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Testing the Efficient Market Hypothesis and the Model-Data Paradox of Chaos on Top Currencies from the Foreign Exchange Market (FOREX)

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Abstract: In this paper, we analyse two interesting applications related to the dynamics of economic phenomena linked to the Efficient Market Hypothesis (EMH), informative surprises, and the Model-Data Paradox of Chaos in certain top currency pairs from the foreign exchange market (FOREX). On the one hand, we empirically show that the FOREX market reacts under the Efficient Market Hypothesis in some cases, creating a significant variation in a short period of time (15, 30, and 60 min) in the quotes of the main currencies from the most important economic regions in the West (the United States, Europe, and the United Kingdom). This variation would depend on the actual deviation of high-impact macroeconomic news reported by these markets in relation to trade balance, unemployment rate, Gross Domestic Product (GDP), retail sales, the Industrial Production Index (IPI), and the Consumer Price Index (CPI). On the other hand, by testing the Model-Data Paradox of Chaos, we empirically verify that if we consider all the information available in the financial markets of currencies (or at least, more desegregated data) instead of daily data, and we apply a robust chaotic behaviour detection method, we can find differences in relation to the detection of chaos on the same series but with different temporal frequencies. This allows us to confirm that behind these financial time series which show an apparently random irregular evolution, there would be a generating system which, although unknown in principle, would be deterministic (and nonlinear), and we could take advantage of that deterministic character to make predictions, even if only in the short term, understanding "short term" as the time it takes for the market to incorporate these informative surprises in the FOREX market analysed.

Keywords: macroeconomic news; efficient market hypothesis; informative surprises; model-data paradox of chaos; Lyapunov exponents; foreign exchange market; financial time series

MSC: 62P20, 37M10, 37M22, 37M25

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1. Introduction

In this paper, we analysed two interesting applications related to the dynamics of economic phenomena linked to the Efficient Market Hypothesis (EMH), informative surprises, and the Model-Data Paradox of Chaos on some of the main currency pairs from the foreign exchange market (FOREX). Macroeconomic and financial data are generated and published every day and are closely followed by major market participants. If we consider the FOREX market, fluctuations in the prices of different currencies provide opportunities for profit, because as in any other financial market, profits in the FOREX market are made by buying low and selling high. Therefore, in order to make a profit when trading in this market, the key is to predict what the future movement of the exchange rate or the price of the currencies will be. So, can one make money in the FOREX market with a consistent strategy? In other words, can one decide when to enter this market in order to generate positive returns and increase the profitability of one's portfolio?

There are two main perspectives regarding how one can operate in the FOREX market. These two approaches differ in the way they make their predictions about the future of the exchange rate. On the one hand, chartists make their predictions without taking into account the evolution of the macroeconomic variables of the countries involved [1,2]. In particular, they base their forecasts on historical exchange rate values, looking for patterns of past behaviour that can be applied to the present. Such patterns of behaviour in the FOREX market can be modelled by trend lines [3–5], Fibonacci series [6,7], bullish and bearish channels [8,9], stochastic processes [10,11] or even more recent machine learning and deep learning techniques [12–14].

On the other hand, fundamentalists consider the exchange rate to be determined according to its fundamental, or theoretical, value. This fundamental value, in the case of exchange rates, will mainly depend on the differences between the evolution of the two economies involved in the exchange rate [15]. Thus, for instance, in the case of the euro/US dollar exchange rate, a higher level of European inflation will lead to an appreciation of the US dollar, as explained by the purchasing power parity theory [16]. Or, also, with regard to interest rate differentials, a higher interest rate in Europe than in the United States will lead to a depreciation of the dollar, as explained by the interest rate parity theory [17]. That is, the fundamental value of the exchange rate will depend on how well or how poorly one economy performs in relation to another. For a review of this approach applied to the FOREX market, see the following latest publications: [18–21].

If we take into account the Efficient Market Hypothesis proposed by E. Fama [22], examining the past of a series does not provide any information that can be used to predict the future, as even information regarding fundamentals cannot affect the prediction of exchange rates, because all of this information, both past and fundamental, is already embedded in currency prices. Then, according to this theory, the study of chartist patterns lacks a theoretical basis, as chartists make their predictions without considering the evolution of the macroeconomic variables of the countries involved. In addition, on the other hand, the fundamentalist approach would only serve to make medium- or long-term predictions, because, in the short term, the exchange rate would only change to the extent that new information regarding the fundamentals of the exchange rate is produced.

Hence, following the EMH theory, only new information that was not previously known, known as information surprises, can affect the exchange rate. These information surprises can change the exchange rate, as by incorporating new information into the price, the FOREX market readjusts the value of the quote, bringing it closer to its new fundamental value; see, e.g., [23–26]. In other words, the only aspect that would impact exchange rate movements in the short term, where "short term" is understood as the time it takes for the market to incorporate this new information into the foreign exchange market, would be information surprises.

The effect of an information surprise generated in the currency markets can also be explained via the prospect theory. This theory introduces the idea that investors, in situations of uncertainty, avoid risk and prefer situations with a higher degree of certainty; see, e.g., [27]. Another complementary alternative to the study of information surprises would be the black swan theory. For a more detailed explanation and application of this theory, see, e.g., [28,29].

In this paper, we statistically tested whether these information surprises affect exchange rate movements (spreads) in a systematic way by considering three of the most important economic regions (the United States, Europe, and the United Kingdom). Then, we analysed whether there is a relationship between news surprises and movements in the exchange rates of the three currencies studied, taking into account the evolution of six macroeconomic news indicators with a high impact on each of them. These were trade balance, unemployment rate, Gross Domestic Product, retail sales, the Industrial Production Index, and the Consumer Price Index. Lastly, we determined which currency pair had the highest correlation with the same set of macroeconomic news to identify when it would be advisable to enter the market for these currencies in order to generate positive returns and increase the profitability of our portfolio.

Moving on to the second phenomenon analysed in this document, we focused on the empirical verification of the Model-Data Paradox of Chaos proposed by Brock et al. [30] in the context considered. This paradox says that, although it is relatively easy to formulate theoretical economic models that demonstrate chaotic dynamics, there is no clear evidence that economic time series behave chaotically. That is, chaos is avoidable in empirical economic data.

For instance, in the FOREX market, chaotic price dynamics could be modelled theoretically by drawing on the behaviour of chartists and fundamentalists operating in such markets, as discussed above; see, e.g., [31–33]. Fundamentalists would purchase (or sale) when the market value is below (or above) its fundamental price, and chartists would wait for the price to go up (or down) if it has gone up (or down) in the past. When these two types of economic agents interact freely in the market, all kinds of solutions and dynamic equilibrium could appear in the proposed theoretical model, including chaotic behaviour; see, e.g., [34–37]. Thus, the key scientific issue in this area is in the analysis of empirical financial data, as it is very difficult to detect chaos in financial time series, as shown in most published papers, where substantial evidence of nonlinearity but relatively weak evidence of chaos has been found; for a review, see, e.g., [38–42].

In this sense, in this paper, we wanted to empirically test whether, if we consider all the information available in financial foreign exchange markets (or at least, more desegregated data) instead of daily data, and if we apply a robust method for the detection of chaotic behaviour, we could find differences in relation to the detection of chaos over the same series but with different time frequencies [43–45]. That is, chaos would be elusive in empirical financial studies due to the loss of information that would occur when using daily quotes. This fact could make it difficult to detect chaos in these time series, as many authors argue; see, e.g., [46–48].

Therefore, if this premise is true, it would allow us to confirm that behind these financial time series that show an apparently random irregular evolution, there would be a generating system that, although unknown in principle, would be deterministic (and nonlinear), and we could take advantage of this deterministic characteristic to make predictions, even in the short term, understanding "short term" as the time it takes for the market to incorporate these informative surprises in the FOREX market analysed. For further discussion regarding the importance of the Efficient Market Hypothesis and its application in nonlinear time series in this context, see, e.g., [49–51].

To summarise, the main contributions of this paper are discussed as follows. On the one hand, we showed that the FOREX market reacts under the Efficient Market Hypothesis, creating a significant variation in a short period of time (15, 30, or 60 min) to the quotation of the main currencies from the most important economic regions in the West. In this sense, it could be interesting to extend this study to other eastern economic regions such as China or Russia. It would also be interesting to make a minute-by-minute correlation analysis, with the aim of finding the maximum correlation point between the news and the movement of currencies.

On the other hand, we empirically verified that if we consider all of the information available in the FOREX market (or at least, more desegregated data) instead of daily data, and if we apply a robust chaotic behaviour detection method, we can find differences in relation to the detection of chaos on the same series but with different temporal frequencies. In this case, we did not intend to generalise this finding to all financial series or even to all FOREX series. We simply wanted to illustrate how choosing a high frequency (1 min, 15 min, 30 min, or 60 min) instead of a daily frequency in order to preserve the dynamic dependence of information could lead to differences in detecting chaotic behaviour. This was confirmed, at least in the particular currency pairs analysed and during the time intervals considered.

The paper is divided into the following sections: In Section 2, a discussion about the justification of the macroeconomic news we considered is provided, as well as a brief description of it. We also present an exploratory statistical analysis regarding certain top currency pairs from the foreign exchange market (FOREX). In Section 3, an overview on the methodology we employed is presented. In Section 4, the main empirical results of this paper are reported. Finally, concluding remarks are made in Section 5.

2. A Brief Exploratory Statistical Analysis of the Dataset Considered

2.1. Descriptive Analysis of the Main Macroeconomic News and Indicators Used for Top Three Economic Regions

In this section, we provide a descriptive analysis of the main macroeconomic news for each economic region considered. Macroeconomic news is an event or occurrence referring to the country in the economic sphere. That is, macroeconomic news is announcements that provide information on the financial health of a given country or region. For a review of the relationship between macroeconomic news and the FOREX market, see, e.g., [52–54].

For this purpose, we selected some of the most relevant news with the greatest economic impact, as can be seen in Table 1. The news was divided into topics in order to create a diversified sample and thus avoid focusing on macroeconomic news of a single typology and provide a more complete and diversified study. In particular, we considered the following categories: employment, trade balance, economic activity, and price index for the economic regions of the United States (USD), Europe (EUR), and the United Kingdom (GBP). It should be noted that with the exception of Gross Domestic Product, which had a quarterly frequency (63 news items), the rest of the macroeconomic variables had a monthly frequency (189 news items) due to the methodology used to collect them. We followed this criterion when selecting the frequencies in order to avoid news that would create noise in our study, thus achieving greater homogeneity in the series analysed.

Table 1. Summary of selected macroeconomic news by topic, region, and frequency from January 2007 to September 2022.

Topic	Currency	Macroeconomic New	Frequency	No. Obs
Employment	USD	Unemployment rate	Monthly	189
	EUR	Unemployment rate	Monthly	190
	GBP	Unemployment rate	Monthly	189
Trade balance	USD	Trade balance	Monthly	189
	EUR	Trade balance	Monthly	187
	GBP	Goods trade balance	Monthly	189
Economic activity	USD	Advance Gross Domestic Product	Quarterly	63
	EUR	Revised Gross Domestic Product	Quarterly	63
	GBP	Final Gross Domestic Product	Quarterly	63
	USD	Retail sales	Monthly	189
	EUR	Retail sales	Monthly	189
	GBP	Retail sales	Monthly	189
	USD	Industrial Production Index	Monthly	189
	EUR	Industrial Production Index	Monthly	189
	GBP	Industrial Production Index	Monthly	189
Price Index	USD	Consumer Price Index	Monthly	189
	EUR	Consumer Price Index	Monthly	189
	GBP	Consumer Price Index	Monthly	189

The period under study was from June 2007 to September 2022, including several economic and social milestones of high impact, such as the great global financial crisis of

2008 initiated by the collapse of Lehman Brothers, the period of international economic recession in 2012–2015, the health crisis caused by the pandemic in 2020 and 2021, the start of the war between Russia and Ukraine in February 2022, and the rise in interest rates by the main central banks from July 2022.

