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Detecting Structural Changes in Time Series by Using the BDS Test Recursively: An Application to COVID-19 Effects on International Stock Markets

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Abstract: Structural change tests aim to identify evidence of a structural break or change in the underlying generating process of a time series. The BDS test has its origins in chaos theory and seeks to test, using the correlation integral, the hypothesis that a time series is generated by an identically and independently distributed (IID) stochastic process over time. The BDS test is already widely used as a powerful tool for testing the hypothesis of white noise in the residuals of time series models. In this paper, we illustrate how the BDS test can be implemented also in a recursive manner to evaluate the hypothesis of structural change in a time series, taking advantage of its ability to test the IID hypothesis. We apply the BDS test repeatedly, starting with a sub-sample of the original time series and incrementally increasing the number of observations until it is applied to the full sample time series. A structural change in the unknown underlying generator model is detected when a change in the trend shown by this recursively computed BDS statistic is detected. The strength of this recursive BDS test lies in the fact that it does not require making any assumptions about the underlying time series generator model. We ilustrate the power and potential of this recursive BDS test through an application to real economic data. In this sense, we apply the test to assess the structural changes caused by the COVID-19 pandemic in international financial markets. Using daily data from the world's top stock indices, we have detected strong and statistically significant evidence of two major structural changes during the period from June 2018 to June 2022. The first occurred in March 2020, coinciding with the onset of economic restrictions in the main Western countries as a result of the pandemic. The second occurred towards the end of August 2020, with the end of the main economic restrictions and the beginning of a new post-pandemic economic scenario. This methodology to test for structural changes in a time series is easy to implement and can detect changes in any system or process behind the time series even when this generating system is not known, and without the need to specify or estimate any a priori generating model. In this sense, the recursive BDS test could be incorporated as an initial preliminary step to any exercise of time series modeling. If a structural change is detected in a time series, rather than estimating a single predictive model for the full-sample time series, efforts should be made to estimate different predictive models, one for the time before and one for the time after the detected structural change.

Keywords: economic dynamics; time series structural change; recursive BDS test; COVID-19 pandemic; world stock financial indices

MSC: 91B55; 91B70; 91G15; 62P05; 62P20

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1. Introduction

The COVID-19 pandemic has introduced new economic restrictions and has created a new landscape for human interactions. Many authors consider that COVID-19 has caused a structural change, a paradigm shift, in our social and economic model of behavior (see, e.g., [1–5]). The main contribution of our paper is to present a formal test to for detecting structural changes in the generating system of a time series, even when that generating process is unknown or when we do not know how to model it. We illustrate the power of this test by applying it to detect the existence of the changes in the structural models of the world's major financial markets caused by the COVID-19 pandemic. Specifically, we seek to identify possible changes in the underlying model of the major international stock exchanges during the transition period from the pre-COVID-19 era to the post-COVID-19 era.

There is no generally accepted definition of the concepts of stability and structural change in the field of economics. From a traditional macroeconomic perspective, this term is related to changes in the sectoral composition patterns of countries throughout their economic development process [6]. In the finance context, the stability of a stock market can be defined as the consistent propagation of systematic shocks in the market under normal and extreme conditions [7]. In this paper, we adopt an alternative time series econometric approach, where stability or structural change refers to a change in the parameters of the econometric model over the sample of data available for estimation, in our case, over time. A structural change occurs when there is a change in the structural parameters that define the model (for a review, see, e.g., [8–10]). Consequently, to detect a structural change, the traditional econometric approach requires the specification and estimation of a regression model to identify changes in the values of its (structural) parameters [11–14]. This approach can have serious implementation difficulties if the true model generating the time series, and therefore the true specification of the model to be estimated, is unknown. This need to specify a model can be quite problematic if the true unknown generating system does not satisfy the assumptions of the specified time series model (e.g., linearity in ARMA models). Our contribution aims to address this problem.

In this paper, we adopt a different perspective based on the application of nonlinear dynamics and chaos theory to time series analysis. In this approach, the concept of instability or structural change in a time series is the same as the traditional econometric one, i.e., a change in the underlying model behind the time series, but with the fundamental difference that now it is not necessary to know or specify any model generating the time series in order to detect a structural change in this unknown model. In contrast to traditional structural change tests, our contribution does not require the estimation or specification of any model. Our test is valid for any linear or nonlinear, deterministic or stochastic model that generates the time series, and is particularly useful when the true model is unknown and one does not want to impose any assumptions on the modeling of the time series. Moreover, it is not necessary to know the exact moment when the structural change occurs, as this approach provides an estimate of the date when the structural change may have occurred. Specifically, we use the methodology originally proposed by Fernández-Díaz et al. [15], which recursively applies the BDS test to detect structural changes in the unknown underlying generator of a time series.

The BDS test, rooted in chaos theory, was originally proposed by Brock, Dechert, and Scheinkman [16,17]. It aims to analyze the existence of independence against the alternative hypothesis of some time dependence in a time series. But instead of using traditional (linear) time correlation coefficients (acf, pacf, Ljung-Box, etc.), it uses the spatial correlation of the system orbits in phase space (for a review, see, e.g., [18–22]). The BDS test is a robust method for testing the null hypothesis that a time series is Independent and Identically Distributed (IID). It is used against the alternative of time dependence (whether linear or nonlinear), even in the absence of precise information about the form of that time dependence. For this reason, the BDS test has been added to the toolkit of techniques for diagnosing residuals in time series modeling.

As will be shown later, we use the BDS test as a structural change test, taking advantage of its ability to test the IID hypothesis. We apply the BDS test repeatedly, starting with a sub-sample of the original time series and incrementally increasing the number of observations until it is applied to the full sample time series. A structural change in the unknown underlying generator model is detected when a change in the trend shown by this recursively computed BDS statistic is detected.

The main contributions of this paper are as follows. First, we extend the proposal of Fernández-Díaz et al. [15] by showing that the pattern of the recursive BDS time series to test for structural changes does not depend on the embedding dimension chosen to reconstruct the phase space. Second, we propose to use the normalized values of the recursive BDS to detect structural changes. Third, we introduce a formal test for the null hypothesis of no structural change, based on the fact that the existence of a structural change in the time series is reflected in a change in the trajectory or pattern of the recursive (normalized) BDS test. These last two issues are important empirical findings.

Fourth, we show the use of this recursive BDS test with an application to detect the potential structural changes caused by the COVID-19 pandemic in the international financial markets. We use an algorithm developed in the R programming language on the time series of daily returns of the top 20 world stock indices. The results show strong and significant evidence of two major structural changes between June 2018 and June 2022, coinciding with the beginning and end of the economic restrictions imposed by various countries in response to the COVID-19 pandemic. This result provides empirical evidence that the underlying dynamics of financial markets, while unknown, experienced significant disruptions due to the COVID-19 pandemic.

