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Coronavirus, Vaccination and the Reaction of Consumer Sentiment in The United States: Time Trends and Persistence Analysis

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Abstract: At the beginning of the COVID-19 pandemic, the entire world was waiting for a medical solution (for example, vaccines) in order to return to normality. Sanitary restrictions changed our consumption behaviors and feelings. Therefore, this paper analyzes the stochastic properties of consumer sentiment during the COVID-19 episode and the appearance of vaccines against the virus in December 2020 in the United States of America. This study adds a new dimension to the literature because it is the first research paper that uses advanced methodologies based on fractional integration and fractional cointegration analysis to understand the statistical properties of these time series and their behavior in the long term. The results using fractional integration methodologies exhibit a high degree of persistence, finding behavior of mean reversion during the pandemic episode. Therefore, the shock duration in consumer sentiment will be transitory, recovering to its previous trend in the short run. Focusing on the cointegrating part, we arrive at two main conclusions. First, an increase in total vaccination produces a positive reaction or impact on the behavior of consumers. On the other hand, an increase in new COVID-19 cases negatively affects the behavior of the consumer.

Keywords: consumer sentiment; COVID-19; vaccination; mean reversion; persistence; fractional integration; FCVAR model

MSC: 62M10; 91B70; 62P20

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1. Introduction

According to [1], routines and task failure are threatened by the uncertainty generated by an unexpected event or crisis. Such events can severely impact people's consumption patterns and routines [2].

According to [3], the need to control or override thoughts, emotions, impulses and behavior increases consumption levels.

There are many research papers that have studied consumption under the assumption of economic and financial crises ([4,5], among others), where consumers adapt their consumption practices in a variety of ways (see [4,6–10], among others).

Other types of crises that have been studied less, such as those related to war or natural disasters, show us compulsive and impulsive behavior regarding consumption (see [4,11–13], among others).

On 11 March 2020, the World Health Organization (WHO) officially declared the coronavirus (COVID-19) outbreak as a global pandemic. According to [14], the unexpected behavior of the virus affected 137 million individuals in 221 countries. This fact caused strong doubts surrounding its extent and its implications on the global economy.

The sentiment in this article is intended to expose the views of economic agents on future economic developments that may influence the economy because they influence the decisions of agents today (see [15]).

Some scientific articles deal with how the pandemic has affected different aspects of consumption. Ref. [16] maintain that the consumer has a lower perception of threat when using food delivery services. Ref. [17] argue that the measures taken by governments, together with the news and advertisements in the media, caused consumers to buy in a panic due to the fear of shortages. Ref. [18] states that the personal and behavioral processes of consumers changed during the pandemic.

Other research papers analyzed how COVID-19 impacted the general economy and consumer and business sentiment (see [19–22], among others).

Ref. [22] stated that the pandemic and the induced restrictions due to this event drastically affected the economic sentiment of households in Europe, being negative for consumption and the market.

On the other hand, ref. [19] stated changes in customers' spending, specifically in the sectors of goods and services directly affected by COVID-19 pandemic-induced restrictions.

Ref. [23] indicated an unprecedented behavior in the economic uncertainty and subjective uncertainty in business-expectation surveys due to the COVID-19 pandemic.

Sentiment and fear are complementary measures of risk aversion that are linked to uncertainty. For this reason, as [24] indicated, understanding the feeling itself is very important. Sentiment indicators are part of the leading indicators used to know in advance or predict financial and economic trends (see [25–32], among others).

After reviewing the existing literature, this research paper adds a new dimension to the scant literature on time trends and persistence. This is the first research paper that uses advanced methodologies based on fractional integration and fractional cointegration analysis to understand the statistical properties of these time series and their behavior in the long term.

