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Modelling profitability of private equity: A fractional integration approach[☆]

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ABSTRACT

This paper analyses the stochastic behaviour of private equity returns (a measure of profitability) applying fractional integration to an extensive dataset including quarterly data spanning the last four decades for various geographical areas (US, Europe, Asia/Pacific, the Rest of the World and Total) and investment types (Buyout & Growth Equity, Venture Capital & Fund of Funds, Infrastructure, Natural Resources, Real Estate, Subordinated Capital & Distressed as well as the aggregate category All Types). The results support the hypothesis of stationarity and mean reversion in all cases; however, there are differences in the degree of persistence across regions, the series for Europe being the closest to a short-memory process, while those for the US exhibit long memory, which implies that shocks have long-lived effects. Differences are also found in the results by asset class. The implications of these findings for private equity management, profit smoothing and return benchmarking are briefly discussed.

1. Introduction

Private equity is an alternative investment class consisting of capital that is not listed on a public exchange. As explained by Kaplan et al. (2005), a private equity firm serves as the 'general partner" (GP) managing a fund which is endowed by the 'limited partners' (LPs). The LPs (mainly institutional investors and wealthy individuals) commit to provide a certain amount of capital to the fund. The GP then has an agreed time period in which to invest (around 5 years) and in which to return capital to the LPs (usually around 10–12 years). Each fund or limited partnership, therefore, is essentially a closed end fund with a finite life. Their illiquidity and the fact that actual profitability is only disclosed at the end of the fund's life raises the issue of how to measure the performance of private equity in terms of its profitability (and also how to define an appropriate benchmark to assess whether it yields larger risk adjusted average returns than traded securities). A standard profitability measure often used to assess profitability relative to costs and expenses is return on assets (ROA), which is defined as net profit or net income (after all costs, expenses and taxes) divided by total assets.

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Various papers have in fact provided evidence on the profitability of private equity vis-à-vis different benchmarks (see, e.g., Phalippou and Gottschalg, 2009, and Cochrane, 2005). However, none of them have examined the stochastic properties of private equity returns for a wide range of countries and investment types and using a general empirical framework that encapsulates a variety of possible stochastic behaviours. The present study aims to contribute to the existing literature in those two respects. More specifically, it analyses data for four specific areas (US, Europe, Asia/Pacific and Rest of the World) as well as the 'Total', and for seven investment types (Buyout & Growth Equity, Venture Capital, Fund of Funds & Secondary Funds, Infrastructure, Natural Resources, Real Estate, Subordinated Capital & Distressed), as well as the aggregate category 'All Types'. For this purpose, fractional integration methods are used that are more general than those based on standard unit root tests (Dickey and Fuller, ADF, 1979; Phillips and Perron, 1988; etc.); in particular, the differencing parameter is allowed to take any real value, including fractional ones. Following this approach, evidence can be obtained on whether mean reversion occurs (which requires the fractional parameter to be significantly smaller than 1), whether despite being mean-reverting the series of interest is non-stationary (which is the case if the differencing parameter is in the range [0.5, 1)), whether the effects of shocks are transitory or permanent, how fast the adjustment towards equilibrium is, how persistent the series is, and other stochastic properties characterising it.

The structure of the paper is as follows: Section 2 briefly reviews the relevant literature; Section 3 outlines the methodology; Section 4 describes the data and presents the empirical results; Section 5 offers some concluding remarks.

2. Literature review

The literature on private equity (PE) examines different issues such as profitability (i.e. performance against a benchmark, which is the focus of the present study), key factors to select companies to invest in, valuation of PE funds, and interaction with limited partners (LP).

The first type of studies have provided estimates of profitability ranging from − 6% (Phalippou and Gottschalg, 2009) to + 32% (Cochrane, 2005). As for benchmarks, Steger (2017) argues that, since a lot of PE funds invest in small- or mid-sized companies, it is appropriate to use a small-cap stock market index such as the Russell 2000 Index.[2]

Furthermore, several measures have been employed for returns, namely: IRR (Internal Rate of Return); [3] TVPI (Total Value to Paid-In, or 'Money Multiple'), which is defined by Phalippou and Gottschalg (2009) as the sum of all cash distributions plus the latest Net Assets Value (NAV) (which is a proxy for future cash flows), divided by the sum of all drawdowns; PME (Public Market Equivalent), which is calculated in Kaplan and Schoar (2005) as the sum of all discounted cash outflows over the sum of the discounted cash inflows, where the discount rate is the total return on the S&P 500 Index.[4]

Roggi et al. (2019) classify PE funds in different categories - Venture Capital (VC), Leveraged Buyout (BO), Growth Equity, Mezzanine Financing, and Distressed Buyout Funds typically invest in early-stage companies with negative cash flows and high growth potential, while BO Funds use high levels of debt to take controlling interest in mature companies, in order to create value improving efficiency and realizing opportunities**.**