Finally, we examine whether the structural changes in financial markets occurred in a coordinated manner and with the same intensity across international markets, or whether, on the contrary, some countries experienced more significant changes. We also examine whether any market led or lagged compared to the others in these structural changes, thereby examining the potential presence of contagion effects in financial markets.

The paper is organized as follows. Section 2 presents the theoretical framework used in this paper. Section 3 presents an exploratory statistical analysis of the financial dataset used. Section 4 reports and discusses the main results. Finally, Section 5 provides concluding remarks.

2. The Recursive BDS as a Structural Change Test

The primary contribution of this paper is to illustrate how the BDS test can be recursively applied to a time series to formally test the hypothesis of no structural change in the unknown model generating the time series.

The BDS test [16,17] is specifically designed to test the null hypothesis of an identically and independently distributed process. It has been widely used to diagnose white noise residuals in time series models (see [20,23–25]). The BDS test originates from chaos theory and is rooted in the estimation of the correlation integral introduced by Grassberger and Procaccia [26,27], which is used to measure the spatial dependence in the evolution of a dynamical system in phase space. This correlation integral quantifies the average frequency with which any two points of an orbit are in close spatial proximity in phase space. The reconstruction of phase space from a time series using the Ruelle and Takens method of delayed coordinates [28] allows the estimation of the correlation integral for the true unknown underlying dynamical system (if it exists). Takens' theorem states that as we increase the embedding dimension, the true (though unknown) dynamics of the generating system are manifested. With a sufficiently large embedding dimension, it becomes possible to reconstruct the orbit of the underlying system. Put simply, by embedding a scalar time series $\{x_t\}$ with a length of T into an m-dimensional space $\left(x_t, x_{t-1}, x_{t-2}, \ldots, x_{t-(m-1)}\right)$ (comprising m histories), the correlation integral can be estimated by counting the number of points in the m-dimensional space that fall within a hypercube of radius ε.

$$C_{m,T}(\varepsilon) = \frac{2}{(T-m+1)(T-m)} \sum_{s=m}^{T} \sum_{t=s+1}^{T} \prod_{j=0}^{m-1} I_\varepsilon(\varepsilon - \|(x_{s-j} - x_{t-j})\|) \tag{1}$$

where I_ε is the Heaviside function.

The idea behind the BDS test is that when a stationary time series exhibits some form of time dependence, it must arise, at least in part, from some dissipative dynamic system. Therefore, for a sufficiently large m, the resulting orbit described by the m-histories will eventually be enclosed within a bounded region of the phase space, which is referred to as an attractor. By contrast, when a time series lacks any form of time dependence, then its orbit will have the tendency to fill the entire phase space, without being confined within any bounded region of that space (i.e., a time series generated by a purely independent stochastic process has no attractor). The BDS test analyzes whether, as the time series is unfolded into higher m-dimensions, the orbit is bounded within an attractor or tends to fill the phase space without stabilizing.

Formally, Brock, Dechert, and Scheiman [16] showed that under the null hypothesis, the time series $\{x_t\}$ is IID, for $m > 1$ and $\varepsilon > 0$, as $T \to \infty$:

$$C_{m,T}(\varepsilon) = C_{1,T}(\varepsilon)^m \tag{2}$$

Brock et al. [16,17] showed that the asymptotic distribution of the difference in the correlation integrals under the null hypothesis of an IID stationary time series is as follows:

$$\lim_{T\to\infty} BDS_{m,T}(\varepsilon) = \sqrt{T-m+1}\frac{C_{m,T}(\varepsilon) - C_{1,T-m+1}(\varepsilon)^m}{\sigma_{m,T}(\varepsilon)} \sim \mathcal{N}(0,1) \tag{3}$$

Here, the estimated standard deviation $\sigma_{m,T}(\varepsilon)$ is calculated as

$$\begin{aligned}\frac{\sigma_{m,T}(\varepsilon)^2}{4} =& k(\varepsilon)^m + 2\sum_{j=1}^{m-1} k(\varepsilon)^{m-j}(C_{1,T}(\varepsilon))^{2j} + \\ &+ (m-1)^2(C_{1,T}(\varepsilon))^{2m} - m^2 k(\varepsilon)(C_{1,T}(\varepsilon))^{2m-2}\end{aligned} \tag{4}$$

where the parameter $k(\varepsilon)$ represents the probability that a triple of observations t, s, r is within a distance ε:

$$\begin{aligned}k(\varepsilon) = \frac{2}{T\cdot(T-1)\cdot(T-2)} \sum_{t=1}^{T}\sum_{s=t+1}^{T}\sum_{r=s+1}^{T} \Big[& I_\varepsilon(\varepsilon - \|(x_t - x_s)\|)\cdot I_\varepsilon(\varepsilon - \|(x_s - x_r)\|) \\ & + I_\varepsilon(\varepsilon - \|(x_t - x_r)\|)\cdot I_\varepsilon(\varepsilon - \|(x_r - x_s)\|) \Big] \\ & + I_\varepsilon(\varepsilon - \|(x_s - x_t)\|)\cdot I_\varepsilon(\varepsilon - \|(x_t - x_r)\|)\end{aligned} \tag{5}$$

and $k(\varepsilon) > C_{1,T}(\varepsilon)^2$.

The main motivation for applying the BDS test to detect structural changes in a time series is its potential ability to test for IID, that is, temporal dependency, but also identical distribution. Fernández-Díaz et al. [15] proposed the iterative or recursive application of the BDS statistic (3), starting with a subset of the original time series and progressively increasing the number of observations until the BDS was applied to the entire data set. The recursive BDS (RBDS) can be defined then as the following sequence of values or time series:

$$RBDS^{\alpha,\rho}_{m,T}(\varepsilon) = \{BDS_{m,\alpha}(\varepsilon), BDS_{m,\alpha+\rho}(\varepsilon), BDS_{m,\alpha+2\rho}(\varepsilon), \ldots, BDS_{m,T}(\varepsilon)\} \tag{6}$$

The procedure to obtain the RBDS begins by estimating the BDS with a sample of α observations. We then estimate the BDS by adding ρ data to the set of observations under analysis. The detection of a structural change in the underlying hidden generator model occurs when a significant change in the trend of the recursively computed BDS statistic is detected. That is, if a structural change occurs after a certain point in time, adding more observations in (6) will result in a change in the temporal pattern shown by the RBDS time series, since the identically distributed hypothesis of the original time series generating process is no longer satisfied after that point.

To illustrate the ability of this method to detect structural breaks, we present a simple numerical example below. We simulate three samples of size 400 each from a normally and independently distributed process with a mean of 0 in all three cases, but with three different variances. More specifically, we simulate 400 data from the distributions $N(0,1)$, $N(0,5)$, and $N(0,0.4)$, respectively. The resulting aggregate time series of 1200 observations exhibits two structural breaks (at time 400 and time 800), as seen in Figure 1a.