Here, we detail the main features of the above-mentioned macroeconomic news. We obtained the information from the following sources accessed at 19/09/2022. United States: Bureau of Labor Statistics (https://www.bls.gov/), Census Bureau (https://www.census.gov/), and Federal Reserve (https://www.federalreserve.gov/); Europe: Eurostat (https://ec.europa.eu/eurostat/web/main/home); United Kingdom: Office for National Statistics (https://www.gov.uk/search/research-and-statistics). Firstly, the unemployment rate measures the percentage of unemployed people in an economy who are actively seeking employment. The unit of measurement is in percentages, and its interpretation and impact on the movements of the currency of each country would be as follows: If the percentage value is high and higher than expected, this means that there are more people unemployed than expected, with a consequent negative effect on the economy, generating a negative and bearish impact on the currency. On the other hand, a lower than expected figure is a sign that employment is better than expected, meaning there should be a positive and bullish impact on the currency. Second, the trade balance is the difference in value between the goods (goods and services) imported and exported during the month. The unit of measurement is in billions, and a higher than expected reading should be taken as positive and bullish for the currency, and a lower than expected reading should be interpreted as negative and bearish for the currency.

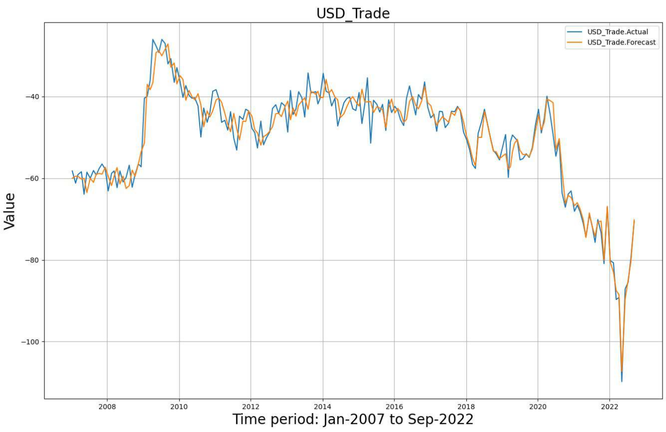
Next, the Gross Domestic Product measures the quarterly change in all goods and services produced by the economy, adjusted for inflation, and is measured as a percentage. A higher than expected reading should be taken as positive and bullish for the currency, and a lower than expected reading should be interpreted as negative and bearish for the currency. Fourth, the retail sales measure all goods sold by retailers based on a sample of retail shops in the country. It is measured as a percentage, and a higher than expected figure can be interpreted as a positive event for the economy and for the currency, while a lower than expected event should lead to a depreciation in the currency in question. Fifth, the Industrial Production Index (IPI) measures the change in the total inflation-adjusted value of output produced in factories, mines, and utilities in the geographical area concerned and is expressed as a percentage. In this case, a higher than expected reading should be interpreted as positive and bullish for the currency, while a lower than expected reading should be taken as negative and bearish for the currency. Lastly, the Consumer Price Index (CPI) measures the change in the prices of goods and services and is also expressed as a percentage. A higher than expected reading should be taken as positive and upward for the currency, as for the IPI. It also behaves in the same way as the IPI in the case of a lower than expected reading (a negative and downward effect on the currency).

Having described the most relevant points of the macroeconomic news, here, we focus on and analyse in more detail the results provided by the news mentioned above for each economic region individually. Then, a comparative commentary is presented, taking into account the results obtained for each area of study, highlighting which are the most relevant indicators, either due to their positive or negative values, in these economies.

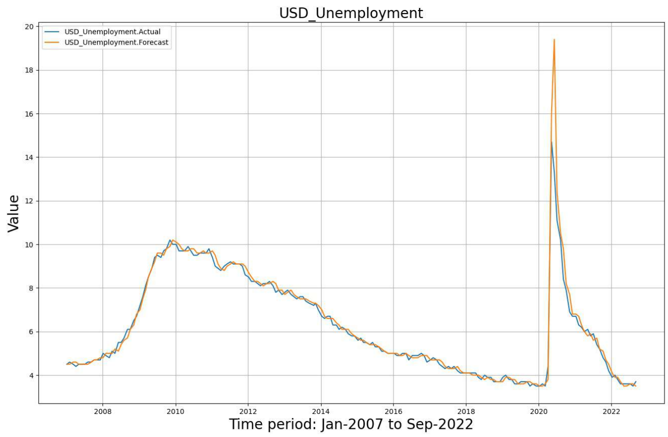
2.1.1. Descriptive Analysis of the United States by Each Macroeconomic New

- Trade balance (Figure 1a). The US trade balance is notable for its consistently negative value, and very significantly so. It started the period at −60 B, and in 2009, a small upward trend began, reaching a peak value of −26 B. From then on, a downward–lateral trend began until the end of 2020. At this stage, the values were between −50 B and −35 B. At the beginning of 2020, the downward trend accelerated, with values reaching −109.8 B. Currently, it seems that exports have recovered some ground, as it is around −50 B. Regarding the forecast value, two phases can be observed. The first was observed until the end of 2017, in which the forecast value was much more erratic and conservative with respect to the current value, and another phase was observed from then until now, in which the forecast value very accurately anticipates the movement of the current data.

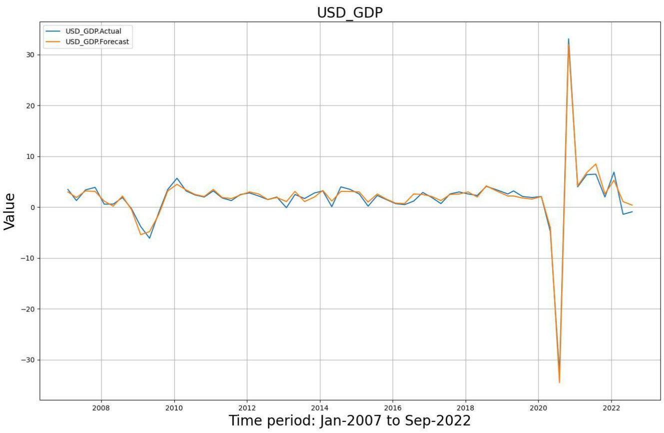
- Unemployment rate (Figure 1b). We can see that the unemployment rate reached its second highest value (within our study period) in 2010. We can note that since then, it was on a downward trend until 2020. Then, with the impact of the pandemic crisis in 2020, the market came to a screeching halt, with unemployment rates climbing to close to 15%. We can see that there is a small difference between the current figure and the one expected by the market (with the exception of 2020, a year in which uncertainty played a major role in the markets). The current and forecast values are quite similar throughout the series; therefore, it seems that this series will not generate any information surprises. Later, we see whether this is the case or not.
- Gross Domestic Product (GDP) [Figure 1c]. The US Gross Domestic Product has been a very stable value, except for during the 2008 crisis, until the arrival of the pandemic in March 2020. From then on, there was a steep fall to −32.9% (the forecast value was expected to fall to −34.5%). Subsequently, it recovered by the same magnitude to 33.1% (and its forecast value of 32%). We can say that the fall in GDP caused by the COVID-19 crisis has fully recovered.
- Retail sales (Figure 1d). The expected value is somewhat more cautious than the actual value, causing information surprises, as we explained. These differences are more marked with increases in volatility, i.e., during the pandemic, as there was a very aggressive reduction in retail sales, with a decrease of more than 16%, and with its consequent rebound effect in the following periods. The pre-2020 values show a very tight range, with values between −3% and +3%.
- Industrial Production Index (IPI) (Figure 1e). The US Industrial Production Index has taken very marked ranges of values in the different crises we have lived through. In 2008, the value fell below −2.5%, while the market expected a much smaller fall, closer to −1%. During the pandemic, we observed a much more marked fall in industrial production, with the value dropping to −11.2%. In this case, the forecast data did predict the magnitude of the impact accurately, predicting a value of −11.3% over the same period. It should be noted that in the two periods of decline noted above, the industrial production data did not recover as easily as they did when this decline was generated. The forecast value is in a much more conservative range than the current value, with the exception of the COVID-19 period, where it correctly anticipated the abrupt market movement.
- Consumer Price Index (CPI) [Figure 1f]. The average value of US inflation stands at almost 2.3%. From 2010 to 2020, inflation was contained between 2 and 4 percent. With the advent of the pandemic, it initially dropped to almost 0.0%, but then it spiked to reach values above 9.0%. As in the case of the unemployment rate, the current and forecast values are quite similar throughout the series; therefore, it seems that this series will not generate any information surprises either.



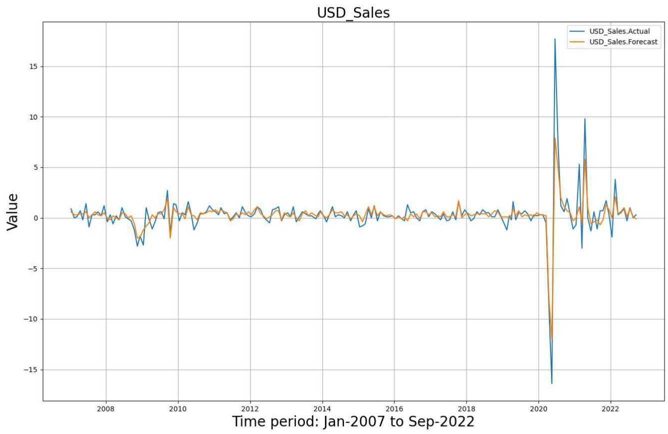
(**a**) Trade balance



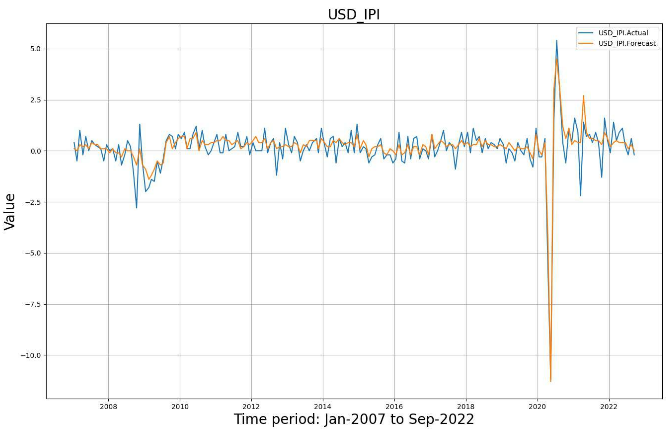
(**b**) Unemployment rate



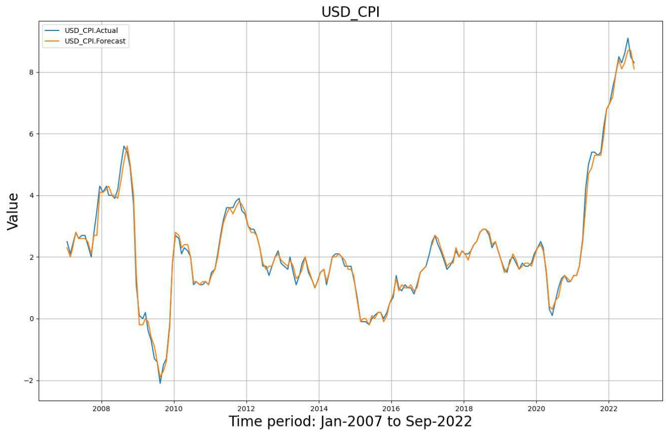
(**c**) Gross Domestic Product



(**d**) Retail sales



(**e**) Industrial Production Index

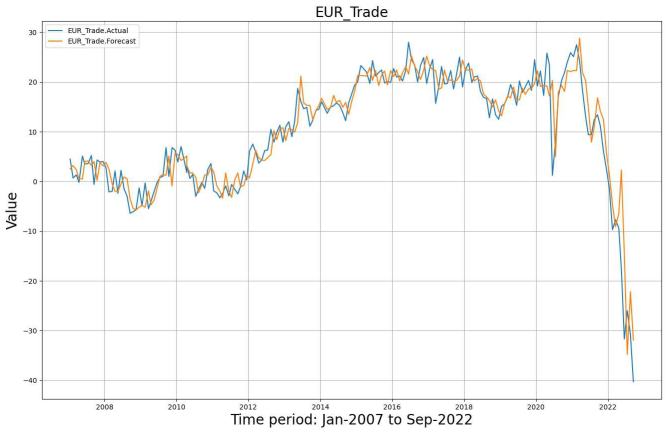


(**f**) Consumer Price Index

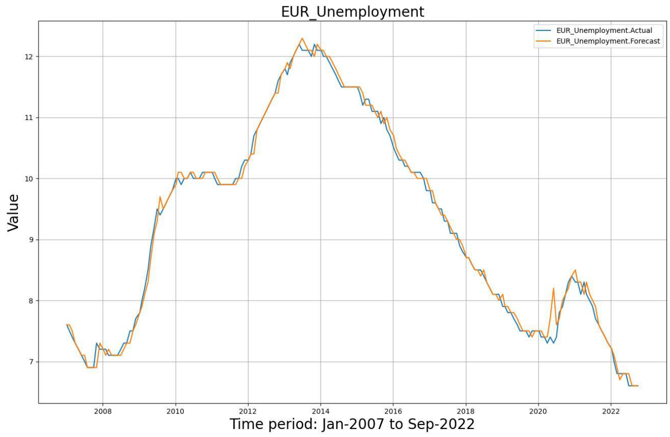
Figure 1. Plot showing the time evolution of the 6 macroeconomic news items from the United States (USA) for the time period analysed between June 2007 and September 2022. The current value of each time period is marked in blue, and the predicted value in orange.