Gompers and Lerner (2000) coined the term 'money-chasing deals' to emphasise that the flow of funds in the private equity industry is the most important factor driving valuation (see also Gohil and Vyas, 2016). Gompers et al. (2005) studied the organizational structure and performance of various venture capital funds. They found a strong positive relationship between the degree of specialization by individual venture capitalists at a firm and the firm's success. They attributed poor performance to inefficient allocation of funding across industries and poor selection of investments within industries. They also concluded that experienced funds outperform inexperienced ones, and that small and inexperienced funds are the main drivers of low performance in private equity funds. The other factors that drive returns are stock market movements, economic trends, vintage year, stage of funding, and the number of companies in the portfolio.

Kaplan et al. (2009) studied 50 venture capital (VC)-financed companies that went public and argued that, at the margin, investors in start-ups should place more weight on 'the horse' (the business) than on 'the jockey' (the management team). Gompers et al. (2010) instead took the view that they should lean on the 'jockey' after showing that entrepreneurs with a track record of success are much more likely to succeed than first-time entrepreneurs and those who have previously failed. Moreover, Gompers et al. (2021) found that general partners (GPs) believe that the strength of the founding team is more important than the start-up's strategy or business model.

Phalippou and Gottschalg (2003) analysed funds that typically have a life of ten years, which can be extended to thirteen, self-report quarterly a Net Asset Value that reflects the value of on-going investments, and basically are non-tradable. They also pointed out that two different assumptions have been made concerning the treatment of final NAVs. The first and most frequent one treats them as a cash inflow of the same amount at the end of the sample time period - that is, NAVs are assumed to be an unbiased measure of the market value of a fund (e.g. Kaplan and Schoar, 2005). The second one only computes cash flows (e.g., Ljungqvist and Richardson, 2003), and is applicable to 'mature' funds and to follow up 'on-going' funds (the median IRR takes 8 years to turn

[2] The Russell 2000 Index is a small-cap stock market index based on the smallest 2000 stocks in the Russell 3000 Index. It is by far the most common benchmark for mutual funds that identify themselves as "small-cap", while the S&P 500 index is used primarily for large capitalization stocks.

[3] The internal rate of return on an investment or project is the "annualized effective compounded return rate" or rate of return that sets the net present value of all cash flows (both positive and negative) from the investment equal to zero. Its formula is: $0 = \mathrm{NPV} = \sum_{t=1}^{T} \frac{C_t}{(1+IRR)^t} - C_0$.

[4] Based on the VC, PE and M&A database, PitchBook, 'Public Market Equivalent' is a metric designed to compare private capital fund performance to public indices. Essentially, the metric adapts public market returns into an IRR-like metric that accounts for irregular and fluctuating cash flows.

positive).

Kaplan and Schoar (2005) concluded that better performing partnerships are more likely to raise follow-on funds and larger funds. Top performing partnerships grow proportionally less than average performers, which is consistent with a concave relationship between fund size and performance. While larger funds have higher PMEs, when their size increases further, performance declines. However, the relationship between fund performance and the sequence number of the fund is convex, although not significantly. Roggi et al. (2019) obtained similar results: there is no fast increase in fund performance until a certain fund sequence is reached, and the marginal increase in the sequence number seems to produce positive effects on fund IRR and PME, even at a high sequence number (i.e., from a 4–5 sequence).

As for the behaviour of PE profits over time, Ang et al. (2018) noted that limited data availability makes it particularly difficult to evaluate their time series properties; however, it appears that the cycles of PE returns for separate classes of Venture Capital, Buyout, and Real Estate are not highly correlated. This suggests that a diversified strategy across sub-asset classes of PE may be beneficial. Moreover, PE returns exhibits negligible serial dependence, in contrast to industry indices. This result is consistent with the smoothing induced by a conservative appraisal process or by a delayed and partial adjustment to market prices, which often arises in illiquid asset markets (see, e.g., Geltner, 1991, and Ross and Zisler, 1991). Kaplan and Schoar (2005) found persistence in private equity, especially in the case of Venture Capital funds. More recently, Harris et al. (2020) obtained similar results using a longer data span, and Bian et al. (2023) examined private equity valuation throughout time inconsistent preferences.

None of the studies discussed above examine the profitability of private equity using fractional integration methods and/or for a wide range of geographical areas/investment types. By contrast, the present paper adopts such a framework to analyse a comprehensive dataset of PE returns and to establish if exogenous shocks to these series have transitory or permanent effects.