The estimation of the RBDS for this simulated time series requires the selection of concrete values for m, ε, α and ρ, the parameters involved in Equation (6). The ρ parameter is the increase in sample size used to estimate the successive values of the BDS statistic, and thus to fit the series of RBDS values in Equation (6). This parameter does not change the value of the BDS statistic estimated with a specific sub-sample of the original series, only the number of points that make up the RBDS series. It is just an option that is used for computational purposes to avoid having to sample BDS over the entire sample interval if, for example, you want to use the recursive BDS test in an exploratory way.

The α parameter is the size of the first sub-sample of the original time series used to initiate the recursive calculation of the BDS in Equation (6). α must to be large enough to allow the robust estimation of BDS at this minimum sample size, but not too large, in order to allow for the construction of a series of RBDSs. The BDS test has been shown to be quite robust when used to test for time dependence on short time series. We suggest to start arbitrarily with $\alpha = 50$. This is the minimum sample size used, for example, by Kanzler [29], one of the most complete studies of the properties of BDS when applied to time series with different sample sizes. $\alpha = 50$ are the minimum values also used to test BDS properties by other authors, such as [30,31] or [32]. They all show that the power of the BDS test increases with sample size. In our case, we observed that from this initial sample size $\alpha = 50$, the following estimates of the BDS statistic ($\alpha = 50$, $\alpha = 51$, $\alpha = 52$, $\alpha = 53\ldots$) remained at stable values. That is, we validated the choice of $\alpha = 50$ by checking the stability of the subsequent BDS estimates in the first sample sizes, the shortest ones, since they are precisely in short series where there could be more problems with estimating the BDS statistic. In other applications, when choosing the appropriate α value, the same exercise of validating the stability of the RBDS series would have to be performed for the shorter values of the sample size.

The recursive BDS (6) is then computed on this simulated time series starting with a data set $\alpha = 50$ observations and adding $\rho = 1$ data to the set of observations under analysis. As can be seen in Figure 1b, when the BDS test is applied recursively, two changes in the trend of the RBDS time series appear graphically, coinciding with each of the changes in the simulated samples.

A practical key to obtaining valid results when computing the BDS in (3) is the proper choice of the parameters ε and m. The ε parameter is the size of the sphere used to compute the correlation integral, i.e., the probability that two points are close to each other in phase space. We follow the criterion proposed by [21,29,33], who compare different approaches to recommend the selection of a radius (ε) as a proportion of the variance of the original time series. They recommend that ε be set between one half and three halves of the standard deviation of the data being analyzed. In this example, we have considered ε to be equal to one half the standard deviation of each time series.

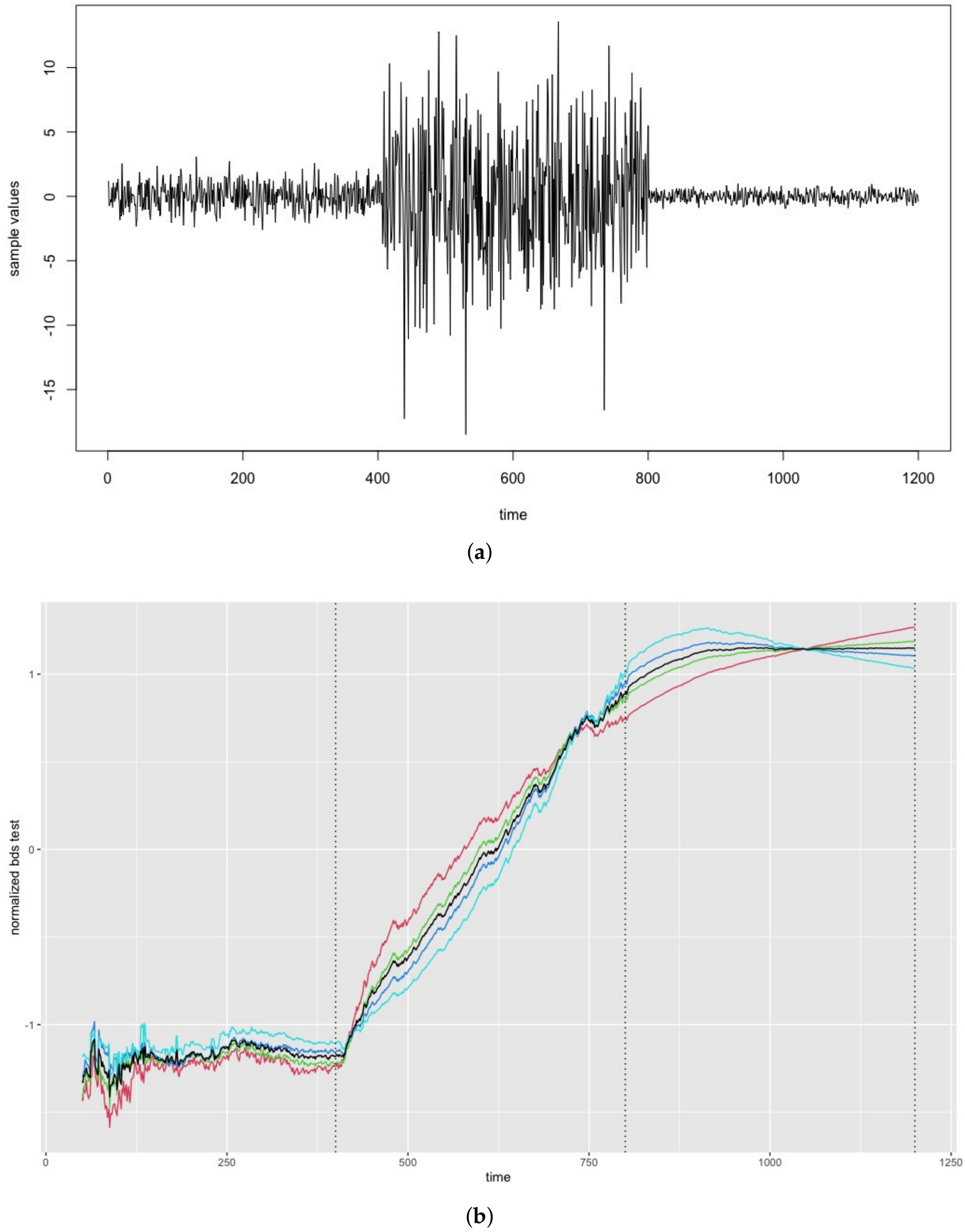

Figure 1. Recursive BDS on a simulated time series with two structural changes. (**a**) Graphical representation of three samples from a normally distributed process with the same mean but different variance $y_{t_{0-400}} \sim N(0,1), y_{t_{401-800}} \sim N(0,5), y_{t_{801-1200}} \sim N(0,0.4)$. (**b**) Graphical representation of the normalised values of the recursive BDS over the three samples considered for an embedding dimension from 2 to 6.