2.1.2. Descriptive Analysis of Europe by Each Macroeconomic New

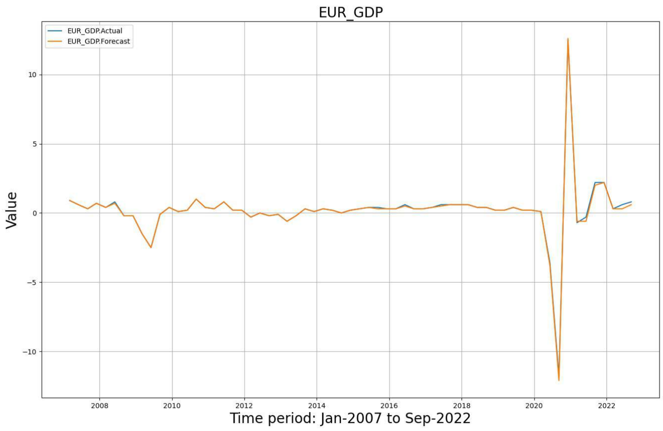
- Trade balance (Figure 2a). The European trade balance has been in positive territory for most of the time period studied, with peak balances of 28 B and the forecast being 28.8 B. Since that peak, it began on a downward trend that does not seem to be over yet. Currently, the minimum value is at −40.3 B, while the expected value was −34.8 B. In general terms, we can say that the forecast value has been more optimistic than the current value, as its average value is 10.28 B, while the current value is 9.91 B.
- Unemployment rate (Figure 2b). The maximum peak of European unemployment was reached in 2013, with values above 12%. Since then, the unemployment rate had been declining steadily until the arrival of COVID-19 in 2020, after which, the unemployment rate has risen somewhat steadily from 7.5% to almost 8.5%. In this period of time, it is worth noting that the market expected a considerably higher unemployment rate than the current value, although throughout our study period, we can observe that the forecast values are quite close to the current value.
- Gross Domestic Product (GDP) (Figure 2c). We can see that the current and forecast data practically coincide over the whole period. It is worth highlighting the fall in GDP during the 2008 crisis, and the much greater fall in 2020 with the onset of the pandemic. Europe was on the verge of recovering the totality of the previous fall due to COVID-19, as this fall was −11.8%, and the subsequent recovery was 12.5%. The current figure has an average value of 0.19%, while the forecast figure is 0.17%.
- Retail sales (Figure 2d). European retail sales had a fairly narrow range of values until 2020, varying between +2% and −2%. In March 2020, due to the COVID-19 pandemic, negative values soared, and it is particularly striking that the market expected a value of −15%, but fortunately for the economy, this value did not exceed 11.7%. For the rest of the values in the series, we can see that the forecast takes much more conservative values than the real value.
- Industrial Production Index (IPI) (Figure 2e). The European industrial production index notably has an average value over the whole period of −0.043%. The forecast value anticipated the magnitude of the current value in most cases, including the two recent crises in 2008 and 2020. It is worth noting that in 2020, the forecast value exceeded both the positive and negative expectations of the indicator, as it estimated a fall in industrial production of −19%, with the actual figure being −17.1%. The same happened when the indicator recovered, as the forecast figure peaked at 14.9%, although the actual value did not exceed 12.4%.
- Consumer Price Index (CPI) (Figure 2f). The European Central Bank has a target inflation rate of 2.71%. The truth is that, even with the recent sharp rise in prices, the average value of inflation is 1.71%. It is worth noting that year-on-year inflation is currently at 9.1%, becoming negative (within our study period) in 2009, 2015, and 2021. The forecast value is very much in line with the current value.



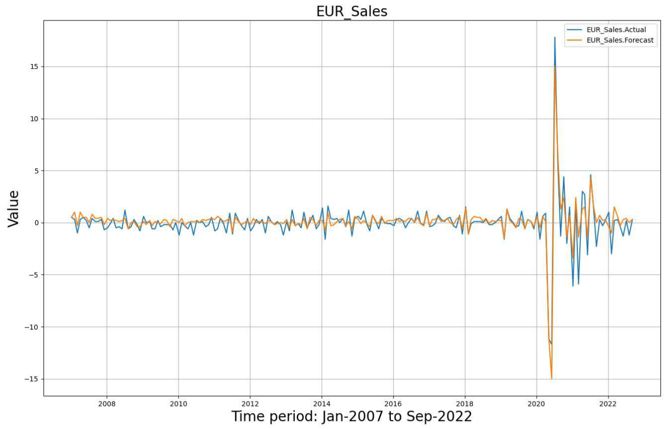
(**a**) Trade balance



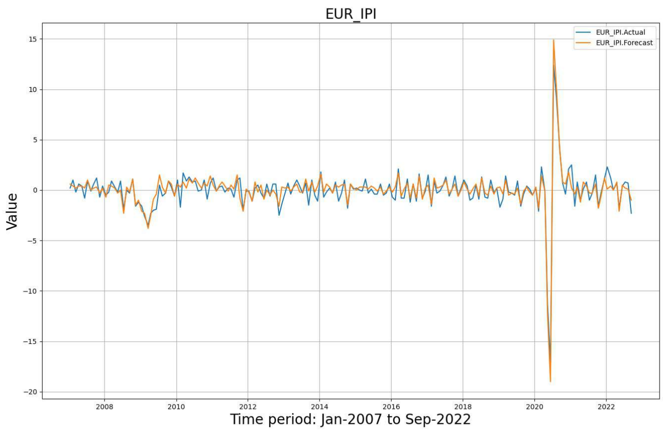
(**b**) Unemployment rate



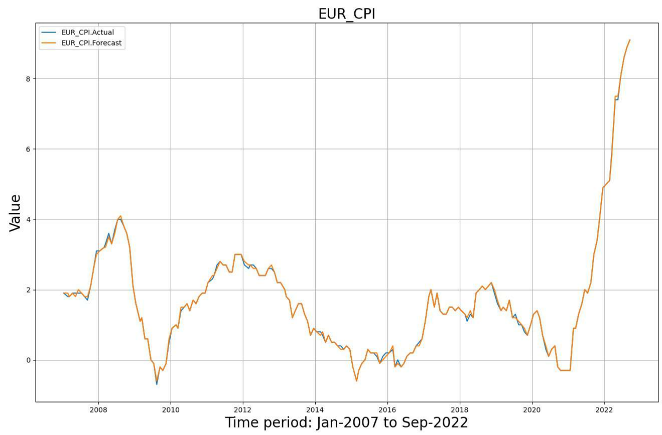
(**c**) Gross Domestic Product



(**d**) Retail sales



(**e**) Industrial Production Index



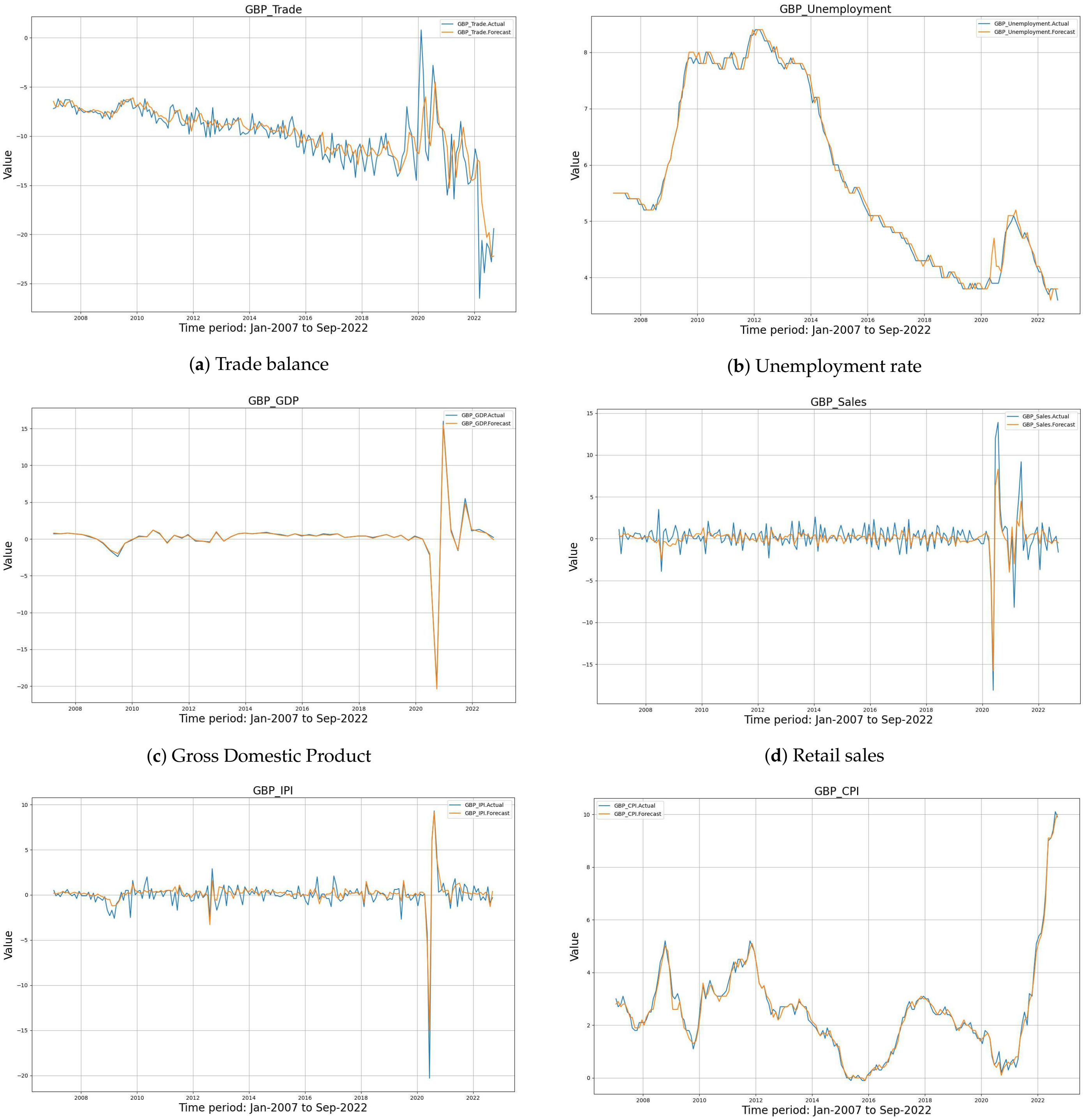
(**f**) Consumer Price Index

Figure 2. Plot showing the time evolution of the 6 macroeconomic news items from Europe (EUR) for the time period analysed between June 2007 and September 2022. The current value of each time period is marked in blue, and the predicted value in orange.