3. Methodology

The standard approach widely used to determine if a series is stationary I(0) (and thus shocks have only temporary effects) or nonstationary I(1) (in which case shocks have permanent effects) is based on unit root tests such as ADF (Dickey and Fuller, 1979), Phillips and Perron (PP, 1988), Kwiatkowski et al. (KPSS, 1992), Elliot et al. (ERS et al., 1996), Ng and Perron (NP, 2001), etc. However, such tests are known to have extremely low power if the true Data Generating Process (DGP) is in fact fractionally integrated (see Diebold and Rudebusch (1991), Hassler and Wolters, 1994 and Lee and Schmidt, 1996). Therefore, in this paper we use an I(d) modelling framework of the following form:

$$(1-B)^d x_t = u_t, t = 1, 2, \ldots, \quad (1)$$

where B stands for a backshift function (i.e., $Bx_t = x_{t-1}$) and u_t is an I(0) process, which is defined as a covariance- or second-order stationary process with a finite sum of its autocovariances. Therefore, u_t can be a white noise process but also have a weak autocorrelation (ARMA) structure. Note that the polynomial on the left-hand side of (1) can be expressed in terms of a Binomial expansion such that

$$(1-B)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j B^j = 1 - dB + \frac{d(d-1)}{2} B^2 - \ldots$$

and thus Eq. (1) can be expressed as

$$x_t = dx_{t-1} - \frac{d(d-1)}{2} x_{t-2} + \ldots + u_t.$$

In this context, if d is a non-integer value, x_t will be a function of all its past history, and the higher the value of d is, the higher is the degree of dependence between the observations. Moreover, if $d > 0$, the series exhibits long memory and mean reversion takes place as long as d is smaller than 1, while stationarity holds if $d < 0.5$.

Allowing d to be any real value allows us to consider a wide range of processes, including:

i) short memory, if $d = 0$,
ii) long-memory stationary behaviour, if $0 < d < 0.5$,
iii) nonstationary though mean reverting processes, if $0.5 \leq d < 1$,
iv) unit roots, if $d = 1$, and
v) explosive patterns, if $d \geq 1$.

The unit root case is therefore a special one when $d = 1$, and several authors such as Diebold and Rudebush (1991), Hassler and Wolters (1994) and others have shown the limited power of unit root tests in the presence of fractional integration.

The estimation of the fractional differencing parameter d is carried out in this study using the Whittle function in the frequency domain as in the test developed by Robinson (1994), which has been shown to be the most efficient method in the Pitman sense against local departures from the null. The specific functional form used here is similar to the one in Gil-Alana and Robinson (1997).

Table 1
Starting dates and maximum and minimum IRRs.

Area	Type	Starting date	Sample size	max_IRR	Date for max IRR	min_IRR	Date for min IRR
USA	VCAP	Q2–81	162	84.72%	Q4–99	-20.16%	Q4–00
	B&GE	Q2–86	142	19.51%	Q4–99	-16.69%	Q4–08
	SC&D	Q4–86	140	14.20%	Q2–09	-18.89%	Q4–08
	NRES	Q4–87	136	19.79%	Q3–97	-22.99%	Q1–20
	RSTATE	Q4–86	140	15.43%	Q4–06	-19.65%	Q4–08
	INFR	Q2–07	58	8.47%	Q4–10	-7.04%	Q4–08
	FoF	Q4–88	132	49.94%	Q4–99	-13.62%	Q4–08
	All Types	Q2–81	162	37.82%	Q4–99	-16.18%	Q4–08
EUROPE	VCAP	Q2–89	130	730.58%	Q4–89	-36.46%	Q3–89
	B&GE	Q3–88	135	25.38%	Q4–96	-24.22%	Q4–08
	SC&D	Q4–94	108	31.52%	Q4–99	-13.65%	Q1–00
	NRES	Q4–03	72	391.93%	Q2–04	-55.47%	Q4–18
	RSTATE	Q1–00	87	22.32%	Q4–05	-28.87%	Q4–08
	INFR	Q3–05	65	44.56%	Q3–05	-16.42%	Q3–08
	FoF	Q3–98	93	18.41%	Q4–06	-18.74%	Q4–08
	All Types	Q4–87	136	98.66%	Q4–99	-22.75%	Q4–08
ASIA/PACIFIC	VCAP	Q3–89	129	34.68%	Q4–14	-16.52%	Q3–01
	B&GE	Q4–89	128	15.88%	Q4–98	-17.66%	Q4–08
	SC&D	Q4–02	76	24.00%	Q4–02	-17.33%	Q4–08
	RSTATE	Q4–03	72	27.13%	Q4–04	-19.68%	Q4–08
	INFR	Q3–98	93	30.07%	Q4–99	-12.58%	Q3–11
	FoF	Q2–03	74	23.97%	Q4–03	-13.54%	Q4–08
	All Types	Q3–89	129	18.20%	Q4–99	-17.27%	Q4–08
REST OF WORLD	VCAP	Q4–93	112	61.18%	Q2–00	-14.95%	Q3–06
	B&GE	Q2–92	118	21.79%	Q4–20	-18.57%	Q3–08
	SC&D	Q4–98	92	22.08%	Q4–04	-16.01%	Q4–08
	NRES	Q4–97	96	22.74%	Q3–04	-18.17%	Q3–20
	RSTATE	Q3–99	91	19.02%	Q4–04	-39.87%	Q4–08
	INFR	Q1–04	71	23.91%	Q3–06	-12.11%	Q2–06
	FoF	Q4–96	100	20.94%	Q4–20	-10.99%	Q3–09
	All Types	Q4–90	124	16.58%	Q4–04	-19.31%	Q4–08
TOTAL	VCAP	Q2–81	162	82.26%	Q4–99	-18.60%	Q4–00
	B&GE	Q2–86	142	17.39%	Q4–99	-18.85%	Q4–08
	SC&D	Q4–86	140	14.36%	Q2–09	-17.25%	Q4–08
	NRES	Q4–87	136	17.60%	Q3–97	-22.30%	Q1–20
	RSTATE	Q4–86	140	15.84%	Q4–06	-23.14%	Q4–08
	INFR	Q3–97	97	23.92%	Q4–99	-13.16%	Q4–08
	FoF	Q4–88	132	43.88%	Q4–99	-13.67%	Q4–08
	All Types	Q2–81	162	33.58%	Q4–99	-17.79%	Q4–08