On the other hand, when the BDS test is used as a test for time independence, it is crucial to make a proper choice of the embedding dimension (m). Broer and Takens [34] showed that with a time delay equal to 1 [35] a value of m equal to two times the degrees of freedom of the underlying generator model would be sufficient to correctly reconstruct the phase space. When the underlying system and therefore its degrees of freedom are unknown, a commonly used strategy for estimating the BDS test is to set the embedding dimensions m in the range of two to six or higher.

An important contribution of this paper is precisely to show that the BDS test used recursively as a structural change test does not depend crucially on the choice of m. In fact, the temporal pattern followed by the RBDS series Equation (6) does not depend on the exact m chosen. In this sense, changing the scale of the RBDS time series estimated for different embedding dimensions (standardizing them to a zero mean with a standard deviation of one) allows us to verify that the evolution of the standardized RBDS values does not depend on m. Therefore, in our practical applications to detect structural changes, we will use the RBDS calculated as the mean of the normalized RBDS series for m = 3 to m = 6.

We have just shown in Figure 1 that the RBDS time series can graphically indicate the presence of a structural change in the underlying model generating the observed time series when a change in its time trend is observed. The RBDS time series will change its trend upward or downward from the moment the structural change occurred. Then, we can test the null hypothesis of no structural change in a time series by detecting changes in the trend of the RBDS estimated with that time series. In this paper, we have used the cumulative sum of a quality characteristic (CUSUM) approach and the Chow test to infer the change in the RBDS trend, for a review see, e.g., [36,37]. These two are traditional structural change tests that require the specification of a model. Our proposal is to use a linear model:

$$RBDS_t = \beta_0 + \beta_1 \times t + \epsilon \tag{7}$$

where $t = \alpha, \alpha + \rho, \alpha + 2 \times \rho \ldots, T$, as shown in Equation (6). A change in the estimated $\hat{\beta}_1$ implies a change in the RBDS trend, and thus a structural change in the original time series.

Table 1 shows that two structural changes have been detected in the simulated time series used in our previous example, specifically at time 404 and time 803, which coincide exactly with the changes in the (known) simulated generator system.

Table 1. Chow and CUSUM test result on the RBDS estimated on simulated time series $y_{t_{0-400}} \sim N(0,1), y_{t_{401-800}} \sim N(0,5), y_{t_{801-1200}} \sim N(0,0.4)$. The last two columns provide information on the points in time at which the structural changes have been empirically detected.

	$Chow_{RBDS}$	$Chow_{pvalue}$	$CUSUM_{RBDS}$	$CUSUM_{pvalue}$	$StructChange_a$	$StructChange_b$
Sup.F	1528.618	<0.01	7.040835	<0.01	t = 404	t = 803

3. Financial Markets, the COVID-19 Pandemic and Structural Changes

On 12 December 2019, the first case of COVID-19 was reported in Wuhan, China [38], which then spread and reached global pandemic status, causing public health problems [39–41], and forcing governments to take urgent public health measures [1,4]. Indeed, the confinement associated with the COVID-19 pandemic has led to a slowdown in production and employment levels that has affected all sectors of the economy; for a review, see, e.g., [2,3,5,42–44]. In particular, the relationship between the COVID-19 pandemic and the financial markets has become an important aspect to study [45–47].

The scientific contributions to this topic can be divided into literature published before the COVID-19 pandemic and focused mainly on the contagion effects of the global financial crisis [48–51], and literature published in 2020 or later. This last group includes contributions that consider the high stock market volatility as a reaction linked to uncertainty about the course of the pandemic and its associated economic losses [52–57]. Regarding the impact of lockouts and other government actions, although agents overreact to unexpected news, the market corrects itself as more information becomes available [58–60]. The results generally show a positive effect of government stimulus packages on stock market returns [61–64].

The analysis of the impact of the pandemic can also be conducted from a sectoral perspective. Based on sectoral indices in the US, the industries identified as most affected by the pandemic are transportation, motor vehicles and components, energy, and travel and leisure [65]. Industries such as natural gas, food, health care, and software stocks earn high positive returns, while on the other hand, equity values in the petroleum, real estate, entertainment, and hospitality sectors fall dramatically [66]. Finally, and especially in this context of uncertainty, financial firms are usually characterized by an important role in the transmission of financial contagion [67].

In the forthcoming section, we will examine whether the COVID-19 outbreak caused a structural shift in the models that govern the primary global stock markets. This examination will be conducted without delving into the specific pathways or mechanisms through which these causal impacts were transmitted. That is, we will not specify any theoretical model of financial market behavior. We will identify the potential changes in the unknown underlying model of the top 20 major international daily stock market returns over the past 5 years (from June 2018 to June 2022), with a particular focus on the period of confinement, and we will analyze the differences between the pre- and post-COVID-19 era. And we will perform this structural change analysis by recursively applying the BDS test as described above.

The financial market return indices considered (1.062 daily observations for each index) and their tickers are as follows: United States (S&P), United States (Dow Jones), United States (NASDAQ), United States (NYSE), United Kingdom (UK), Europe 50 (EU5), Europe 100 (EU10), Germany (GE), France (FR), Belgium (BE), Spain (SP), Italy (IT), Switzerland (SW), Norway (NO), Russia (RU), Japan (JA), China (CH), South Korea (SK), Brazil (BR), and Mexico (ME). Figures 2 and 3 show the time evolution of the 20 selected world stock market indices and returns, respectively, over the last 5 years showing the daily closing prices. As can be seen, almost all markets experienced a sharp drop in price levels coinciding with the beginning of the pandemic, with a greater or lesser intensity and duration of the downturn. After these crashes, international stock markets generally entered a new recovery phase, which again varied in intensity and duration depending on the market under consideration. Figure 2 also shows that the stock index series are generally not stationary at the mean. For this reason, and in order to apply the RBDS test, we need to work with the logarithmic return series (Figure 3). The fall in the price level at the beginning of the pandemic (Figure 2) is reflected in a sharp increase in the volatility of the return series. However, there is no significant change in the patterns of the return series immediately before and after the sharp increase in the volatility of the series (early 2020), at least visually.

Table 2 presents the main descriptive statistics of the daily returns series of the 20 major stock market indices from June 2018 to June 2022. The results from this table show that for all stock markets, the mean returns are non-negative over this period. For some time series, e.g., USA (S&P), USA (NA), Norway (NO), or Brazil (BR), the returns are slightly positive, but the differences with zero means are not too clear. Initially, there are no significant differences among markets in return volatility (standard deviations), with the highest variability found in Brazil, Italy, and Russia (RU). Moving on to other descriptive statistics, skewness can be interpreted as a second (next to volatility) major proxy for measuring financial market stability. During the analyzed period, the return of all stock prices was negatively skewed, which means that there was a higher probability of large negative returns than positive returns. Based on the results of the kurtosis coefficients (K), we can conclude that the histograms of the daily return series for all stock indices are leptokurtic ($K > 0$). This means that the daily price fluctuations are more pronounced than estimated for the normal distribution. In this sense, the Jarque–Bera test rejects the null hypothesis that the return series follow a normal distribution.