2.1.3. Descriptive Analysis of the United Kingdom by Each Macroeconomic New

- Trade balance (Figure 3a). The UK trade balance has been negative throughout the period, with the only notable moment of positive balance being at the beginning of 2020. Thereafter, the volatility of the data increased considerably. At the beginning of 2022, the trade balance reached its minimum value of −26.5 B. It should be noted that the forecast value is much more moderate than the actual value, especially in periods where volatility increases, as the expected data do not match the actual value.
- Unemployment rate (Figure 3b). We can observe an upward trend starting in mid-2008, which was sustained until 2013, when the unemployment rate began to fall until 2020, when the economy was damaged by the arrival of the COVID-19 pandemic, with its consequent increase in the unemployment rate throughout the year. It is currently beginning a downward trend, with values below 4%. We can see this indicator has very similar real and expected values, with the exception at the beginning of the pandemic, where the market expected a much higher rise in unemployment than the real increase.
- Gross Domestic Product (GDP) (Figure 3c). The forecast figure tracks the current figure fairly closely over the whole period; therefore, it seems likely that there will be no significant information surprises. It is worth noting that the UK fell some way short of recovering from its full fall in 2020 due to the pandemic, which was −19.8%, and the subsequent recovery was 16%.
- Retail sales (Figure 3d). We see in the graph how the predicted data are always more conservative than the actual data, indicating that analysts expected (for both positive and negative values) lower values than the actual data. We can see that before the pandemic, retail sales were very limited between −5% and 5%, but this ceased to be the case with the arrival of the COVID-19 pandemic, during which, sales initially fell to −18.1%, largely recovering in the following months after this fall, with values close to 8%. We can see that the predicted value of retail sales is below the actual value, even during the pandemic and in the following years.
- Industrial Production Index (IPI) (Figure 3e). It is striking that with the arrival of the pandemic in 2020 and the consequent fall in the British consumer price index, the forecast value estimated a much smaller fall than what actually occurred. The figure was an estimated 15 percent, compared with the actual figure of just over 20 percent. However, the predicted value of the rise did coincide with the actual value, being 9.2% and 9.3%, respectively. Once again, we can see that the fall was much more abrupt than the subsequent recovery, and the forecast values are much more moderate than the actual values, generating real information surprises.
- Consumer Price Index (CPI) (Figure 3f). The minimum inflation value was recorded during 2015, while temporary upward peaks were reached in 2009 and late 2011. Currently, UK prices have been rising at a frenetic pace from 2021 to the present, reaching increases of 10%. We can see that the dispersion between the current and forecast data is very low. This indicates that the data expected by analysts are very similar, if not the same as the actual data that appears later, which is likely to have little or no effect on the currency market as there are not many information surprises in the data. We examine this in this paper.

  To conclude this section, we compare the results of the three economic regions analysed. Europe stands out as the only one with a positive average balance of trade. At the same time, it has the highest average unemployment rate, the lowest average GDP growth rate, and a negative average retail sales balance. In terms of industrial production, it stands out for having achieved the highest value among the three regions, although it has a negative average value (close to zero). Regarding the United States, it has the most negative trade balance of the three regions and the highest and the lowest unemployment rate and GDP of the three regions, respectively. In terms of the consumer price index, the United States stands out for having the lowest average value. Finally, the UK stands out for having the lowest unemployment rate compared with the other geographical areas. It also had the lowest value for retail sales and industrial production during the pandemic and the highest value for inflation in both absolute and average terms.



(**a**) Trade balance

(**b**) Unemployment rate

(**c**) Gross Domestic Product

(**d**) Retail sales

(**e**) Industrial Production Index

(**f**) Consumer Price Index

Figure 3. Plot showing the time evolution of the 6 macroeconomic news items from the United Kingdom (UK) for the time period analysed between June 2007 and September 2022. The current value of each time period is marked in blue, and the predicted value in orange.

2.2. Descriptive Analysis of the Top Currencies Rates Used from the Foreign Exchange Market

In this section, we provide an exploratory analysis of the three currency pairs used in this paper: euro/US dollar (EURUSD), euro/pound (EURGBP), and pound/US dollar (GBPUSD). The data were extracted from the Free FOREX Historial Data repository available at http://www.histdata.com/ and consulted on 19/09/2022. The open bid price series was collected. The frequency of the data extracted was minute by minute. Due to the granularity of the data, data were missing. To deal with this, we used a simple deterministic imputation technique called Last Observation Carried Forward (LOCF), which involved filling the missing data with the previous value that we did have [55].

In Table 2, a summary of the main descriptive statistics for each currency pair—EURUSD, EURGBP, and GBPUSD—from January 2007 to September 2022 is presented. The number of data in all cases is equal to 8,238,061 observations. Regarding the EURUSD rate, the average price is 1.2486, with a maximum price of 1.6036 and a minimum of 0.9669 throughout the period analysed. The Kurtosis coefficient indicates that the distribution is platykurtic, i.e., there is a low degree of concentration around the central values. Moreover, with respect to the Skewness coefficient, we see that the distribution is skewed to the right.

The EURGBP rate has a lower value for both the mean (0.8347878) and the dispersion of the series (0.06080891) compared with the EURUSD exchange rate. The Kurtosis coefficient indicates that the distribution is leptokurtic and the distribution is asymmetric to the left according to the Skewness coefficient. Lastly, the average value of the GBPUSD exchange rate is 1.50584, showing the highest dispersion of all (0.2154754). The Kurtosis coefficient shows that the distribution is leptokurtic as well, although to a lesser degree than in the EURGBP currency pair, while on the other hand, this currency pair has a more right-skewed distribution, according to the Skewness coefficient, compared with the other currency pairs.

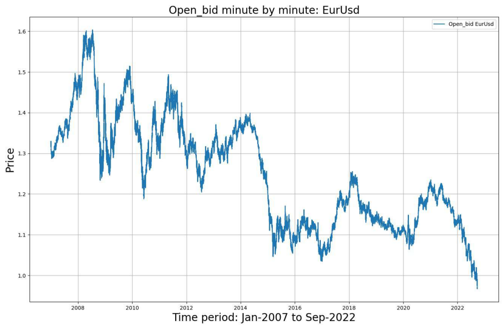
Table 2. Summary of the main descriptive statistics for each currency pair—EURUSD, EURGBP, and GBPUSD—from January 2007 to September 2022. Data shown are from the open bid price rates.

Currency	No. Obs	Mean	Stand. Dev.	Min	Max	Kurtosis	Skewness
EURUSD	8,238,061	1.248645	0.134039	0.966964	1.603625	−0.794841	0.347385
EURGBP	8,238,061	0.834787	0.060808	0.657782	0.980500	0.417404	−1.027335
GBPUSD	8,238,061	1.505842	0.215475	1.084358	2.116291	0.216963	0.815237

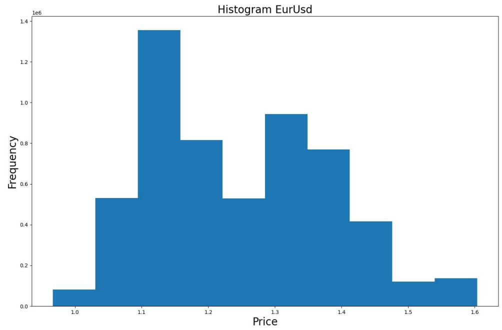
Here, we plot the time evolution and the histogram of the three currencies pairs, as shown in Figure 4. Regarding the EURUSD rate (Figure 4a), we can see that since 2008, when the price peaked, the market has been immersed in a downward trend for over 13 years, with the occasional sideways period between 2015 and 2017. This market behaviour is explained by the monetary policies carried out mainly by the Federal Reserve (FED) and the European Central Bank (ECB). The recent rise in interest rates due to the increase in inflation is having an impact on the price of the currency pair, as well as on the values of the trade balance. We can also see in the histogram (Figure 4b) how the most frequently repeated price value is in the range of 1.10 and 1.15.

The EURGBP rate (Figure 4c) showed an upward trend in January 2007 until 2009, where it peaked at 0.9805. Later, it started a downtrend that ended in 2015. Between 2015 and 2016, it marked a triple bottom at the price of 0.70, putting an end to the downtrend. From 2017 to 2021, the quote was immersed in a fairly marked sideways band, mainly between 0.85 and 0.95. According to the histogram (Figure 4d), the price is mostly between 0.80 and 0.90. Thirdly, we can see that the GBPUSD exchange rate (Figure 4e) showed a very marked sideways trend between 2010 and 2016, with values around 1.45–1.65. Then, there was a decline in the pair's price to the range of 1.20–1.40 from 2016 until almost mid-2022. These two marked sideways trends are reflected in the histogram (Figure 4f), as the most repeated values are between 1.30 and 1.40 and 1.50 and 1.60. This is mainly due to these two previously mentioned sideways periods. Currently, since the beginning of

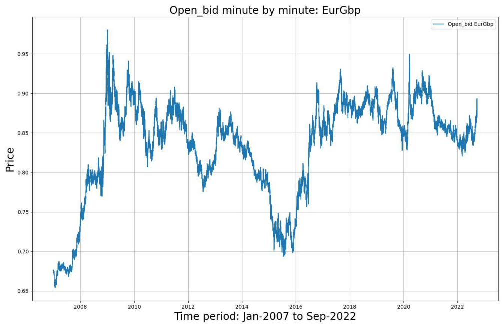
2022, the rate is in a downtrend, where it has marked new historical lows, mainly due to the macroeconomic policy decisions of both the Bank of England and the Federal Reserve.



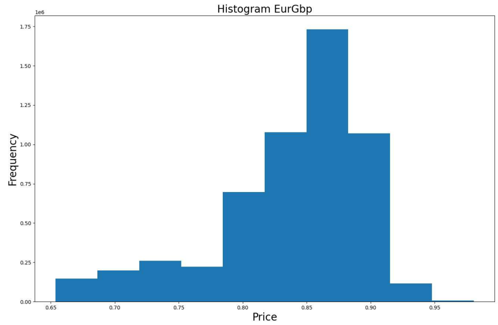
(**a**) EURUSD: Time evolution



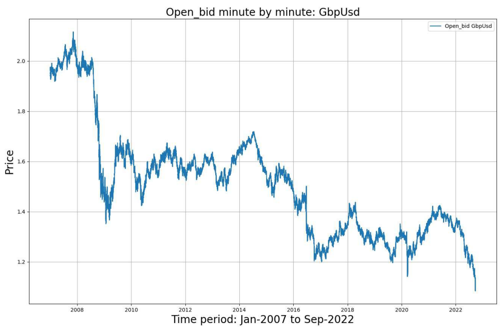
(**b**) EURUSD: Histogram



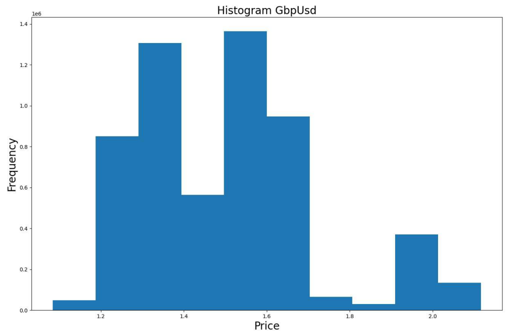
(**c**) EURGBP: Time evolution



(**d**) EURGBP: Histogram



(**e**) GBPUSD: Time evolution



(**f**) GBPUSD: Histogram

Figure 4. Plot showing the time evolution of the three currencies pairs. On the right, we also included a histogram for each of them considering the time period analysed between June 2007 and September 2022.

3. Methodology

3.1. How Can the Efficient Market Hypothesis Be Tested through the Correlation between the Information Surprises and Market Movements?

In this section, we introduce the methodology we followed to determine how to test the Efficient Market Hypothesis by correlating information surprises and the movements of major currency pairs in the FOREX market. If we take into account the Efficient Market Hypothesis proposed by E. Fama [22], examining the past of a series does not provide any information that can be used to predict the future, as even information regarding fundamentals cannot affect the prediction of exchange rates, because all of this information, both past and fundamental, is already embedded in currency prices. Hence, following the EMH theory, the only aspect that would impact exchange rate movements in the short term, where "short term" is understood as the time it takes for the market to incorporate this new information into the FOREX market, would be information surprises.

In this context, different publications have contrasted the Efficient Market Hypothesis in the FOREX market; see, e.g., [56–59]. In this paper, we assumed that this hypothesis was satisfied based on the results of these articles. Particularly, we focused on analysing whether or not there was a correlation between new information surprises, as these comprise information that has not been previously known and the short-term movement (in 15, 30, and 60 min) that occurs at the time of the macroeconomic news, considering the main currency pairs. In addition, if this correlation exists, we wanted to determine which item of news generates the greatest variation in the price movements of the different currency pairs.

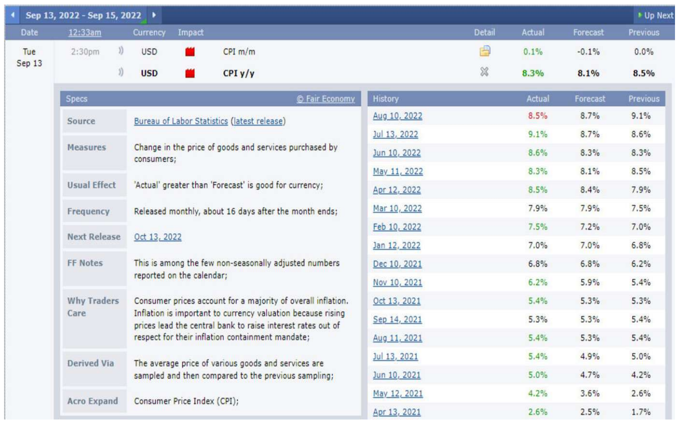
An alternative methodology to the one used in this paper is the technique proposed by Ball and Brown [60], called event-study. This statistical approach is widely used to examine empirical financial data. It consists of evaluating the specific impact of an event (in our case, an information surprise) on the valuation of a financial asset such as a FOREX currency pair; for a review, see, e.g., [61–63].