4. Data and empirical results

Quarterly series constructed by Cambridge Associates LLC (CA) have been retrieved from the Eikon-Reuters database, and have been divided by geographical area (USA, Europe, Asia/Pacific, Rest of World, and Total) and asset class (Venture Capital, Buyout & Growth Equity, Subordinated Capital & Distressed, Natural Resources, Real Estate, Infrastructure, Fund of Funds & Secondary Funds, and All Types). The sample period varies depending on the geographical area and the investment type: it ends in all cases in 2021Q3 but the starting date is 1981Q2 in some cases and later in others (more details are provided in Table 1) – in total the sample includes 4154 observations. The chosen metric is 'Pooled IRR' instead of other IRRs (average or weighted) or TVPI.[5]

Table 2 reports some descriptive statistics. It can be seen that the median quarterly IRR for the PE industry was 3.15% during the period spanning from Q2–81 to Q3–21[6] (3.22% in the USA, 3.78% in Europe, and 1.79% in Asia/Pacific), whilst by type of investment it was 3.62% for 'Buyout & Growth Equity', 2.36% for 'Venture Capital', and 3.29% for 'Fund of Funds & Secondary Funds'.

Right skewness (which is indicated by a positive value) characterises the data for all areas, except the Rest of World, and the Total, and leptokurtosis (with a value > 3, indicating that the observations are largely concentrated around the mean) is exhibited by all series. Shapiro-Wilk tests (see Table 3) reject the hypothesis of the pooled IRR stemming from a normal distribution, and Runs tests[7] suggest that only European returns follow a random process.

[5] Another metric extensively used is TVPI (Total Value to Paid In), which is calculated as the ratio of the current value of remaining investments within a fund, plus the total value of all distributions to date, relative to the total amount of capital paid into the fund to date. Hence, values larger than 1.0 indicate that an investment has gained value.

[6] In the same period of time, capitalization soared from 178 millions of dollars to 6.5 trillions of dollars, what entails a cumulative annual growth of a 29.8%.

[7] Runs tests is a non-parametric statistical test that checks the randomness hypothesis for a two-valued data sequence. More precisely, it tests the hypothesis that the elements of the sequence are mutually independent.

Table 2
Descriptive statistics.