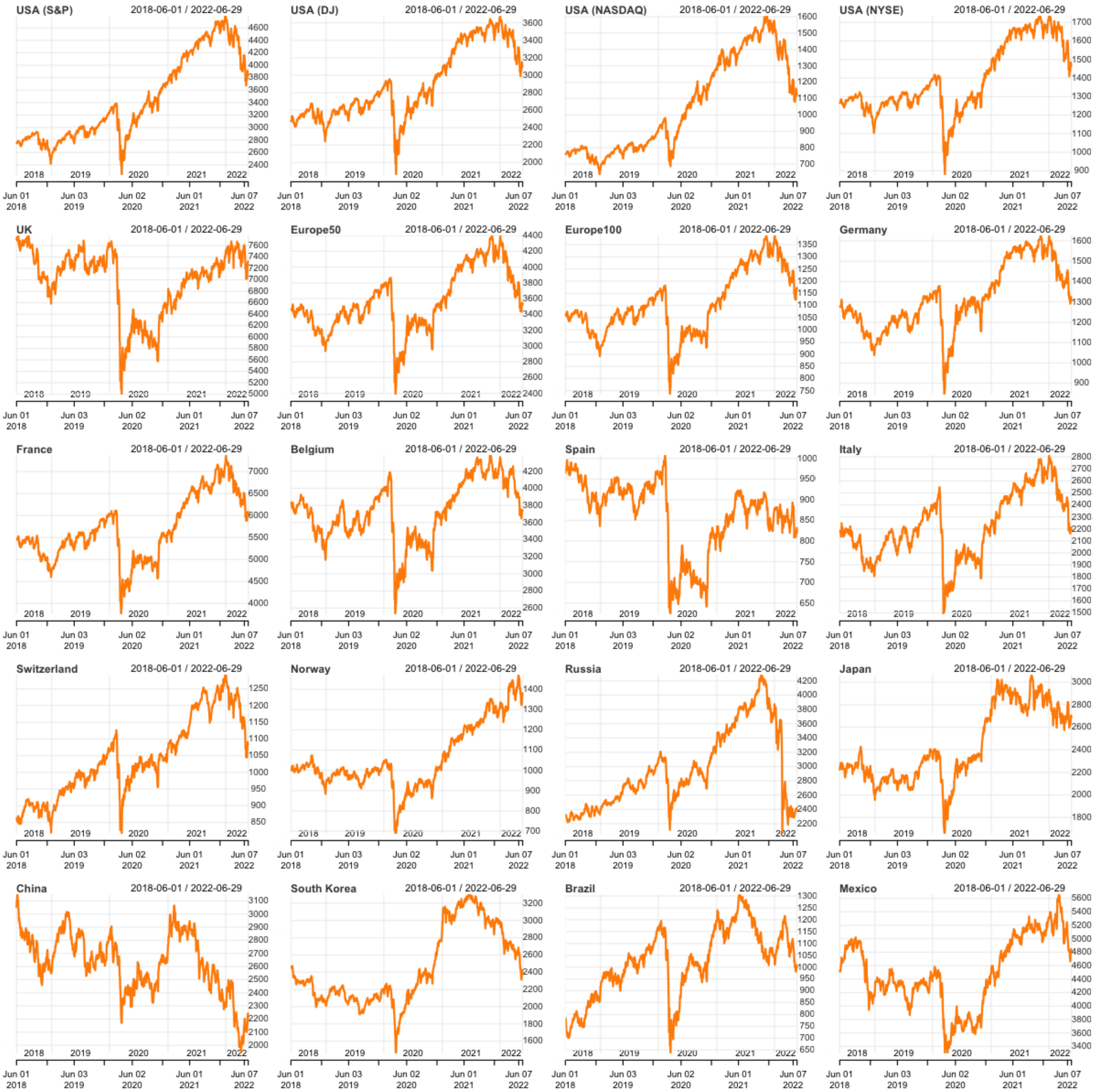

Figure 2. Graphical representation of the time evolution of the top 20 world stock markets over the last 5 years. The values shown are daily closing price indices from June 2018 to June 2022.