The EMH hypothesis is uniquely valid for macroeconomic news, which contains information that can alter the fundamental value of currencies. There is a classification of news according to the impact it has on the foreign exchange market when the event is known. This classification is divided into three types of news: high impact, medium impact, and low impact. We considered medium- and high-impact news. As we can see in Figure 5a, macroeconomic news is composed of three values following the methodology proposed by [64]:

- Previous: These data are revisions of the current data of the previous month, i.e., the macroeconomic data published the previous month are revised and updated to account for any modifications, as often, due to the acceleration of the publication of data, some data are left unchecked.
- Forecast: These data are expected by economic analysts based on surveys of financial organisations, such as Reuters and Bloomberg.
- Actual: These are the actual data published. They express the evolution of the economy over the last month in the case of monthly news. The difference between these data and the consensus will determine the direction of the fluctuation of the currency market, at least in the short term.

Date	6:29pm	Currency	Impact		Detail	Actual	Forecast	Previous
Tue Sep 27	10:00am	EUR		M3 Money Supply y/y		6.1%	5.4%	5.7%
		EUR		Private Loans y/y		4.5%	4.5%	4.5%
	1:30pm	EUR		ECB President Lagarde Speaks				
		USD		Fed Chair Powell Speaks				
	2:30pm	USD		Core Durable Goods Orders m/m		0.2%	0.3%	0.2%
		USD		Durable Goods Orders m/m		-0.2%	0.1%	-0.1%
	3:00pm	USD		HPI m/m		-0.6%	0.0%	0.1%
		USD		S&P/CS Composite-20 HPI y/y		16.1%	17.1%	18.6%
	3:35pm	GBP		MPC Member Pill Speaks				
	3:55pm	USD		FOMC Member Bullard Speaks				
	4:00pm	USD		CB Consumer Confidence		108.0	104.0	103.6
		USD		New Home Sales		685K	500K	532K
		USD		Richmond Manufacturing Index		0	-10	-8
Wed Sep 28	1:01am	GBP		BRC Shop Price Index y/y		5.7%		5.1%
	8:00am	EUR		German GfK Consumer Climate		-42.5	-38.9	-36.8
	9:15am	EUR		ECB President Lagarde Speaks				
	10:15am	GBP		MPC Member Cunliffe Speaks				
	2:30pm	USD		Goods Trade Balance		-87.3B	-88.9B	-90.2B
		USD		Prelim Wholesale Inventories m/m		1.3%	0.4%	0.6%
	4:00pm	USD		Pending Home Sales m/m		-2.0%	-0.9%	-0.6%
	4:10pm	USD		FOMC Member Bullard Speaks				
	4:15pm	USD		Fed Chair Powell Speaks				
	4:30pm	USD		Crude Oil Inventories		-0.2M	2.0M	1.1M
	5:00pm	USD		FOMC Member Bowman Speaks				
	8:00pm	GBP		MPC Member Dhingra Speaks				

(**a**)



Sep 13, 2022 - Sep 15, 2022 Up Next

Date	12:33am	Currency	Impact		Detail	Actual	Forecast	Previous
Tue Sep 13	2:30pm	USD		CPI m/m		0.1%	-0.1%	0.0%
		USD		**CPI y/y**		**8.3%**	**8.1%**	**8.5%**

Specs	© Fair Economy
Source	Bureau of Labor Statistics (latest release)
Measures	Change in the price of goods and services purchased by consumers;
Usual Effect	'Actual' greater than 'Forecast' is good for currency;
Frequency	Released monthly, about 16 days after the month ends;
Next Release	Oct 13, 2022
FF Notes	This is among the few non-seasonally adjusted numbers reported on the calendar;
Why Traders Care	Consumer prices account for a majority of overall inflation. Inflation is important to currency valuation because rising prices lead the central bank to raise interest rates out of respect for their inflation containment mandate;
Derived Via	The average price of various goods and services are sampled and then compared to the previous sampling;
Acro Expand	Consumer Price Index (CPI);

History	Actual	Forecast	Previous
Aug 10, 2022	8.5%	8.7%	9.1%
Jul 13, 2022	9.1%	8.7%	8.6%
Jun 10, 2022	8.6%	8.3%	8.3%
May 11, 2022	8.3%	8.1%	8.5%
Apr 12, 2022	8.5%	8.4%	7.9%
Mar 10, 2022	7.9%	7.9%	7.5%
Feb 10, 2022	7.5%	7.2%	7.0%
Jan 12, 2022	7.0%	7.0%	6.8%
Dec 10, 2021	6.8%	6.8%	6.2%
Nov 10, 2021	6.2%	5.9%	5.4%
Oct 13, 2021	5.4%	5.3%	5.3%
Sep 14, 2021	5.3%	5.3%	5.4%
Aug 11, 2021	5.4%	5.3%	5.4%
Jul 13, 2021	5.4%	4.9%	5.0%
Jun 10, 2021	5.0%	4.7%	4.2%
May 12, 2021	4.2%	3.6%	2.6%
Apr 13, 2021	2.6%	2.5%	1.7%

(**b**)

Figure 5. (**a**) A sample calendar including macroeconomic news from the three economic regions under consideration for 27–28 September 2022; (**b**) data collection process for identifying the US Consumer Price Index macroeconomic news in September 2022.

Given this classification, we define an information surprise as the difference that exists, after the publication of macroeconomic news, between the actual data that have just been published (Actual) and the value that the market expected as the final result (Forecast);

see [54]. Therefore, this new variable, the information surprise, is created as follows: Actual–Forecast. In this sense, we wanted to analyse whether different information surprises would be able to provoke a strong movement in the quotation of the main currency pairs in the short term, that is to say, in the 15, 30, and 60 min after the news is known, so that the appreciating or depreciating fluctuations of the currency pair would be produced by the difference between the real data and the expected data of the macroeconomic news.

We only used the 15, 30, and 60 min windows because, in technical analyses, chartists of financial markets usually use this frequency of the candles (movements generated in that time interval collecting the opening, closing, maximum, and minimum price information) to indicate the movement in the short term, disregarding the noise that is generated in the market minute by minute. Indeed, other time frames could have been used, such as, e.g., 120 or 240 min, but we were interested in the movements generated in the short term.

In this sense, we created new time series data to work with, with these being the differences in the pips of EURUSD, EURGBP, and GBPUSD in 15, 30, and 60 min (a pip in the quotation of the currencies is the variation in the fourth decimal place of this price). As the data we obtained were minute by minute, we had to examine the difference between the minute in which the macroeconomic news was issued and the subsequent 15, 30, and 60 min for each of the three currency pairs mentioned above. The calculation of these movements was made 15, 30, and 60 min after the announcement was made with respect to the open bid price, which was the opening bid price of the candle at that minute for each currency pair.

In order to illustrate this step, the result of a particular news item and its subsequent market reaction is explained graphically below. To do so, we took the US Consumer Price Index; see Figure 5b. In this image, we see the current, forecast, and previous value for September 2022. In this case, the information surprise would be: actual deviation = 8.3% − 8.1% = 0.2%. In other words, an information surprise equal to 0.2% would have been generated and will be one of the observations of the time series generated from the differences in this particular news item. We repeated the process for the rest of the macroeconomic news considered and the three economic regions. Here, we analyse the impact this news has had on the FOREX market; see Figure 6.

For our example, we decided to examine the distance in the 60 min after the news announcement, i.e., from 14:30 to 15:30. In that time interval, we measured the difference in pips in terms of the opening price of the EURUSD currency pair. In Figure 6, it can be seen that there was a quite considerable price reduction during our 60 min interval, dropping from 1.01684 to 1.00278. That is, there was a price reduction of − 0.01406. Recall that a pip is the fourth decimal place of a quote of the selected currency pairs, meaning that, using the original result of − 0.01406, we had to multiply it by 10,000 to obtain that amount expressed in pips, being in this case equal to 140.6 pips. These data would correspond to an observation of the time series that we would have to generate to collect the information about the impact (through differences in pips) of the publication from this news on the EURUSD price movements. We repeated the process to generate the rest of the time series from the macroeconomic news, the three currency pairs, and the three time frequencies considered (15, 30, and 60 min).

Then, as we just explained, the macroeconomic news calendar allowed us to determine the exact moment when the announcement was going to be made in order to calculate both the information surprise generated by the news and the impact in pips that it had on the movements in the different currencies at 15, 30, and 60 min. In order to test the Efficient Market Hypothesis, we carried out a correlation analysis between the actual deviation or information surprise of each news item and the movement generated in the currency pairs in our study for each of the frequencies under consideration. Hence, the correlation analysis required the previous calculation of the information deviation in each news item, as well as the difference between the prices of the currency pairs for each of the time frequencies mentioned above. These data series generated were the ones we used to make the following correlation hypothesis test.

Figure 6. The data shown are the candlesticks corresponding to the EURUSD price from minute to minute. The time period selected corresponds to the publication of the macroeconomic news under consideration. In this case, the U.S. Consumer Price Index was published at 14:30 h in September 2022. Source: ProRealTime Platform.

The null hypothesis is $H_0 : r_{a,b} = 0$, where $r_{a,b}$ is the Pearson's correlation coefficient and its equation is $r_{a,b} = \frac{Cov(a,b)}{\sigma_a \sigma_b}$, with $Cov(a,b)$ being the covariance and σ_a, σ_b the standard deviation. In this case, the variable a is the information surprise data series, and the variable b is the exchange rate deviation series for the frequencies considered. Once all the correlation coefficients and their corresponding p-values were estimated, we applied the decision rule to decide whether or not to reject H_0. That is, we compared the p-values with the significance level (α) used equal to 5%. If the p-values were less than or equal to α, then we would reject H_0. Therefore, in this assumption, we would be affirming that there is a relationship between the information surprise and the movement generated for a given time interval. In the case of not rejecting H_0 because the p-values were greater than α, the correlation coefficient $r_{a,b}$ would be equal to 0, and there would be no relationship.

Once we compared the different p-values and the level of significance α, we identified those cases where the null hypothesis was rejected. After that, we analysed the correlation values, both the sign and the coefficient. The higher the coefficient (close to 1 or -1), the stronger the relationship. The closer the correlation was to 0, the weaker it was. The sign of the correlation indicated whether the correlation was direct (positive sign) or inverse (negative sign). A positive sign of the correlation would mean that the two data series have a direct relationship, i.e., if the information surprise increases, it would strengthen the price of that currency. Likewise, if it decreases or even if the information surprise is negative, this would be a sign of weakness in the price of that currency. The negative sign of the correlation would imply that the two data series have an inverse relationship, i.e., if the information surprise increases, it would be understood as a weakness in the price of that currency. Similarly, if it decreases, or even if the information surprise is negative, it would be understood as a sign of strength in the price of that currency.

Continuing with the example of the monthly U.S. Consumer Price Index, a value above the expected value (actual deviation or positive information surprise) would mean that prices are higher than expected, and to reduce them, interest rates would have to be raised. This would possibly attract more foreign investment into the country, thus revaluing the currency. In short, it would be interpreted as a strength or bullish signal for the dollar. Recall that in the EURUSD currency pair, the US dollar is in the denominator, meaning that strength or bullish momentum for the US dollar would translate into a decline in the EURUSD. Based on this macroeconomic argument, the correlation coefficient should be negative, as the relationship between the information surprise and the USD currency is positive, but with respect to the EURUSD currency pair, it is negative. This is displayed in the results section.

3.2. How Can the Paradox of Chaos Be Tested through the Estimation of the Lyapunov Exponent Considering Different Time Frequencies in the Financial Series Used?

In this section, we provide the methodology used to test the Model-Data Paradox of Chaos in this context. Our major motivation was that chaos is evasive in world-real financial studies, mainly due to the loss of information that happens when daily prices are used. This fact could make it more difficult to detect chaos in these time series. Chaotic systems are sensitive to initial conditions, which means that the time dependence is lost if chaotic time series are sampled at too long time intervals, showing up as independent even though they are from a (chaotic) dynamic system. For FOREX time series, where quotes change almost continuously up or down in the markets, daily sampling may be too long. To avoid this problem, high-frequency data may be used to detect chaos in the foreign exchange market. For this reason, we compared daily, 1 h, 30 min, 15 min, and minute-by-minute time series for the three currency pairs considered. Normally, when studying the chaotic behaviour of financial time series, log returns data are used.