Area	Type	avg_IRR	median_IRR	stddev_ IRR	skew_ IRR	kurt_ IRR
USA	VCAP	3.867%	2.377%	9.719%	3.948	32.619
	B&GE	3.710%	3.951%	4.900%	-0.431	5.365
	SC&D	2.682%	3.049%	3.952%	-1.490	9.777
	NRES	2.789%	2.774%	5.812%	-0.615	5.968
	RSTATE	2.201%	2.418%	3.940%	-1.524	11.075
	INFR	2.614%	2.957%	2.720%	-0.405	4.699
	FoF	3.609%	3.442%	6.811%	2.339	18.420
	All Types	3.243%	3.217%	5.270%	1.428	14.874
EUROPE	VCAP	8.481%	2.816%	65.038%	10.646	118.719
	B&GE	3.650%	3.899%	8.031%	-0.322	4.592
	SC&D	2.943%	2.832%	6.990%	1.080	6.524
	NRES	6.022%	-0.856%	49.147%	6.898	54.489
	RSTATE	1.445%	2.381%	7.755%	-0.807	6.070
	INFR	2.854%	2.636%	7.869%	1.842	14.076
	FoF	2.779%	2.657%	6.008%	-0.246	4.367
	All Types	3.863%	3.777%	11.196%	4.274	39.307
ASIA/PACIFIC	VCAP	2.751%	1.056%	7.019%	1.366	7.301
	B&GE	2.331%	1.786%	5.640%	-0.036	3.834
	SC&D	2.333%	1.752%	5.489%	0.616	7.859
	RSTATE	1.838%	1.819%	5.436%	0.398	11.527
	INFR	0.942%	0.467%	6.470%	1.073	6.417
	FoF	3.290%	2.823%	6.120%	0.580	4.878
	All types	2.025%	1.789%	5.050%	0.065	4.546
REST OF WORLD	VCAP	2.025%	1.789%	5.050%	0.065	4.546
	B&GE	2.506%	2.441%	6.479%	-0.220	4.834
	SC&D	2.812%	3.248%	4.842%	-0.265	7.948
	NRES	1.960%	1.897%	6.333%	-0.168	5.156
	RSTATE	2.485%	2.770%	7.680%	-2.271	14.226
	INFR	2.174%	2.056%	5.054%	1.188	10.193
	FoF	2.903%	3.050%	5.199%	0.030	4.471
	All Types	2.332%	2.617%	4.733%	-0.675	6.960
TOTAL	VCAP	3.782%	2.364%	9.375%	3.982	33.247
	B&GE	3.614%	3.619%	4.955%	-0.589	6.066
	SC&D	2.657%	2.836%	3.821%	-1.114	8.968
	NRES	2.757%	2.702%	5.584%	-0.696	6.067
	RSTATE	2.102%	2.201%	4.159%	-1.859	13.849
	INFR	1.527%	2.075%	5.208%	0.482	6.217
	FoF	3.360%	3.294%	6.256%	1.976	15.615
	All Types	3.163%	3.149%	5.029%	0.896	12.302

In the estimated model, we assume that x_t in (1) corresponds to the errors in a regression model incorporating an intercept and a linear time trend (Barghava, 1986; Schmidt and Phillips, 1992):

$$y_t = \beta_0 + \beta_1 t + x_t; t = 1, 2, \ldots, \tag{2}$$

where β_0 and β_1 denote the unknown coefficients on these deterministic terms. More precisely, the estimated model is:

$$y_t = \beta_0 + \beta_1 t + x_t; (1 - B)^d x_t = u_t, t = 1, 2, \ldots. \tag{3}$$

As mentioned before, we use Robinson's (1994) procedure, which is valid for the computation of d independently of the deterministic terms used in (3). We report the estimates of the differencing parameter d for three specifications: i) no deterministic components, i.e., setting β_0 and β_1 equal to 0 in Eq. (3); ii) a constant only, i.e. $\beta_1 = 0$ and β_0 being freely estimated; iii) both β_0 and β_1 being freely estimated from the data together with d. In each case a preferred model is selected on the basis of the significance of the regressors. In addition, we assume that the error term u_t in (3) follows either a white noise process (Table 4) or an autocorrelated one (Table 6); in the latter case, we use the exponential spectral approach of Bloomfield (1973), which is non-parametric in the sense that it does not require to specify a functional form for u_t but only its spectral density function, which is very similar (in logs) to the one produced by AR structures. Finally, given the quarterly frequency of the data, a seasonal AR(1) process is also assumed (Table 8).

Concerning the results by geographical area based on white noise errors (Table 4), it can be seen that the time trend is not required in any case, and the intercept coefficient is significant only in the case of Europe. As for the degree of integration, the values of d range from − 0.09 in Europe to 0.43 in the USA. The null hypothesis of short memory or I(0) behaviour cannot be rejected for Europe, although it is in the remaining cases in favour of long memory (d > 0) or fractional integration, the estimated value of d being 0.25 for Asia-Pacific, 0.35 for the Rest of the World, and 0.41 for the Total.

Concerning the results by asset class based on the same white noise assumption, the time trend is significant for 'Infrastructure' (INFR) in the Total and the USA, and also for 'Natural Resources' (NRES) and 'Subordinated Capital & Distressed' (SC&D) in Europe. In

Table 3
Shapiro-Wilk test.