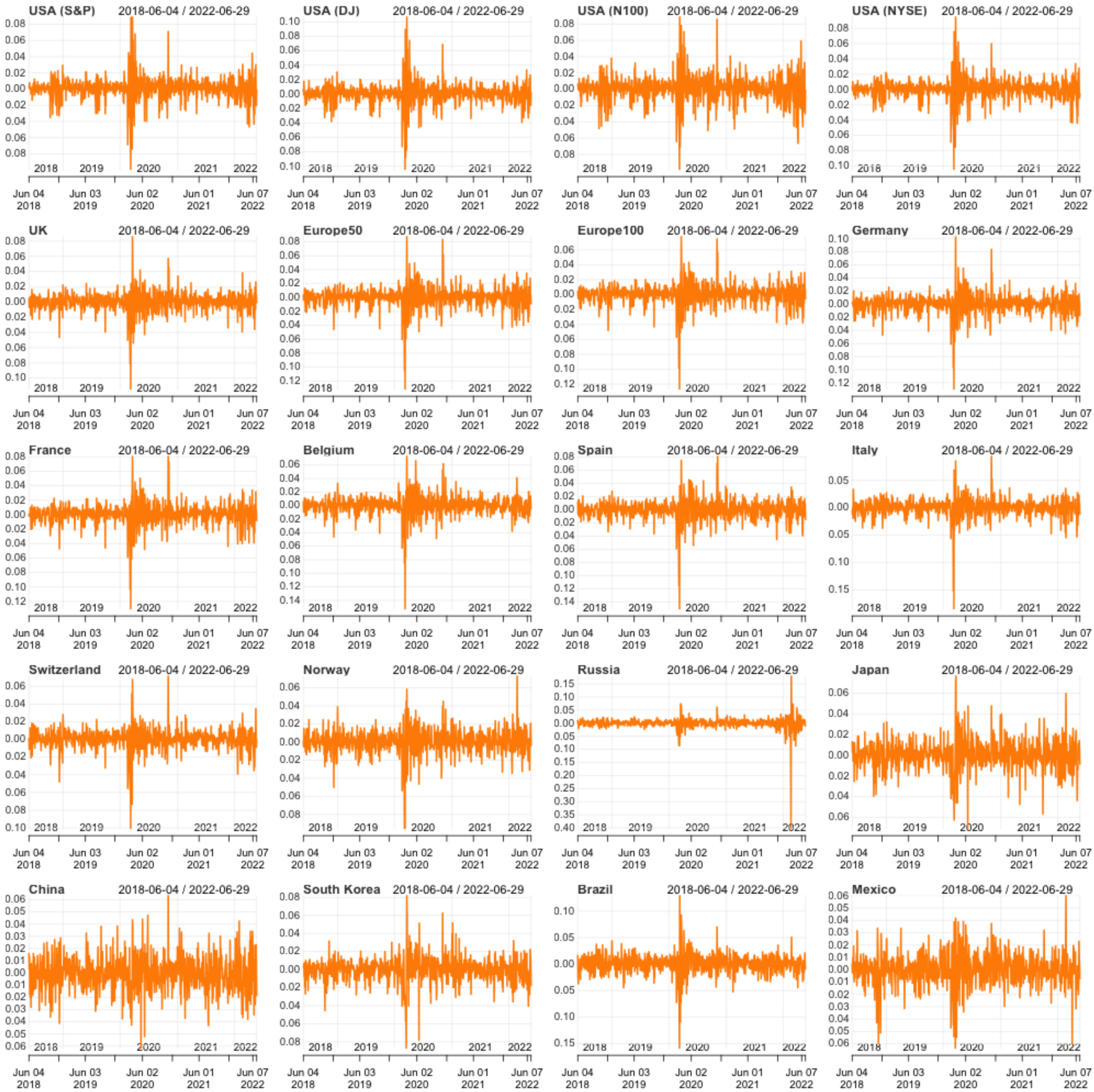

Figure 3. Graphical representation of the time evolution of the top 20 world stock markets over the last 5 years. The values shown are daily returns from June 2018 to June 2022.

Table 2. Summary of the descriptive statistics for daily returns time series of major stock exchange indices from June 2018 to June 2022.

	Min	Max	Mean	Stdev	Skewness	Kurtosis	JB (*p*-Value)
USA (SP)	−0.103	0.092	0.001	0.011	−0.443	11.823	7244 (<0.01)
USA (DJ)	−0.106	0.105	0.000	0.010	−0.481	14.956	11,573 (<0.01)
USA (NA)	−0.091	0.087	0.001	0.013	−0.399	6.617	2281 (<0.01)
USA (NY)	−0.107	0.091	0.000	0.015	−0.745	13.049	8865 (<0.01)
United Kingdom (UK)	−0.105	0.082	0.000	0.011	-0.917	12.686	8458 (<0.01)
Europe50 (EU5)	−0.172	0.081	0.000	0.013	−1.159	13.440	9582 (<0.01)
Europe100 (EU10)	−0.134	0.084	0.000	0.014	−1.263	13.885	10,273 (<0.01)
Germany (GE)	−0.121	0.111	0.000	0.013	−0.827	12.681	8424 (<0.01)
France (FR)	−0.129	0.092	0.000	0.013	−1.153	13.047	9002 (<0.01)
Belgium (BE)	−0.132	0.086	0.000	0.012	−1.627	18.660	18,476 (<0.01)
Spain (SP)	−0.168	0.097	0.000	0.014	−1.673	17.381	16,133 (<0.01)
Italy (IT)	−0.191	0.081	0.000	0.018	−1.988	21.501	24,612 (<0.01)
Switzerland (SW)	−0.114	0.074	0.000	0.013	−0.984	11.588	7120 (<0.01)
Norway (NO)	−0.085	0.077	0.001	0.012	−0.915	7.544	3098 (<0.01)
Russia (RU)	−0.414	0.189	0.000	0.018	−0.831	19.138	11,382 (<0.01)
Japan (JA)	−0.132	0.075	0.000	0.016	−0.522	7.438	2913 (<0.01)
China (CH)	−0.081	0.062	0.000	0.015	−0.276	2.371	315 (<0.01)
South Korea (SK)	−0.080	0.080	0.000	0.013	−0.419	8.111	3428 (<0.01)
Brazil (BR)	−0.162	0.132	0.001	0.020	−0.832	11.015	6390 (<0.01)
Mexico (ME)	−0.068	0.066	0.000	0.015	-0.512	4.227	977 (<0.01)

4. Results

In this section, we present the main results of this paper. We apply the recursive BDS test to analyze the extent to which COVID-19 has brought about a structural change in the top stock markets described in the previous section.

First, we estimate the RBDS time series Equation (6) using the daily returns in each stock market (Figure 4) We use the bds.test function in the tseries R-library [68]. To run the RBDS algorithm, we need to choose a concrete value for m, ε, α, and ρ, the parameters involved in Equation (6). We set ε = 0.5 times the standard deviation of each original series, which is one of the values generally accepted as valid in the specific literature [21,29,33]. We performed several tests estimating RBDS with ρ = 50, 10, 5, and 1, and no significant changes in the results were observed in terms of detecting structural changes in the selected financial time series analyzed. For this reason, without loss of generality and since we are using daily series of stock market indices, we decided to estimate the BDS every day, that is, with $\rho = 1$. We started by estimating the BDS statistic for a first sub-sample of size $\alpha = 50$. We checked the stability of the first values estimated for the RBDS series ($\alpha = 50$, $\alpha = 51$, $\alpha = 52$, $\alpha = 53 \ldots$) to avoid problems in estimating the BDS statistic with the sorted time series. Then, the procedure to obtain the RBDS time series in Equation (6) began by the estimation of the BDS statistic Equation (3) with a sample of $\alpha = 50$ daily observations and repetition of the BDS estimates by adding $\rho = 1$ data to the sample in each iteration. For each return time series, we estimated five RBDS time series (embedding dimensions $m = 2$ to $m = 6$). Each of these RBDS time series was standardized, and, finally, for each stock market, we obtained the mean time series of the five standardized (Z-score) RBDS time series (m ranging from two to six) , that is, the RBDS series used to check for structural change in each stock market.

The main empirical findings are as follows: (i) All of the world's stock market indices have behaved very similarly, even though the incidence of coronavirus cases has been different in each country. (ii) We can clearly identify two major structural changes during the period under study. Figure 4 provides graphical evidence of how the recursive BDS test is able to detect these changes in the behavior of the analyzed stock markets, with the evolution of the RBDS statistic being mainly stable for most countries until March 2020. At this date, an increase in the trend of the RBDS (reflecting a structural change) begins, which lasts until about August of the same year, when the RBDS stabilizes again (the trend changes again). (iii) The dates on which we have mainly detected the structural changes for the majority of countries correspond precisely to the beginning and the end of the period of the restrictions on mobility imposed in most of the countries and its corresponding impact on the productive sectors. This evidence therefore shows that the COVID-19 pandemic was accompanied by at least two structural changes in international financial markets