Hence, to carry out our analysis, we needed to differentiate the log quotes series in order to obtain our series of lagged returns by [48]:

$$x_t = \log(p_t) - \log\left(p_{t-lag}\right) \tag{1}$$

where p_t is the open bid price series collected in Table 2, and lag is equal to $1, 15, 30, 60$, and 1440, corresponding to the frequencies of 1 min, 15 min, 30 min, 1 h, and 1 day, respectively. Now, we show the necessary steps in the procedure to detect chaotic signals within the time series of lagged returns, if any. We focused on methods derived from chaos theory which estimate the complexity of a dataset by exploring the structure of the attractor; for a review, see, e.g., [65–67]. Particularly, we were interested in the so-called *Lyapunov exponent* (λ_k) as an attractor invariant measurement [68]. In general terms, the Lyapunov exponent indicates how fast a shock at a point moves along the trajectory in a finite number of steps. In this paper, we considered the definition proposed by [69]. It can be defined as follows: Let $X_t = F(X_{t-1})$ be a difference equation where $F : \mathbb{R}^k \to \mathbb{R}^k$ for $t = 1, 2, 3, \ldots, T$. For this k-dimensional system, there will be k Lyapunov exponents, which are given by

$$\begin{aligned}\lambda_i(X_0) &= \lim_{T\to\infty} \frac{1}{T}\{\log(|DF(X_T)| \cdot \ldots \cdot |DF(X_0)|)\} \\ &= \lim_{T\to\infty} \frac{1}{T}\left\{\log\left(\left|DF^T(X_0)\right|\right)\right\}\end{aligned} \tag{2}$$

where $DF^T(X_0)$ is the Jacobian evaluated along the trajectory $\{X_0, X_1, \ldots, X_T\}$ and $i = 1, 2, \ldots, k$. Following the ideas provided by [69,70], testing the hypothesis of chaotic behaviour in dissipative systems is equivalent to testing the hypothesis that at least one of these Lyapunov exponents is positive. We were thus interested in testing the chaos hypothesis, which is defined by,

$$\begin{array}{c} H_0 : \hat{\lambda}_i \leqslant 0 \\ H_1 : \hat{\lambda}_i > 0 \quad (Chaotic\ behaviour) \end{array} \tag{3}$$

for some $i = 1, 2, 3, \ldots, k$. Rejecting the null hypothesis $H_0(\hat{\lambda}_i \leqslant 0)$ means that chaotic behaviour exists.

In this paper, we focused on an empirical approach. Thus, the principal objective was to use an observed time series to test whether chaotic behaviour, Equation (3), exists in the underlying unknown data-generating system. In particular, we considered the time series regarding the three currency pairs for all of the above-mentioned frequencies.

Following the ideas proposed by [71–74], we had to take into account an observable function including a noise term ε_t which generates observations as $x_t = f(X_t, \varepsilon_t)$. Then, we make up a sequence of lagged vectors by associating a vector in a reconstructed state space $\mathbb{R}^m$ for each time period whose coordinates satisfy $x_t^m = \left(x_t, x_{t-\tau}, x_{t-2\tau}, \ldots, x_{t-(m-2)\tau}, x_{t-(m-1)\tau}\right)$.

Assuming that there exists a pseudodynamical system $G : \mathbb{R}^m \to \mathbb{R}^m$ such that $x_t^m = G\left(x_{t-\tau}^m\right)$, where x_t^m are the uniform delay coordinate embedding vectors, see [75], such a dynamical system G could be expressed as a matrix that depends on a function v, which is a function of the lags.

$$\begin{pmatrix} x_t \\ x_{t-\tau} \\ \vdots \\ x_{t-(m-2)\tau} \\ x_{t-(m-1)\tau} \end{pmatrix} = G \begin{pmatrix} x_{t-\tau} \\ x_{t-2\tau} \\ \vdots \\ x_{t-(m-1)\tau} \\ x_{t-m\tau} \end{pmatrix}$$

$$\begin{pmatrix} x_t \\ x_{t-\tau} \\ \vdots \\ x_{t-(m-2)\tau} \\ x_{t-(m-1)\tau} \end{pmatrix} = \begin{pmatrix} v\left(x_{t-\tau}, x_{t-2\tau}, \ldots, x_{t-(m-2)\tau}, x_{t-(m-1)\tau}, x_{t-m\tau}\right) \\ x_{t-\tau} \\ \vdots \\ x_{t-(m-2)\tau} \\ x_{t-(m-1)\tau} \end{pmatrix} \tag{4}$$

The Jacobian for the reconstructed G dynamical system can be given by the following way,

$$\partial G = \begin{pmatrix} \frac{\partial v}{\partial x_{t-\tau}} & \frac{\partial v}{\partial x_{t-2\tau}} & \frac{\partial v}{\partial x_{t-3\tau}} & \cdots & \frac{\partial v}{\partial x_{t-(m-1)\tau}} & \frac{\partial v}{\partial x_{t-m\tau}} \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix} \tag{5}$$

Then, the estimation of the Lyapunov exponent via this method is reduced to the estimation of the unknown nonlinear function $v : \mathbb{R}^m \to \mathbb{R}$. This method is called the *Jacobian indirect approach*; for an in-depth review, see, e.g., [76–80].

Following the ideas proposed by [81–83], we focused on approximating the unknown nonlinear function v by a single-layer hidden feedforward neural net.

$$v \approx \hat{v} = \Phi_0 \left[\hat{\alpha}_0 + \sum_{q=1}^{h} \hat{\omega}_{q0} \Phi_q \left(\hat{\alpha}_q + \sum_{j=1}^{m} \hat{\omega}_{jq} x_{t-j\tau} \right) \right] \tag{6}$$

where h is the number of nodes, $\Phi_0 \in I$, $\hat{\alpha}_0$ are the bias of the neural net from the input, $\hat{\omega}_{q0}$ are the weights from the input to the hidden layer, $\hat{\alpha}_q$ are the bias from the hidden layer, $\hat{\omega}_{jq}$ are the weights from the hidden layer to the output, and Φ_q is the transfer function.

Then, applying the chain rule to Equation (6), we obtained the partial derivates of the Jacobian in Equation (5) by

$$\frac{\partial \hat{v}}{\partial x_{t-j\tau}} = \Phi'_0(z_0) \sum_{q=1}^{h} \hat{\omega}_{q0} \Phi'_0(z_q) \hat{\omega}_{jq} \tag{7}$$

where

$$z_0 = \hat{\alpha}_0 + \sum_{q=1}^{h} \hat{\omega}_{q0} \Phi_q(z_q), \quad z_q = \hat{\alpha}_q + \sum_{j=1}^{m} \hat{\omega}_{jq} x_{t-j\tau}$$

The partial derivatives are determined by

$$\partial \hat{G} = \begin{pmatrix} \frac{\partial \hat{v}}{\partial x_{t-\tau}} & \frac{\partial \hat{v}}{\partial x_{t-2\tau}} & \frac{\partial \hat{v}}{\partial x_{t-3\tau}} & \cdots & \frac{\partial \hat{v}}{\partial x_{t-(m-1)\tau}} & \frac{\partial \hat{v}}{\partial x_{t-m\tau}} \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix} \tag{8}$$

The kth Lyapunov exponent can be estimated as follows:

$$\hat{\lambda}_k = \lim_{M \to \infty} \frac{1}{M} \log \mu_k \left(\left| \partial \hat{G}^M \right| \right) \tag{9}$$

where μ_k is the kth largest eigenvalue provided by the global Jacobian $\partial \hat{G}^M = \partial \hat{G}(x_M) \cdot \partial \hat{G}(x_{M-1}) \cdot \ldots \cdot \partial \hat{G}(x_1)$ for $k = 1, 2, 3, \ldots, m$, where M is the block size, and $\partial \hat{G}(\cdot)$ are the estimated partial derivatives calculated from Equation (8). For a review of the criteria for the choice of block size see, e.g., [84–86].

Finally, regarding the asymptotic properties of the Lyapunov exponent estimator, the statistical test can be defined as follows [83]:

$$\hat{t}_k = \frac{\hat{\lambda}_k}{\sqrt{\hat{\varphi}_k / M}} \sim N(0, \hat{\varphi}_k) \tag{10}$$

Considering the ideas proposed by [87], we estimated the asymptotic variance (φ_k). We then calculated the standard error of the Lyapunov exponent estimator and used it to test its significance under the null hypothesis. For instance, at a significance level of 5%, if we obtained a p-value above this value, we would not reject the null hypothesis, which would mean that there would be no chaotic behaviour.

4. Results and Discussion

4.1. Analysing Correlations between Information Surprises and Market Movements in Europe, the United States, and the United Kingdom to Test the Efficient Market Hypothesis

In this section, we present the main results in order to test one of our main hypotheses regarding whether or not there is a relationship between information surprises in macroeconomic news and the different movements generated in each of the frequencies considered on the three currency pairs. That is, we wanted to analyse and contrast whether, when there is an information surprise (there is a difference between the expected and the actual data), there is a prolonged reaction in the currency market (movements of the EURUSD, EUGBP, and GBPUSD currency pairs in the following 15 min, 30 min, and 60 min) since the news is known.

Particularly, we estimated the correlation coefficients and the associated p-values for each news item and currency movement considered. These were six news items per economic region, for a total of three zones (Europe, the United States, and the United Kingdom), with respect to three movements of each currency pair, EURUSD, EURGBP, and GBPUSD. This gave us a total of 48 correlation coefficients and p-values per geographical area, and as there were three regions in our study, we had to estimate a total of 144 coefficients and p-values.

The following time series were taken into account when estimating these coefficients. As the macroeconomic news was monthly, we obtained a total of 189 observations in each of them, with the exception of GDP, which, being quarterly, had a total of 63 observations in each series. In the 1 min time series ($EURUSD$-Dif15m, $EURGBP$-Dif15m, and $GBPUSD$-Dif15m), there was a total of 549,204 observations. In the 30 min time series ($EURUSD$-Dif30m, $EURGBP$-Dif30m and $GBPUSD$-Dif30m), there was a total of 274,602 observations. Finally, in the 60 min time series ($EURUSD$-Dif60m, $EURGBP$-Dif60m and $GBPUSD$-Dif60m), there was a total of 137,301 observations in each one.

These differences in observations between the various currency movements and the macroeconomic news was not a problem, as the correlation was calculated individually for each news and currency movement, i.e., only on the exact dates and times at which the news was released. For each correlation analysis, we generated different subseries of data, as the information we worked with for each currency pair was different due to the difference in the time of publication on the macroeconomic calendar. We also focused on the correlations that were significant, that is, those in which we rejected the null hypothesis when the p-value was less than or equal to 5%, as our significance level α was 5%. The analysis was carried out by economic region.

The data shown in Table 3 provide the following comments: First, if we consider the macroeconomic news from Europe, the EUR_Unemp in $EURUSD$-Dif15m stands out, where the p-value is 0.0035. This correlation is 0.0036; therefore, although significant, it is practically zero. All of the other coefficients are not significant.

Table 3. Correlation coefficients for each news considered and currency movements for the three economic regions are shown from June 2007 to September 2022. p-values are written in parentheses. We marked in bold the correlation coefficients with values statistically different from 0. Those that are statistically significant at the 95% confidence level are marked with an asterisk *.