Area	W	p-value
USA	0.6831	0.0000
	0.9583	0.0003
	0.8912	0.0000
	0.9519	0.0001
	0.8085	0.0000
	0.9594	0.0502
	0.8230	0.0000
	0.8555	0.0000
Europe	0.1752	0.0000
	0.9617	0.0008
	0.9279	0.0000
	0.3262	0.0000
	0.9223	0.0001
	0.8163	0.0000
	0.9761	0.0859
	0.6884	0.0000
ASIA/PACIFIC	0.8832	0.0000
	0.9793	0.0470
	0.8621	0.0000
	0.8174	0.0000
	0.9421	0.0004
	0.9453	0.0030
	0.9772	0.0286
REST OF WORLD	0.8260	0.0000
	0.9627	0.0023
	0.8972	0.0000
	0.9466	0.0007
	0.7732	0.0000
	0.8001	0.0000
	0.9657	0.0105
	0.9342	0.0000
TOTAL	0.6844	0.0000
	0.9520	0.0001
	0.9019	0.0000
	0.9475	0.0000
	0.7838	0.0000
	0.9396	0.0002
	0.8532	0.0000
	0.8778	0.0000

the first two cases, the time trend coefficient is positive while it is negative for the two European asset classes (see Table 5 that reports the estimated coefficients of the selected models). As for the values of d, most of them are significantly positive, which supports the hypothesis of long memory and fractional integration. However, there are some exceptions: for INFR, the I(0) hypothesis of short memory cannot be rejected in any region; the same holds for SC&D in the case of Europe, Asia and the Rest of the World, and for 'Venture Capital' (VCAP) in Europe. For the other series, the estimated values of d are significantly positive and range from 0.20 (SC&D, in the USA) to 0.67 (RSTATE, in the Rest of the World). Generally, the lowest degrees of integration are found for INFR and RS&D, and the highest for Funds of Fund & Secondary Funds (FoF) and RSTATE.

Under the assumption of autocorrelation as in the exponential spectral model of Bloomfield (1973) (Tables 6 and 7), the time trend coefficient is statistically significant for Europe and Asia-Pacific, in the former case with a negative coefficient and in the latter with a positive one (see Table 7). Regarding the order of integration, the estimated value of d is negative for Europe and Asia-Pacific, for which the I(0) hypothesis cannot be rejected along with the Rest of the World (d = 0.03). However, for the Total and the USA, it is significantly positive, which supports again the hypothesis of long memory (it is equal to 0.28 for the Total and 0.30 for the USA). As for the results by asset class, the time trend is found to be statistically significant for INFR (for the Total and the Rest of the World), SC&D (for Europe, Asia and the Rest of the World), and in some other cases such as NRES (for Europe), B&GE and RSTATE (for Asia) and FoF (for the Rest of the World). These coefficients are positive for all assets except for RS&D (in Europe, Asia and the Rest of the World) and RSTATE (in Asia). Concerning d, there are many cases where the I(0) hypothesis cannot be rejected: B&GE, INFR and SC&D for the Total; B&GE, NRES and SC&D for the USA; all asset classes except NRES in Europe and Asia, and finally all except RSTATE for the Rest of the World. Thus, there is less evidence of long memory under the assumption of Bloomfield errors, this being found only for FoF in the Total (with d = 0.44) and in Europe (0.47); NRES in the Total (0.23); RSTATE (in the Total, 0.65: USA, 0.79, and Europe and the Rest of the World, 0.36); also in INFR for the USA (with d = 0.37) and finally for VCAP (the Total, 0.46, and the USA, 0.49).

Finally, when a seasonal AR process is assumed for the errors (Tables 8 and 9), the results are very similar to those based on white noise errors (Tables 4 and 5): there are only three series for which the time trend is statistically significant (INFR in the Total and the

Table 4
Estimates of d based on white noise errors.