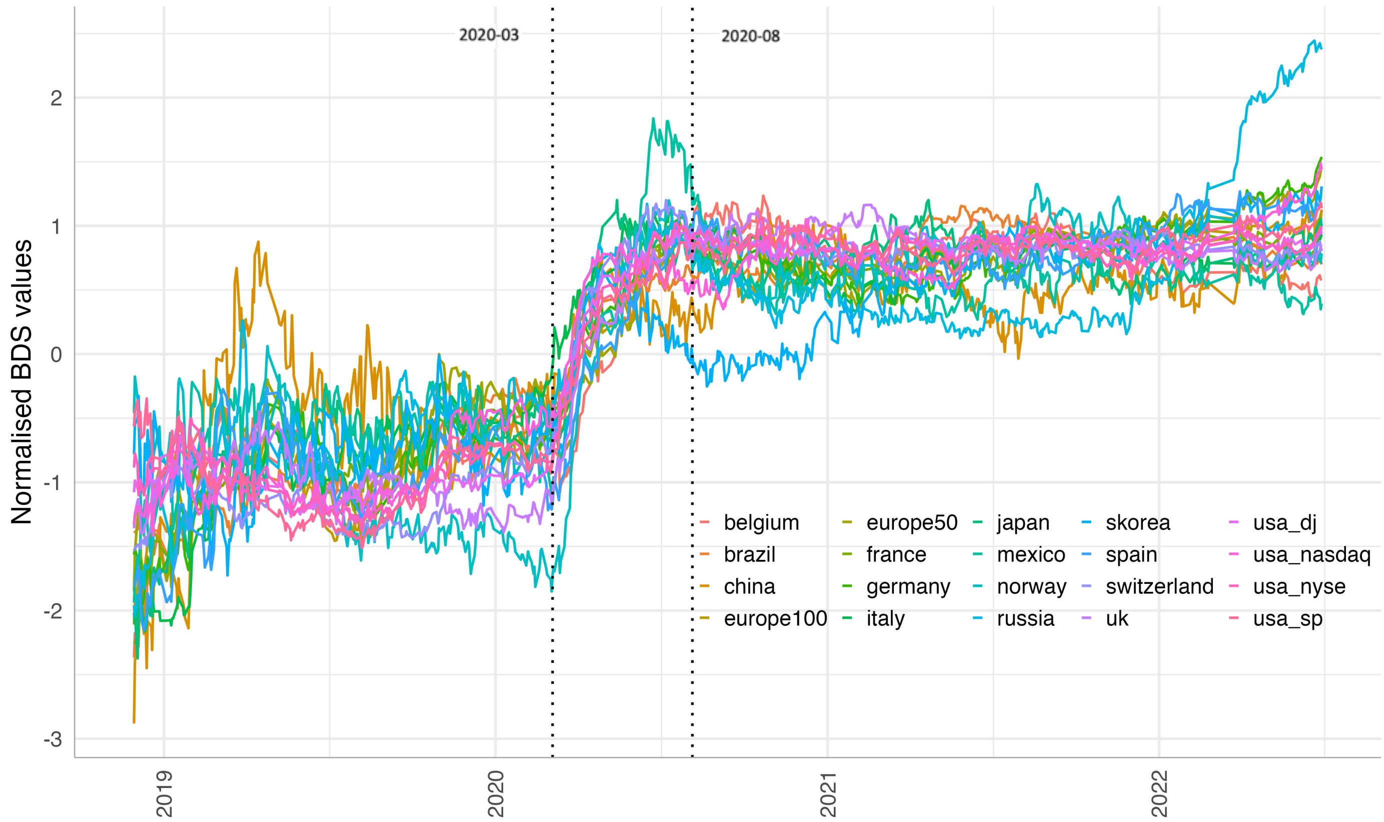

Figure 4. Graphical representation of the mean normalised RBDS Equation (6) time series of the 20 top world market daily returns (June 2018 to June 2022). The RBDS time series are obtained by estimating the BDS statistic Equation (3) with an initial sample of $\alpha = 50$ daily observations and repeating the BDS estimates by adding $\rho = 1$ data in each iteration, repeating this procedure for an embedding dimension from $m = 2$ to $m = 6$, and averaging the obtained normalized RBDS time series.

Second, in order to obtain statistical evidence to validate the results obtained graphically, we carried out the appropriate hypothesis testing applying the traditional Chow and CUSUM tests to the RBDS series to capture any change in the trend of its time evolution according to the specification of the linear model of the Equation (7). We used the strucchange R-library [69].The results in Table 3 show that both tests reject the hypothesis of no structural change, dating the two structural changes detected between mid to late March 2020 for the first structural change and mid to late August 2020 for the second structural change. These dates mark the beginning and the end of a period during which economic activity faced various restrictions. These restrictions ranged from basic precautions such as wearing masks and social distancing to more severe measures such as full-scale confinements, widespread lockdowns, and the forced temporary shutdown of non-essential businesses (see [70]).

Table 3. Chow and CUSUM for RBDS structural change test in financial markets. The null hypothesis of both tests is no structural change during the period June 2018 to June 2022. The last two columns indicate the point in time at which structural changes are detected, estimated as the time where the supremum test statistic is reached.

	$Chow_{bds}$	$Chow_{pvalue}$	$CUSUM_{bds}$	$CUSUM_{pvalue}$	$StructChange_a$	$StructChange_b$
USA (SP)	2925.58	<0.01	7.30	<0.01	2020-Mar-22	2020-Aug-28
USA (DJ)	1300.99	<0.01	6.68	<0.01	2020-Mar-26	2020-Aug-26
USA (NA)	1539.25	<0.01	6.85	<0.01	2020-Mar-24	2020-Aug-30
USA (NY)	5887.85	<0.01	7.96	<0.01	2020-Mar-23	2020-Aug-24
United Kingdom (UK)	1378.67	<0.01	6.65	<0.01	2020-Mar-30	2020-Aug-25
Europe50 (EU5)	471.32	<0.01	6.09	<0.01	2020-Mar-27	2020-Aug-30
Europe100 (EU10)	483.70	<0.01	5.82	<0.01	2020-Mar-23	2020-Aug-28
Germany (GE)	3437.25	<0.01	6.96	<0.01	2020-Mar-22	2020-Aug-25
France (FR)	2534.63	<0.01	7.39	<0.01	2020-Mar-23	2020-Aug-28
Belgium (BE)	1097.41	<0.01	6.41	<0.01	2020-Mar-29	2020-Aug-29
Spain (SP)	1702.92	<0.01	6.26	<0.01	2020-Mar-21	2020-Aug-26
Italy (IT)	1098.73	<0.01	4.65	<0.01	2020-Mar-19	2020-Aug-25
Switzerland (SW)	491.09	<0.01	6.89	<0.01	2020-Mar-27	2020-Aug-31
Norway (NO)	2428.49	<0.01	8.18	<0.01	2020-Mar-31	2020-Aug-31
Russia (RU)	539.13	<0.01	4.33	<0.01	2020-Feb-14	2020-Jun-14
Japan (JA)	1783.52	<0.01	6.47	<0.01	2020-Feb-27	2020-Jun-20
China (CH)	984.89	<0.01	7.03	<0.01	2020-Jan-19	2020-Apr-07
South Korea (SK)	2957.81	<0.01	8.20	<0.01	2020-Mar-20	2020-Aug-18
Brazil (BR)	1423.99	<0.01	8.21	<0.01	2020-Apr-06	2020-Sep-17
Mexico (ME)	908.56	<0.01	6.79	<0.01	2020-Apr-15	2020-Sep-26

Third, since the incidence of the coronavirus was not uniform across countries, nor were the restrictions on economic activity that were imposed, we examined whether the structural changes in the financial markets occurred in a coordinated manner across international markets, or whether, on the contrary, some markets led or lagged compared to the others in these structural changes, thereby examining the potential presence of contagion effects in the financial markets.