	***EUR*_Trade**	***EUR*_Unemp**	***EUR*_GDP**	***EUR*_Sales**	***EUR*_IPI**	***EUR*_CPI**
$EURUSD$-Dif15m	0.0583	**0.0036**	−0.0747	0.0608	−0.0295	0.0639
	(0.4810)	(0.0035) *	(0.7010)	(0.4147)	(0.7020)	(0.3884)
$EURUSD$-Dif30m	0.0226	0.009	0.0601	0.0422	−0.073	0.0091
	(0.7757)	(0.9020)	(0.5227)	(0.5688)	(0.3657)	(0.9105)
$EURUSD$-Dif60m	−0.0204	0.0414	−0.0114	0.0503	−0.0823	0.0231
	(0.7984)	(0.5731)	(0.8719)	(0.5029)	(0.3221)	(0.7578)
$EURGBP$-Dif15m	−0.0495	0.029	−0.153	−0.0122	0.0163	0.037
	(0.5053)	(0.6930)	(0.2598)	(0.8705)	(0.8316)	(0.6189)
$EURGBP$-Dif30m	−0.0303	0.0245	0.1069	−0.0347	−0.1083	0.0348
	(0.8478)	(0.7386)	(0.6882)	(0.6444)	(0.1263)	(0.6419)
$EURGBP$-Dif60m	−0.0001	0.0322	0.192	−0.0633	−0.1151	0.0791
	(0.8815)	(0.6609)	(0.1792)	(0.3932)	(0.1089)	(0.2843)
$GBPUSD$-Dif15m	0.0886	−0.0267	0.0845	0.051	−0.0696	0.0024
	(0.2329)	(0.7159)	(0.4175)	(0.4954)	(0.3588)	(0.9758)
$GBPUSD$-Dif30m	0.0169	−0.0178	−0.0088	0.0705	0.0022	−0.0408
	(0.8202)	(0.8084)	(0.6403)	(0.3427)	(0.8853)	(0.5780)
$GBPUSD$-Dif60m	−0.0286	0.007	−0.1698	0.0899	0.0267	−0.0568
	(0.7003)	(0.9235)	(0.2063)	(0.2263)	(0.6298)	(0.4396)

Table 3. *Cont.*

	USD_Trade	USD_Unemp	USD_GDP	USD_Sales	USD_IPI	USD_CPI
$EURUSD$-Dif15m	−0.0727	0.0856	**−0.3296**	−0.0584	**−0.1721**	**−0.2116**
	(0.3206)	(0.2417)	(0.0147) *	(0.4054)	(0.0162) *	(0.0037) *
$EURUSD$-Dif30m	−0.0937	0.0708	−0.1756	−0.1025	−0.0734	**−0.1981**
	(0.2026)	(0.3420)	(0.2495)	(0.1517)	(0.3131)	(0.0066) *
$EURUSD$-Dif60m	−0.0322	0.0441	−0.0975	−0.1377	0.0417	**−0.1867**
	(0.6644)	(0.5552)	(0.5661)	(0.0593)	(0.5716)	(0.0106) *
$EURGBP$-Dif15m	**−0.1473**	0.0645	−0.0349	**−0.1791**	−0.0522	0.0329
	(0.0443) *	(0.3837)	(0.8424)	(0.0136) *	(0.4812)	(0.6558)
$EURGBP$-Dif30m	**−0.1537**	0.0786	0.0531	−0.1048	−0.0484	0.0343
	(0.0358) *	(0.2847)	(0.5449)	(0.1542)	(0.5029)	(0.6418)
$EURGBP$-Dif60m	−0.1373	0.0896	0.0465	−0.1063	0.0594	0.0259
	(0.0615)	(0.2222)	(0.7343)	(0.1455)	(0.4237)	(0.7786)
$GBPUSD$-Dif15m	0.0142	0.0673	**−0.3388**	0.0262	−0.1547	**−0.2529**
	(0.8521)	(0.3567)	(0.0118) *	(0.7442)	(0.0298)	(0.0004) *
$GBPUSD$-Dif30m	0.0266	0.0394	−0.2462	−0.0632	−0.0369	**−0.2479**
	(0.7173)	(0.5899)	(0.0769)	(0.3681)	(0.6147)	(0.0006) *
$GBPUSD$-Dif60m	0.0815	−0.0014	−0.1396	−0.0981	−0.0129	**−0.2379**
	(0.2672)	(0.9711)	(0.3862)	(0.1811)	(0.8623)	(0.0010) *
	GBP_Trade	**GBP_Unemp**	**GBP_GDP**	**GBP_Sales**	**GBP_IPI**	**GBP_CPI**
$EURUSD$-Dif15m	0.0642	−0.0123	0.1059	−0.1052	−0.0834	**−0.2134**
	(0.3380)	(0.9517)	(0.4161)	(0.1521)	(0.2397)	(0.0032) *
$EURUSD$-Dif30m	0.0592	0.0783	0.0502	−0.0822	−0.0936	**−0.2615**
	(0.4355)	(0.2635)	(0.7039)	(0.2676)	(0.2075)	(0.0002) *
$EURUSD$-Dif60m	0.1414	0.0833	0.063	−0.0423	**−0.1498**	**−0.2149**
	(0.0937)	(0.2482)	(0.6039)	(0.5935)	(0.0508) *	(0.0025) *
$EURGBP$-Dif15m	−0.0039	**0.1882**	−0.1013	**−0.3641**	**−0.3202**	**0.3406**
	(0.9685)	(0.0136) *	(0.4649)	(0.0001) *	(0.0001) *	(0.0001) *
$EURGBP$-Dif30m	0.0165	**0.1975**	−0.0995	**−0.3289**	**−0.2632**	**0.2481**
	(0.8733)	(0.0084) *	(0.4701)	(0.0001) *	(0.0002) *	(0.0005) *
$EURGBP$-Dif60m	0.0529	0.137	**−0.2551**	**−0.2711**	**−0.2136**	**0.1559**
	(0.5671)	(0.0742)	(0.0473) *	(0.0001) *	(0.007) *	(0.0345) *
$GBPUSD$-Dif15m	0.0723	**−0.157**	0.1443	**0.3206**	**0.2517**	**−0.4474**
	(0.3044)	(0.0428) *	(0.2800)	(0.0001) *	(0.0005) *	(0.0001) *
$GBPUSD$-Dif30m	0.0534	−0.1246	0.1461	**0.2753**	**0.1822**	**−0.4101**
	(0.4486)	(0.1040)	(0.2796)	(0.0001) *	(0.0127) *	(0.0001) *
$GBPUSD$-Dif60m	0.0773	−0.0631	**0.2895**	**0.266**	0.0764	**−0.2895**
	(0.3406)	(0.4312)	(0.0218) *	(0.0002) *	(0.2741)	(0.0001) *

Second, taking into account the macroeconomic news from the United States, we obtained several significant correlation values. USD_trade has significant correlation with $EURGBP$-Dif15m and $EURGBP$-Dif30m. The correlation value we see is −0.1473 and −0.1537, respectively. The USD_GDP news has significant correlation with $EURUSD$-Dif15m and with $GBPUSD$-Dif15m. It is noteworthy that both correlations are at 15 min and that their correlation values are quite close. These are −0.3296 and −0.3388, respectively. USD_Sales reaches a significant correlation with the $EURGBP$-Dif15m currency movement, with this value being −0.1791. It is striking that this correlation is reached with a pair in which the country's currency is not found, although it is weak. The USD_IPI news has a significant correlation with $EURUSD$-Dif15m and $GBPUSD$ at 15 m again. These values are quite close to each other and to the previous correlation of USD_Sales and $EURGBP$-Dif15m. These values are −0.1721 and −0.1547, respectively. Lastly, in the USD_CPI news, we see several very significant correlations. For the movements of $EURUSD$-Dif15m, $EURUSD$-Dif30m, and $EURUSD$-Dif60m, the correlation values are -0.2116, -0.1981, and -0.1867, respectively. For the movements of $GBPUSD$-Dif15m, $GBPUSD$-Dif30m, and $GBPUSD$-Dif60m, these values are equal to -0.2529, -0.2479, and -0.2379.

Then, from a macroeconomic and global point of view, for the US, we see that all correlations are negative; this is due to the US dollar being the currency pair's denominator. This clarification is important, as the real correlation (and the one that makes macroeconomic sense) is a positive relationship with respect to the movement of that currency alone. The explanation for each news item where the correlation is significant is as follows:

- Trade balance: If the trade balance data are higher than expected, it will be a sign that a higher demand for US dollars is needed to pay for exports, which will increase the price of that currency.
- Gross Domestic Product: A higher than expected figure is a sign that the US economy is in better economic health, indicating a strengthening currency.
- Retail sales: Higher than expected retail sales indicate stronger domestic demand, a sign of greater confidence and economic strength in the country.
- Industrial Production Index: A higher than expected figure indicates stronger manufacturing strength in the country. This should be reflected in an appreciating and stronger currency.
- Consumer Price Index: Higher than expected inflation will indicate strong economic activity and the possible future need to raise interest rates to reduce inflation. Higher interest rates in the country will lead to higher foreign investment in the country, which will lead to higher demand for the currency, with consequent appreciation.

Third, if we consider the macroeconomic news from the United Kingdom, again, we obtain several significant correlation values. For instance, GBP_Unemp has significant correlation with $GBPUSD$-Dif15m, $EURGBP$-Dif15m, and $EURGBP$-30m. The values of these correlations are -0.157, 0.1882, and 0.1975. The explanation for the difference in sign of the correlations is due to the position of the pound in the various currency pairs, but this does not make these values economically meaningless. A higher unemployment rate will be a sign of a cooling economy, meaning the currency will tend to depreciate. Regarding GBP_GDP, we can identify two correlation values that are significant, $GBPUSD$-Dif60m and $EURGBP$-Dif60m. These values are 0.2895 and -0.2251. It is remarkable that both of these significant correlations are reached in 60 min (again, there was a difference in signs due to the position of the currency). From a macroeconomic point of view, the value of the correlations makes sense, because if the GDP is higher than expected, this will be a sign of economic strength and will imply an appreciation of the currency.

As far as the GBP_Sales news is concerned, we obtained significant correlations for all the movements in which the pound finds itself. The movements and their correlations are as follows: $GBPUSD$-Dif-15m with a correlation equal to 0.3206, $GBPUSD$-Dif30m with a correlation of 0.2753, and $GBPUSD$-Dif60m with a correlation of 0.266. Regarding the $EURGBP$ currency pair, the correlations are negative because the pound is in the denominator. Its correlations are $EURGBP$-Dif15m with a value of -0.3641, $EURGBP$-Dif30m with a coefficient equal to -0.3289, and $EURGBP$-Dif60m with a value of -0.2711. It is very interesting to see that the correlations decrease with increasing time in the currency movements. Again, the correlation values make sense from a macroeconomic point of view, as higher retail sales will mean higher demand for the currency.

According to the GBP_IPI news, significant correlations were found in $EURUSD$-Dif60m, which is surprising as this pair does not contain the currency in question. It should be noted that this correlation is not high, being -0.1498. We also found correlation in $GBPUSD$-Dif15m and $GBPUSD$-Dif30m, with values equal to 0.2517 and 0.1822. Additionally, correlations were found in $EURGBP$-Dif15m, $EURGBP$-Dif30m, and $EURGBP$-Dif60m, with coefficients equal to -0.3202, -0.2632, and -0.2136, respectively. These values are in line with the macroeconomic interpretation, as a stronger than expected development in the manufacturing sector will lead to an increase in the appreciation of the currency in question.

Finally, the results regarding the Consumer Price Index in the UK (GBP_CPI) are striking, as the correlation is significant for all of the values of all of the currency pairs. We found the following correlations: −0.2134, −0.26155, and −0.2149, respectively, for the $EURUSD$-Dif15m, $EURUSD$-Dif30m, and $EURUSD$-Dif60m values. The correlations for $GBPUSD$-Dif15m, $GBPUSD$-Dif30m, and $GBPUSD$-Dif60m are 0.3406, 0.2481, and 0.1559. In addition, the correlations for $EURGBP$-Dif15m, $EURGBP$-Dif30m, and $EURGBP$-Dif60m are −0.4474, −0.4101, and −0.2895, respectively. Note that for the currency pairs in which the pound is found, again, we observed that the correlation values decrease as the time of the movement increases. This correlation makes macroeconomic sense, and the interpretation is the same as that outlined above. If the rise in prices is higher than expected, this will generate an expectation of higher interest rates for the State, causing more foreign investment in the country, with the consequent appreciation of the currency.

To summarise, we showed that the FOREX market reacts under the Efficient Market Hypothesis, creating a significant variation in a short period of time (15, 30, or 60 min) in the price of the main currencies of the most important economic regions in the West (the United States, Europe, and the United Kingdom) depending on the actual deviation in the high-impact macroeconomic news communicated by these markets in relation to trade balance, unemployment rate, Growth Domestic Production, retail sales, the Industrial Production Index, and the Consumer Price Index. Our results are in line with those obtained by Égert and Kočenda [88] and Kočenda and Moravcová [89], as we see that certain macroeconomic news has a significant correlation with the movement of different currency pairs.

4.2. Analysing Possible Chaotic Behaviour by Estimating Lyapunov Exponents in the EURUSD, EURGBP, and GBPUSD Exchange Rates to Test the Paradox of Chaos

In this section, we report the main results regarding the second application of this paper. That is, we wanted to test the proposal related to the chaos model-data paradox in this context. The hypothesis is that chaos is difficult to detect in empirical financial studies due to the loss of information that occurs when using daily quotes. This may make it difficult to detect chaos in these time series, as explained above. Then, we wanted to prove whether, by taking into account all the information available in the FOREX market (full sample information on quotes) instead of daily data, we could find differences in relation to the detection of chaotic behaviour on the same series but with different time frequencies.