Area: TOTAL			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	**0.41 (0.28, 0.57)**	0.41 (0.28, 0.57)	0.41 (0.28, 0.57)
Buyout & Growth Equity	0.29 (0.16, 0.48)	**0.29 (0.16, 0.48)**	0.29 (0.17, 0.48)
Fund of Funds & Secondary Funds	**0.47 (0.34, 0.64)**	0.47 (0.33, 0.64)	0.48 (0.35, 0.64)
Infrastructure	0.14 (0.02, 0.33)	0.15 (0.02, 0.33)	**0.12 (−0.03, 0.32)**
Natural Resources	0.29 (0.17, 0.43)	**0.28 (0.17, 0.40)**	0.27 (0.15, 0.41)
Real Estate	**0.45 (0.34, 0.59)**	0.45 (0.33, 0.59)	0.45 (0.34, 0.59)
Subordinated Capital & Distressed	0.21 (0.03, 0.44)	**0.21 (0.03, 0.43)**	0.21 (0.04, 0.44)
Venture Capital	**0.53 (0.41, 0.68)**	0.53 (0.41, 0.69)	0.53 (0.41, 0.69)
Area: USA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	**0.43 (0.30, 0.59)**	0.43 (0.30, 0.59)	0.43 (0.30, 0.59)
Buyout & Growth Equity	0.30 (0.17, 0.45)	**0.29 (0.17, 0.45)**	0.30 (0.17, 0.45)
Fund of Funds & Secondary Funds	**0.48 (0.34, 0.64)**	0.47 (0.35, 0.64)	0.48 (0.34, 0.64)
Infrastructure	-0.03 (−0.12, 0.11)	-0.05 (−0.21, 0.16)	**-0.17 (−0.36, 0.11)**
Natural Resources	0.28 (0.16, 0.43)	**0.26 (0.15, 0.42)**	0.26 (0.14, 0.42)
Real Estate	**0.42 (0.31, 0.56)**	0.42 (0.31, 0.55)	0.42 (0.31, 0.55)
Subordinated Capital & Distressed	0.20 (0.02, 0.44)	**0.20 (0.02, 0.43)**	0.21 (0.03, 0.43)
Venture Capital	**0.54 (0.41, 0.69)**	0.53 (0.41, 0.69)	0.54 (0.41, 0.69)
Area: EUROPA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	-0.09 (−0.23, 0.11)	**-0.08 (−0.22, 0.11)**	-0.09 (−0.25, 0.11)
Buyout & Growth Equity	0.11 (0.00, 0.25)	**0.11 (0.00, 0.26)**	0.13 (0.02, 0.29)
Fund of Funds & Secondary Funds	0.21 (0.05, 0.41)	**0.21 (0.05, 0.42)**	0.21 (0.05, 0.42)
Infrastructure	0.09 (−0.07, 0.32)	**0.09 (−0.07, 0.31)**	0.16 (−0.02, 0.76)
Natural Resources	0.02 (−0.10, 0.19)	0.02 (−0.09, 0.17)	**-0.12 (−0.25, 0.07)**
Real State	**0.32 (0.18, 0.50)**	0.32 (0.18, 0.52)	0.36 (0.21, 0.57)
Subordinated Capital & Distressed	-0.06 (−0.22, 0.13)	-0.05 (−0.16, 0.11)	**-0.10 (−0.24, 0.09)**
Venture Capital	-0.06 (−0.17, 0.09)	**-0.06 (−0.15, 0.08)**	-0.06 (−0.16, 0.07)
Area: ASIA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.25 (0.11, 0.44)	**0.26 (0.11, 0.44)**	0.22 (0.06, 0.43)
Buyout & Growth Equity°	0.24 (0.10, 0.44)	**0.25 (0.10, 0.44)**	0.23 (0.07, 0.44)
Fund of Funds & Secondary Funds	0.15 (0.00, 0.35)	**0.14 (0.00, 0.35)**	0.17 (0.02, 0.42)
Infrastructure	**0.01 (−0.11, 0.17)**	0.01 (−0.11, 0.17)	0.01 (−0.11, 018)
Natural Resources	—	—	—
Real Estate	**0.36 (0.14, 0.66)**	0.33 (0.12, 0.65)	0.32 (0.10, 0.66)
Subordinated Capital & Distressed	0.06 (−0.14, 0.31)	**0.05 (−0.10, 0.28)**	0.04 (−0.14, 0.32)
Venture Capital	**0.30 (0.16, 0.50)**	0.31 (0.17, 0.51)	0.27 (0.09, 0.49)
Area: REST OF THE WORLD			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	**0.35 (0.18, 0.55)**	0.35 (0.19, 0.55)	0.35 (0.19, 0.55)
Buyout & Growth Equity	**0.25 (0.11, 0.43)**	0.25 (0.11, 0.43)	0.25 (0.10, 0.43)
Fund of Funds & Secondary Funds	0.33 (0.16, 0.54)	**0.32 (0.16, 0.54)**	0.32 (0.15, 0.54)
Infrastructure	-0.11 (−0.22, 0.11)	**-0.12 (−0.28, 0.11)**	-0.13 (−0.30, 0.11)
Natural Resources	**0.22 (0.09, 0.41)**	0.22 (0.09, 0.40)	0.20 (0.06, 0.40)
Real Estate	**0.67 (0.49, 0.91)**	0.67 (0.48, 0.92)	0.67 (0.49, 0.92)
Subordinated Capital & Distressed	0.20 (−0.01, 0.46)	**0.17 (−0.01, 0.43)**	0.13 (−0.08, 0.43)
Venture Capital	**0.31 (0.16, 0.51)**	0.31 (0.16, 0.51)	0.31 (0.16, 0.51)

Note: the values in bold correspond to the selected specification on the basis of the statistical significance of the deterministic terms. The confidence intervals at the 95% level are reported in brackets.