The upper triangle shows the p-values to test for the hypothesis of simultaneous co-movement between two financial markets. If simultaneous co-movement is rejected, the bottom triangle shows the maximum lead (positive) or lag (negative) between the row and column markets.

For this purpose, the possible co-movements in the structural changes of the main world stock markets are analyzed estimating the cross-correlation coefficient of the RBDS time series (see Table 4). The null hypothesis is simultaneous structural changes between two financial markets. If the null hypothesis is rejected, it means that one market is moving ahead of the other. In the upper triangle on Table 4, we include the *p*-values to test for the hypothesis of simultaneous co-movement between two financial markets RBDS time series. When the simultaneous co-movement is rejected at the 5% significance level, the lower triangular section indicates the optimal lead (positive) or lag (negative) between the row and column markets. This occurs when the maximum absolute cross-correlation is reached. For example, in the case of Russia and the United States (SP), the series of Russia would be 36 days ahead of the series of the United States (SP) on average. A negative value corresponds to an optimal crossing between RBDS series in the opposite direction (columns-rows). For example, the Brazilian market (BR) lags the US market (SP) by an average of 15 days.

Table 4. Cross-correlation between financial markets' structural changes. We have marked with an asterisk * those coefficients that are statistically significant at the 99% confidence level. The upper triangle shows the *p*-values to test for the hypothesis of simultaneous co-movement between financial markets. When the simultaneous co-movement is rejected, the lower triangle below the main diagonal shows the maximum lead (positive) or lag (negative) between the row and column markets.

	SP	DJ	NA	NY	UK	EU5	EU10	GE	FR	BE	SP	IT	SW	NO	RU	JA	CH	SK	BR	ME
SP		>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
DJ	-		>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
NA	-	-		>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
NY	-	-	-		>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
UK	-	-	-	-		>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
EU5	-	-	-	-	-		>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
EU10	-	-	-	-	-	-		>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
GE	-	-	-	-	-	-	-		>0.1	>0.1	>0.1	>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
FR	-	-	-	-	-	-	-	-		>0.1	>0.1	>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
BE	-	-	-	-	-	-	-	-	-		>0.1	>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
SP	-	-	-	-	-	-	-	-	-	-		>0.1	>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
IT	-	-	-	-	-	-	-	-	-	-	-		>0.1	>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
SW	-	-	-	-	-	-	-	-	-	-	-	-		>0.1	<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
NO	-	-	-	-	-	-	-	-	-	-	-	-	-		<0.01 *	<0.01 *	<0.01 *	>0.1	<0.01 *	<0.01 *
RU	36	40	38	37	44	41	37	36	37	43	35	33	41	45		<0.01 *	<0.01 *	<0.01 *	<0.01 *	<0.01 *
JA	23	27	25	24	31	28	24	23	24	30	22	20	28	32	−14		<0.01 *	<0.01 *	<0.01 *	<0.01 *
CH	62	66	64	63	70	67	63	62	63	69	61	59	67	71	25	39		<0.01 *	<0.01 *	<0.01 *
SK	-	-	-	-	-	-	-	-	-	-	-	-	-	-	−34	−21	−60		<0.01 *	<0.01 *
BR	−15	−11	−13	−14	−7	−10	−14	−15	−14	−8	−16	−18	−10	−6	−51	−38	−77	−17		>0.1
ME	−24	−20	−22	−23	−16	−19	−23	−24	−23	−17	−25	−27	−19	−15	−60	−47	−86	−26	-	

The results show that, in general, the European and US markets underwent structural change simultaneously during the COVID-19 pandemic. In contrast, China, Russia, and Japan anticipated the structural change, while countries such as Brazil, Mexico, and South Korea lagged behind. In other words, we have detected a clear contagion effect in the financial market structural changes during the COVID-19 pandemic. The financial shocks have spread rapidly between European and US markets, with China, Russia, and Japan anticipating the subsequent changes seen in other markets.

5. Discussion

The results of our analysis show, as expected, that the COVID-19 pandemic caused a significant structural change in the financial time series. According to the results, all analyzed stock markets behaved similarly in the period from June 2018 to June 2022, with some markets moving first and demonstrating a pull effect. In this sense, it is clear that the structural change starts in Southeast Asia with the involvement of China, Russia, and Japan, and then extends to the European continent and the United States simultaneously, and finally to South America. The main implications of our paper, however, derive less from the specific application to the effect of COVID-19 on financial markets and more from the proposed methodology for detecting structural changes.

The traditional tests for structural change consist of detecting changes in the parameters or residuals of the estimated model for the time series before and after the change. The problem with using these traditional tests is that they require specifying and estimating a model for the time series even if the true generating model of the time series is not known. This need to specify a model can be quite problematic if the true generating system does not satisfy the assumptions of the specified time series model. Unlike these traditional structural change tests, our proposal does not require the estimation or specification of a model. This test is valid for any linear or non-linear, deterministic or stochastic model that generates the time series, and is especially useful when the true model is unknown and one does not want to impose any assumptions in modeling the time series.

The main contribution of our paper is to propose a formal test based on the BDS test for the hypothesis of no structural change, but without the need to specify or estimate any model. The BDS test has already been widely used as a test of independence (against the alternative hypothesis that there is a linear or nonlinear dynamic system behind the time series). In this sense, it has been used as an important tool for diagnosing residuals to detect temporal dependence remaining in the residual when modeling time series [32,71–75].

We propose to use the same BDS statistic, but now focus on testing for identical distribution in the process or system generating the time series, even if this generating process is not known or we do not know how to model it. This formal test for structural change applies the BDS statistic repeatedly, starting with a sub-sample of the original time series and incrementally increasing the number of observations until it is applied to the full sample time series. A structural change in the underlying generator model is detected when a change in the trend shown by this recursively computed BDS statistic is detected.

This recursive BDS test for structural change does not depend crucially on the parameter m (embedding dimension) used to reconstruct the orbits in phase space. And, as noted above, it does not require the knowledge or specification of any model for the underlying system generating the time series. The recursive BDS is easy to implement and can have important practical implications for time series modeling. The recursive BDS should be used as a first step in any time series modeling exercise (linear or nonlinear). Any analysis or exercise in predictive time series modeling should consider implementing this recursive BDS test for structural change as a preliminary step. When a structural change is detected in a time series, rather than estimating a single predictive model for the full sample time series, an effort should be made to estimate different predictive models, one for the time before and one for the time after the detected structural change.

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