Specifically, we used the top three currency pairs from the FOREX market (EURUSD, EURGBP, nGBPUSD) considering daily, 1 h, 30 min, 15 min, and minute-by-minute frequencies. Thus, we analysed 15 financial series. Note that these series do not correspond to the ones we used previously as they were not generated from the publication of the macroeconomic news but from Equation (1), in which these are lagged returns, with lag equal to 1, 15, 30, 60, and 1440 min. In this case, we compared the results obtained by the Jacobian indirect method considering four different blocking methods: full sample ($\hat{\lambda}_F$), nonoverlapping ($\hat{\lambda}_N$), equally spaced ($\hat{\lambda}_E$), and bootstrap ($\hat{\lambda}_B$). The algorithms of those methods are implemented in the R package called *DChaos* developed by [90]. We considered the following set of parameters: $2 \leqslant h \leqslant 10$, $1 \leqslant \tau \leqslant 10$, and $3 \leqslant m \leqslant 10$. To obtain the results shown in Table 4, we estimated 720 different neural net models from the EURUSD, EURGBP, and GBPUSD returns time series for each time frequency considered over the last 15 years (10,800 regression models).

Table 4 provides the following comments. First, we only obtained positive (and significant) Lyapunov exponents for the high-frequency series (1 min, 5 min, and 30 min) from the three currency pairs. In the case of the daily series, in all cases, we obtained p-values greater than the significance level (5%), not rejecting the null hypothesis, and therefore not detecting chaotic behaviour.

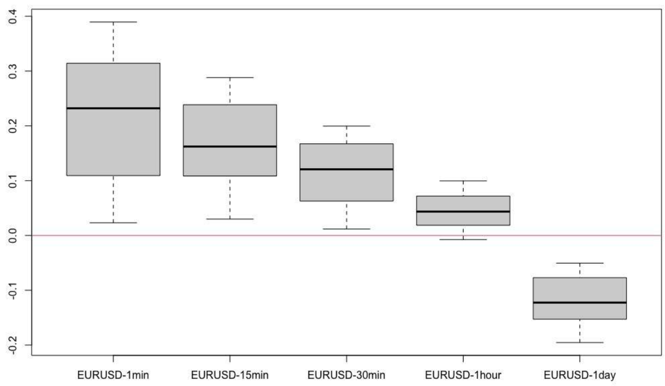
Table 4. Lyapunov exponents from best fit neural net model for each blocking method ($\hat{\lambda}_F$=full, $\hat{\lambda}_N$=nonoverlapping, $\hat{\lambda}_E$=equally spaced, and $\hat{\lambda}_B$=bootstrap) from the EURUSD, EURGBP, and GBPUSD returns time series for each time frequency considered over the last 15 years. Standard errors are given in parentheses, and the set of optimal parameters $[m, \tau, h]$ are indicated in brackets. We highlight Lyapunov exponents with positive values in bold and those that are statistically significant at the 99% confidence level with asterisks $*$. The median values of all blocks used for estimation based on blocking methods are presented. The block length M was chosen following [83].

	$[m,\tau,h]$	$\hat{\lambda}_F$	$\hat{\lambda}_N$	$\hat{\lambda}_E$	$\hat{\lambda}_B$
EURUSD-1m	[9,1,6]	**0.20437**	**0.21658**	**0.21571**	**0.23997**
		(5.8×10^{-3}) *	(6.1×10^{-3}) *	(6.1×10^{-3}) *	(6.4×10^{-3}) *
EURUSD-15m	[7,1,3]	**0.14892**	**0.15218**	**0.15844**	**0.18127**
		(0.5×10^{-3}) *	(0.7×10^{-3}) *	(0.8×10^{-3}) *	(1.0×10^{-3}) *
EURUSD-30m	[8,1,10]	**0.10881**	**0.11293**	**0.11477**	**0.12673**
		(2.7×10^{-3}) *	(3.0×10^{-3}) *	(3.0×10^{-3}) *	(3.2×10^{-3}) *
EURUSD-1h	[8,1,4]	**0.04167**	**0.04922**	**0.04471**	**0.05593**
		(1.0×10^{-2}) *	(0.8×10^{-2}) *	(0.8×10^{-2}) *	(0.5×10^{-2}) *
EURUSD-1d	[8,1,3]	−0.12621	−0.12993	−0.12618	−0.13057
		(0.2406)	(0.2481)	(0.2402)	(0.2467)
EURGBP-1m	[9,1,2]	**0.25176**	**0.26492**	**0.26381**	**0.28308**
		(7.4×10^{-3}) *	(7.8×10^{-3}) *	(7.8×10^{-3}) *	(8.1×10^{-3}) *
EURGBP-15m	[7,1,5]	**0.16933**	**0.17836**	**0.17391**	**0.19923**
		(4.6×10^{-3}) *	(4.8×10^{-3}) *	(4.9×10^{-3}) *	(4.9×10^{-3}) *
EURGBP-30m	[8,1,8]	**0.15002**	**0.15271**	**0.15883**	**0.16640**
		(3.5×10^{-3}) *	(3.6×10^{-3}) *	(3.6×10^{-3}) *	(3.7×10^{-3}) *
EURGBP-1h	[7,1,9]	**0.000974**	**0.00109**	**0.00101**	**0.00124**
		(0.0493)	(0.0410)	(0.0405)	(0.0421)
EURGBP-1d	[8,1,1]	−0.06211	−0.06455	−0.06372	−0.07530
		(0.1399)	(0.1468)	(0.1429)	(0.1582)
GBPUSD-1m	[7,1,6]	**0.13965**	**0.14663**	**0.14419**	**0.15117**
		(2.7×10^{-3}) *	(2.9×10^{-3}) *	(2.9×10^{-3}) *	(3.0×10^{-3}) *
GBPUSD-15m	[10,1,3]	**0.16005**	**0.17708**	**0.17391**	**0.18194**
		(3.0×10^{-3}) *	(3.4×10^{-3}) *	(3.4×10^{-3}) *	(3.6×10^{-3}) *
GBPUSD-30m	[8,1,9]	**0.12806**	**0.13507**	**0.13956**	**0.14987**
		(2.2×10^{-3}) *	(2.6×10^{-3}) *	(2.6×10^{-3}) *	(2.9×10^{-3}) *
GBPUSD-1h	[8,1,9]	−0.03056	−0.03702	−0.03301	−0.04197
		(0.0677)	(0.0759)	(0.0703)	(0.0882)
GBPUSD-1d	[9,1,5]	−0.15907	−0.16020	−0.16341	−0.17022
		(0.25919)	(0.26220)	(0.26608)	(0.2717)

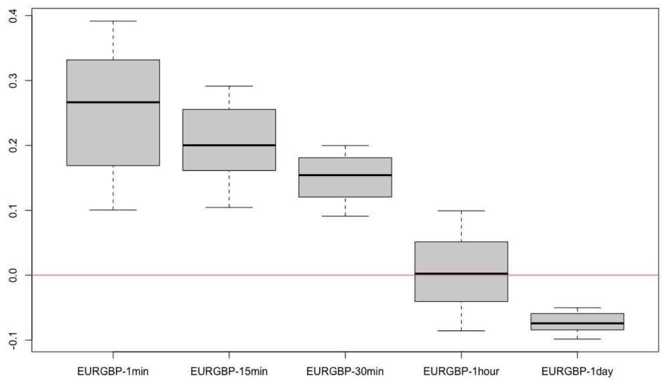
Second, as we added information at lower time frequencies (more aggregated), the estimated Lyapunov exponent tended to decrease, even becoming negative in the case of 1 h and, above all, daily frequencies for all three currencies. This issue would support the thesis that chaos is often avoided in empirical financial studies because of the loss of information that occurs when using daily quotes. This could make it difficult to detect chaos in these time series, as many authors argue; see, e.g., [43,44,46,47].

Third, we can observe similar behaviour between the three currency pairs, with some significant differences in the case of GBPUSD. Fourth, with respect to the optimal parameters of the best neural networks for all cases, we identified that the embedding dimension m is greater than or equal to 7 and that the time-delay τ is equal to 1, as suggested by [71]. Fifth, the results given using the blocking methods are better than those provided using the full sample.

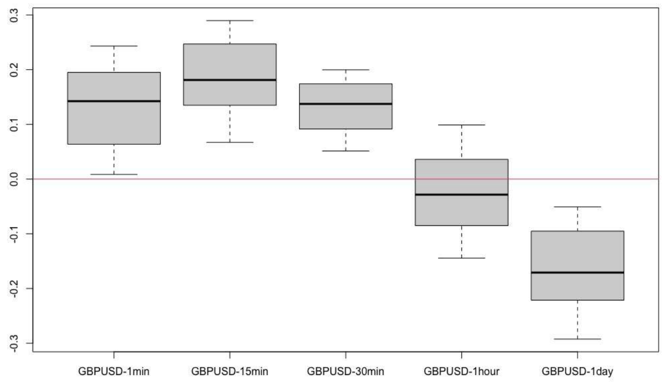
Finally, we show box plots in Figure 7 considering the largest Lyapunov exponents obtained from all the neural net models by each exchange rate and time frequency based on the bootstrap blocking method. As we can see, the mean value (bold line) of the Lyapunov exponents from 720 different neural net regressions were positives in all the high-frequency exchange rates (1 min, 15 min, and 30 min) for the three currency pairs considered; see Figure 7. We can also clearly observe that in all three cases for the daily series, the 1000 exponents that were estimated for each of the 720 neural networks considered were negatives. Regarding the series taken with a frequency equal to 1 h, we found certain differences depending on the type of currency exchange. What is common to all of them, as we also saw in the previous results, is that as we added the information at a lower time frequency (more aggregated), the average Lyapunov exponent tended to decrease, even becoming negative in the case of 1 hour and, above all, daily frequencies.



(**a**)



(**b**)



(**c**)

Figure 7. Results pertaining to the returns time series from (**a**) EURUSD, (**b**) EURGBP, and (**c**) GBPUSD for each time frequency considered over the last 15 years are shown for the bootstrap blocking method. The number of bootstrap iterations is $B = 1000$.

5. Conclusions

The main conclusions of this paper are as follows: On the one hand, we applied the correlation contrast by estimating Pearson's correlation coefficient between macroeconomic news information surprises and the movements of currency pairs at 15, 30, and 60 min. In this sense, we showed that the FOREX market reacts under the Efficient Market

Hypothesis, creating a significant variation in a short period of time (15, 30, or 60 min) to the quotation of the main currencies from the most important economic regions in the West (the United States, Europe, and the United Kingdom).

Then, we showed how the information surprises generated by different macroeconomic news have a different impact on short-term exchange rate movements, where "short term" is understood as the time it takes for the market to incorporate this new information into the foreign exchange market. It could be interesting to extend this study to other eastern economic regions such as China or Russia. It would also be useful to make a minute-by-minute correlation analysis, with the aim of finding the maximum correlation point between the news and the movement of currencies. On the other hand, we empirically verified that if we consider all the information available in the FOREX market (or at least, more desegregated data) instead of daily data, and we apply a robust chaotic behaviour detection method, we can find differences in relation to the detection of chaos on the same series but with different temporal frequencies. That is, as we added information at lower time frequencies (more aggregated), the estimated Lyapunov exponent tended to decrease, even becoming negative in the case of 1 hour and, above all, daily frequencies for all three currencies. This finding supports the thesis that chaos is usually elusive in financial empirical studies because of the loss of information that occurs when daily quotes are used.

Note that we do not intend to generalise this finding to all financial series or even to all FOREX series. Our main interest was to illustrate that by choosing a high frequency (1 min, 15 min, 30 min, or 60 min) instead of a daily one with the purpose of preserving the dynamic dependence on information, we could find chaos, at least in the specific currency pairs analysed and during the time intervals considered.

Thus, we can confirm that behind the financial time series which show an apparently random irregular evolution, there would be a generating system which, although unknown in principle, would be deterministic (and nonlinear), and we could take advantage of that deterministic character to make predictions, even if only in the short term, understanding "short term" as the time it takes for the market to incorporate these information surprises in the FOREX market analysed. These results could open up a new line of research in which new contributions could be published, considering other periods, financial assets, and/or methods to estimate Lyapunov exponents.

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