To the best of our knowledge, this paper tries to answer two questions from fractional methodologies. First, how has COVID-19 impacted consumer sentiment? Second, how is consumer sentiment affected by the appearance of vaccines? We consider that these questions are very important in understanding expected consumer behavior and the subsequent effects. See, for example, the research paper by [33], which suggests that commodity prices increased due to high demand after COVID.

The structure of this paper is as follows. Section 2 describes the data used for our study. Section 3 explains the methodologies used to carry out the research. The results are discussed in Section 4. Finally, the conclusion can be found in Section 5.

2. Data

The database related to COVID-19 cases and the total vaccination numbers have been obtained from Ritchie et al. (2020) (https://ourworldindata.org/coronavirus, accessed on 24 September 2022), which is published and managed by researchers at the Blavatnik School of Government at the University of Oxford. To measure consumer sentiment, we use the index provided by University of Michigan through the Federal Reserve Bank of Sant Louis (https://fred.stlouisfed.org/series/UMCSENT, accessed on 24 September 2022) that is displayed in Figure 1.

As we can observe from Figure 1, consumer sentiment has dropped about 30% since the start of the pandemic, and the trend has not recovered even with the appearance of vaccines.

For our analysis, we use monthly frequency data. For the variable 'new cases of COVID-19', we take into account the dates of January 2020 to the present. The month of December 2020 is when the vaccines against COVID-19 began to be administered. Finally, both dates are taken into account for the study of consumer sentiment.

Table 1 reports some descriptive statistics for the series under examination. It can be seen that new cases of COVID-19 per month were 102.653 during the period analyzed, and the standard deviation was 131.547.

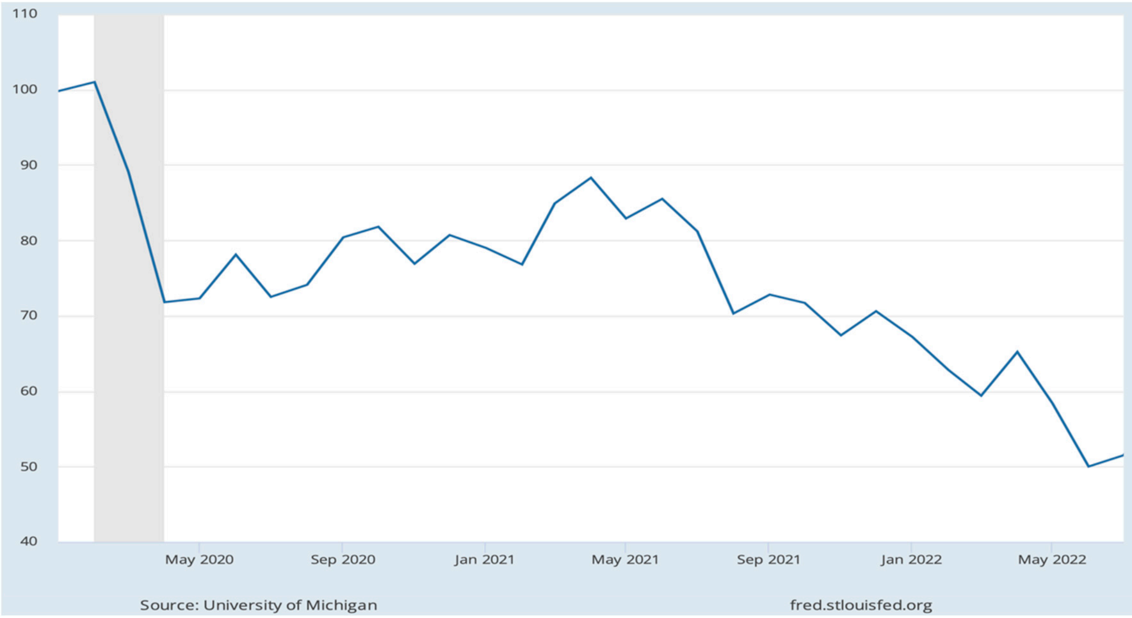

Figure 1. Consumer Sentiment.

