-3.4* [1.0]	-4.2* [1.0]	-4.9* [1.0]	-5.5* [1.0]	-6.1* [1.0]	(0.96-1.20)
PT	25.1* [1.6]	19.6* [1.6]	14.5* [1.6]	10.4* [1.8]	6.8* [1.8]	4.1* [2.0]	2.0* [2.0]	0.4 [2.0]	-0.9 [2.0]	-2.0* [2.0]	-2.9* [1.8]	-3.7* [1.8]	(1.34-1.56)
UK	6.7* [1.0]	5.0* [1.0]	3.8* [1.0]	2.7* [1.0]	1.9* [1.8]	1.1 [1.8]	0.5 [1.8]	-0.1 [1.8]	-0.5 [1.8]	-1.0 [1.8]	-1.5 [1.8]	-2.0* [1.8]	(1.14-1.70)

Note: * indicates significance at the 5 percent level

Finally, the ANN-FI test would incorporate the Yaya-Ogbonna-Furuoka-Alana (YOFA) test framework (Yaya *et al*. 2021) into the context of fractional integration analysis. The test is based on the Equation 14. where *q* is the order of hidden unit which is set to 1 in this empirical application. As Table 5 shows, unemployment rates in Luxembourg would be mean reverting. For the remaining four countries, namely France, Ireland, Portugal and the United Kingdom, the findings were indecisive.

Table 5: Findings from the ANN-FI test

	Fractional integration parameters												*95%CI*
	0.2	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*	*1.1*	*1.2*	*1.3*	
FR	2.1*	1.7*	1.3	0.9	0.6	0.2	-0.1	-0.3	-0.7	-1.0	-1.3	-1.7*	(0.38-1.28)
IR	3.9*	3.0*	2.3*	1.6	1.0	0.4	-0.1	-0.6	-1.0	-1.4	-1.7*	-1.9*	(0.50-1.16)
LX	3.1*	2.4*	1.7*	1.0	0.2	-0.4	-0.9	-1.4	-1.7*	-2.0*	-2.2*	-2.3*	(0.42-0.96)
PT	4.2*	2.9*	1.9*	1.3	0.7	0.3	-0.1	-0.4	-0.8	-1.2	-1.6	-1.8*	(0.46-1.20)
UK	1.9*	1.5	1.3	1.1	0.8	0.4	0.1	-0.4	-0.8	-1.2	-1.7*	-1.9*	(0.28-1.20)

Note: * indicates significance at the 5 percent level

5. Conclusions

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