Table 1. Descriptive statistics.

Variable	N	Date	Max	Min	Mean	Std. Dev.
COVID-19 New Cases	31	January 2020–July 2022	536.642	2	102.653097	131.546492
Consumer Sentiment	31	January 2020–July 2022	101	50	74.98	11.91
Total Vaccination (variation)	20	December 2020–July 2022	0.85	0	0.34	0.26

From the Consumer Sentiment time series, we observe that from June 2020 to July 2022, the average value is around 74.98, having a monthly deviation of 11.91 during this period. We also analyzed for this time series the period from December 2020 to July 2022, which was when the vaccines appeared. We notice that the mean (71.33) and the standard deviation (11.20) drop slightly.

Finally, we observe that the total vaccination (variation) has an average behavior of 0.34 and presents a low volatility.

3. Methodology

3.1. Unit Roots

Unit roots can be tested in many different ways. For this research, we use an ADF test based on [34]. Ref. [35] method has been considered because it is a non-parametric methodology that has greater calculation power. Also, considering deterministic trends, we use the methodology based on [36,37], producing essentially the same results.

3.2. ARFIMA (p, d, q) Model

Following authors such as [38–40] and others, it is now an established fact that all unit root methods have very low power if the true data-generating process displays long memory or if it is fractionally integrated. Thus, in what follows, fractional orders of differentiation are allowed.

This methodology is important because, until the 1980s, the standard approach was to apply deterministic functions of time where the residuals on the regression model were I(0) stationary. After research conducted by [41], the consensus about the non-stationary component of most series was stochastic, and the use of unit roots of first differences I(1) was considered to be the most appropriate way to proceed. On the other hand, to achieve stationary I(0), the number of differences does not necessarily have to be an integer value since it can be any point on the real line and, therefore, fractional I(d).

For this reason, we use the ARFIMA (p, d, q) model where the mathematical notation is:

$$(1 - L)^d x_t = u_t,\ t = 1,\ 2, \ldots, \tag{1}$$

In Equation (1), x_t refers to the time series that has an integrated process of order d $(x_t \approx I(d))$, d refers to any real value, L is the lag operator $(Lx_t = x_{t-1})$, and u_t is the covariance stationary process where the spectral density function, which is positive and finite at zero frequency, displays a type of time dependence in weak form. For this reason, we can state if u_t is ARMA $(p,\ q)$, x_t is ARFIMA $(p,\ d,\ q)$. The Akaike information criterion ([42]) and Bayesian information criterion ([43]) were used to select the appropriate AR and MA orders in the models.

The d parameter has been estimated considering all combinations of AR and MA terms $(p;\ q\ \leq\ 2)$ for the time series and for the subsamples taking into account their confidence bands at 95%.

Given the parameterization in (1), we can differentiate between various cases depending on the value of the parameter d, and several specifications based on (1) can be noted. If $d > 0$ in (1), then x_t is said to be a long-memory process since the autocorrelations decay hyperbolically, and the higher the value of d, the slower the rate of decay. Note that allowing d to be any real value enables one to consider a wide range of cases, including short memory $(d = 0)$, stationary long memory $(0 < d < 0.5)$; non-stationary mean reverting processes $(0.5 \leq d < 1)$; unit roots $(d = 1)$ or even explosive pattern $(d > 1)$ (see [44]).

3.3. FCVAR Model

The Fractionally Cointegrated Vector AutoRegressive (FCVAR) model is a generalization of [45] Cointegrated Vector AutoRegressive (CVAR) model to allow for a fractional process of order d that cointegrates to order $d - b$ with $b > 0$.

Following [46], we use their multivariate Fractional Cointegrated VAR (FCVAR) model to check the relationship of the variables in the long term. The advantage of this model is the ability to use stationary and non-stationary time series. The FCVAR model is notated in the next equation:

$$\Delta^d X_t = \alpha\beta' L_b \Delta^{d-b} X_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b^i Y_t + \varepsilon_t \tag{2}$$

where zero mean and the p-dimensional independent and identically distributed variance-covariance matrix (Ω) is defined by the term ε_t; The terms α and β are $p \times r$ matrices where $0 \leq r \leq p$. The long-term equilibria between variables are represented by β. The short-term behavior of the variables is represented by the parameter Γ_i. The parameter α suggests the deviations from the equilibria and the speed of the adjustment. Y_t refers to a p-dimensional I(1) time series.

4. Empirical Results

Section 4 shows the results of applying the previously explained methodologies.

To assess the statistical properties of the time series to achieve robust outcomes, we start the analysis by conducting the Augmented Dickey–Fuller test ([34]), Phillips and Perron ([35]), and Kwiatkowski et al. ([37]) to study the stationarity of our dataset.

Table 2 displays the results, which suggest that the cases of COVID-19 present a non-stationary behavior I(1), while vaccination and consumer behavior present a different behavior—stationary I(0).

Table 2. Unit root tests.

	ADF			PP		KPSS	
	(i)	(ii)	(iii)	(ii)	(iii)	(ii)	(iii)
Original Data							
COVID-19 new cases	−2.7215 *	−4.168 *	−4.4454 *	−3.1068 *	−3.1336	0.2878 *	0.0543
Consumer Sentiment Index	−1.6817	−2.0433	−2.8667	−1.4709	−2.2411	0.7736	0.1473
Total Vaccination	−0.3231	−3.3669 *	−2.7304	−3.2679 *	−0.6459	0.7334	0.1808

*(i) No deterministic components; (ii) intercept, (iii) linear time trend. * Statistic significant at the 5% level.*

Following the results obtained in Table 2 and due to the lower power of the unit root methods under fractional alternatives, we also employed ARFIMA (p, d, q) models to study the persistence of the COVID-19 new cases, consumer sentiment and total vaccination.

Table 3 presents the results for the ARFIMA (p, d, q) model for each time series, following the maximum likelihood estimator proposed by [47] to get the fractional parameter d and considering all combinations of $p,\ q \leq 2$ for the AR (autoregressive) and MA (moving-average) terms for the ARFIMA $(p,\ d,\ q)$ model.

Table 3. Results of long memory tests.

Data Analyzed	Sample Size (Month)	Model Selected	d	Std. Error	Interval	I(d)
Original Time Series						
COVID-19 new cases	31	ARFIMA (2, d, 2)	0.41	0.354	[−0.17, 1.00]	I(0), I(1)
Consumer Sentiment Index	31	ARFIMA (0, d, 0)	0.86	0.190	[0.55, 1.17]	I(1)
Total Vaccination	20	ARFIMA (0, d, 0)	1.43	0.087	[1.29, 1.58]	I(1)

From Table 3, we observe that the estimates of the differencing parameter d are lower than 1 $(d < 1)$ in COVID-19 new cases and in the consumer sentiment index. Only in this last case do we observe a high degree of persistence with all values in the confidence bands in the interval $[0.5,\ 1)$ and showing non-stationary though mean-reverting behavior.

Also, we observe that the parameter d in each of these time series is 0.41 and 0.86, respectively. These results suggest mean reversion. Thus, shocks are expected to be transitory. Observing the interval for the new COVID-19 cases, we cannot reject the I(0) hypothesis, nor the I(1), where the shock is expected to be permanent, causing a change in trend. For the case of the Consumer Sentiment Index, again, we cannot reject hypothesis I(1), where the shock is expected to be permanent.

On the other hand, the total number of vaccines has a behavior of non-reversion to the mean $(d = 1.43)$, because once you have been injected with a vaccine dose, this number cannot tend to zero, as it represents a measure to deal with the pandemic shock.

Next, we use the FCVAR model to study the possible existence of persistence in the long-run co-movement of the series. Table 4 summarizes the results of the FCVAR model.

Once we have the results from the FCVAR model, we are going to focus on: the integrating and cointegrating part $(d \neq b)$ and the beta term to analyze the behavior of the time series.

In Panel I, we observe that the order of integration of the individual series is lower than 1 $(d < 1)$, obtaining the same magnitude in the reduction in the degree of integration in the cointegrating regression. This result implies I(0) cointegrating errors $(d - b = 0)$. Therefore, with the results of our cointegration analysis, we do not rule out the hypothesis that the effects of the shock disappear in the short run. Focusing on Panel II, where we use cointegration analysis to analyze the long-term effect of new cases of COVID-19 on consumer sentiment, we observe that the order of integration of the individual series is about $d = 1.401$, while the reduction in the degree of integration in the cointegrating regression is $b = 1.134$, implying an order of integration of about $d - b = 0.267$ for the cointegrating relationship. Thus, we conclude that the long-term relationship between time series follows a long memory process. Analyzing the values that we get from the FCVAR model, we conclude that the duration of the shock is long-lived.

Table 4. Results of the FCVAR model.

	$d \neq b$	Cointegrating Equation Beta	
		Var1	**Var2**
Panel I: Total Vaccination (Var1) on Consumer Sentiment (Var2)	$d = 0.014\ (0.248)$ $b = 0.014\ (0.000)$	1.000	1.778
	$\Delta^d\left(\begin{bmatrix} Var.\ Vacc. \\ C.S. \end{bmatrix} - \begin{bmatrix} -0.960 \\ 76.460 \end{bmatrix}\right) = L_d \begin{bmatrix} 203834.760 \\ 3937.342 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta^d L_d^i (X_t - \mu) + \varepsilon_t$		
Panel II: COVID-19 new cases variation (Var1) on Consumer Sentiment (Var2)	$d = 1.401\ (0.232)$ $b = 1.134\ (0.218)$	1.000	−13.237
	$\Delta^d\left(\begin{bmatrix} New\ Cases \\ C.S. \end{bmatrix} - \begin{bmatrix} 376.532 \\ 100.357 \end{bmatrix}\right) = L_d \begin{bmatrix} -1.630 \\ -0.007 \end{bmatrix} \nu_t + \sum_{i=1}^{2} \hat{\Gamma}_i \Delta^d L_d^i (X_t - \mu) + \varepsilon_t$		

Additionally, according to the cointegrating equation beta from Table 4, we observe that an increase of one unit in the percentage of the variation of the total number of vaccines against COVID-19 corresponds to an increase of 1.778 in the consumer sentiment index ($\beta = 1.778$). Focusing on the cointegrating equation beta of Panel II, we conclude that the increase in new cases of COVID-19 reduces the consumer sentiment index by a -13.237 ($\beta = -13.237$).

5. Concluding Comments

Since the beginning of 2020, Western economies have been living with a deadly virus called COVID-19 that brought health restrictions imposed by all governments. This crisis brought about a change in routines and the interruption of many tasks that could not be addressed digitally, impacting people's consumption patterns (see [2]) and also impacting consumer sentiment.

For this reason, this research article analyzes the behavior of consumer sentiment during COVID-19 and vaccination using methodologies based on fractional integration and cointegration.

The results of the long memory tests show that there is a high degree of persistence in the Consumer Sentiment Index ($d = 0.86$). This result suggests mean reversion. Therefore, the COVID-19 shock is expected to be transitory for consumer sentiment.

Once we completed the univariate analysis, we focused on the FCVAR model. We observed that an increase of one unit in the percentage of the variation of the total number of vaccines against COVID-19 corresponds to an increase of 1.778 in the consumer sentiment index ($\beta = 1.778$). Also, we conclude that the increase in new cases of COVID-19 reduces the consumer sentiment index by a -13.237 ($\beta = -13.237$).

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