USA along with SC&D in Europe, in the latter case with a negative coefficient). Evidence of short memory or I(0) behaviour is found for INFR in all regions, but also for all other asset types except FoF and RSTATE in Europe. In general, RSTATE and VCAP display the highest degrees of integration in all regions except Europe.

As a robustness check, we also apply two widely used semi-parametric estimation methods, namely the log-periodogram estimator (Geweke and Porter-Hudak, 1983), and the local Whittle estimation approach of Künsch (1987) (Table 10). In both cases, a bandwidth parameter specifying the number of Fourier frequencies must be chosen between 0 and 1 - we follow Weijie et al. (2021) who suggest the interval (0.58, 0.67) for the GPH estimator for a sequence length of 100, and (0.59, 0.68) when the length is 300. The results shown in Table 10 are for 'All types' of assets only and are consistent with those reported in Tables 4 – 9, evidence of long memory being found in all cases except for Europe. When following a parametric approach as in Haslett and Raftery (1989), the results (Table 11) are still broadly consistent with the previous ones since long memory is found in various cases in the other geographical areas but in Europe only in the case of Real Estate.

Our evidence concerning the stochastic properties of the variables under examination implies that the effects shock will not be permanent; this can be seen, for instance, in the case of the Covid-19 pandemic, when the initial negative impact on returns was quickly

Table 5
Estimates of d based on white noise errors.

Area: TOTAL			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.41 (0.28, 0.57)	—	—
Buyout & Growth Equity	0.29 (0.16, 0.48)	0.03478 (2.51)	
Fund of Funds & Secondary Funds	0.47 (0.34, 0.64)	—	—
Infrastructure	0.12 (−0.03, 0.32)	-0.00758 (−1.61)	0.00045 (1.76)
Natural Resources	0.28 (0.17, 0.40)	0.02619¡0 (1.80)	—
Real Estate	0.45 (0.34, 0.59)	—	—
Subordinated Capital & Distressed	0.21 (0.03, 0.43)	0.02551 (3.19)	—
Venture Capital	0.53 (0.41, 0.68)	—	—
Area: USA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.43 (0.30, 0.59)	—	—
Buyout & Growth Equity	0.29 (0.17, 0.45)	0.03604 (2.54)	—
Fund of Funds & Secondary Funds	0.48 (0.34, 0.64)	—	—
Infrastructure	-0.17 (−0.36, 0.11)	0.01937 (3.55)	0.00036 (2.71)
Natural Resources	0.26 (0.15, 0.42)	0.02650 (1.81)	—
Real Estate	0.42 (0.31, 0.56)	—	—
Subordinated Capital & Distressed	0.20 (0.02, 0.43)	0.02604 (3.15)	—
Venture Capital	0.54 (0.41, 0.69)	—	—
Area: EUROPA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	-0.08 (−0.22, 0.11)	0.03851 (6.06)	—
Buyout & Growth Equity	0.11 (0.00, 0.26)	0.03354 (2.79)	—
Fund of Funds & Secondary Funds	0.21 (0.05, 0.42)	0.02710 (1.93)	—
Infrastructure	0.09 (−0.07, 0.31)	0.03171 (2.39)	—
Natural Resources	-0.12 (−0.25, 0.07)	0.32114 (4.18)	-0.00735 (−3.86)
Real Estate	0.32 (0.18, 0.50)	—	—
Subordinated Capital & Distressed	-0.10 (−0.24, 0.09)	0.04399 (4.60)	-0.00027 (−1.71)
Venture Capital	-0.06 (−0.15, 0.08)	0.08005 (1.77)	—
Area: ASIA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.26 (0.11, 0.44)	0.01937 (1.77)	—
Buyout & Growth Equity	0.25 (0.10, 0.44)	0.02342 (1.82)	—
Fund of Funds & Secondary Funds	0.14 (0.00, 0.35)	0.019233 (2.32)	—
Infrastructure	0.01 (−0.11, 0.17)	—	—
Natural Resources	—	—	—
Real State	0.36 (0.14, 0.66)	—	—
Subordinated Capital & Distressed	0.05 (−0.10, 0.28)	0.02390 (3.32)	—
Venture Capital	0.30 (0.16, 0.50)	—	—
Area: REST OF THE WORLD			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.35 (0.18, 0.55)	—	—
Buyout & Growth Equity	0.25 (0.11, 0.43)	—	—
Fund of Funds & Secondary Funds	0.32 (0.16, 0.54)	0.02906 (1.68)	—
Infrastructure	-0.12 (−0.28, 0.11)	0.002177 (6.07)	—
Natural Resources	0.22 (0.09, 0.41)	—	—
Real Estate	0.67 (0.49, 0.91)	—	—
Subordinated Capital & Distressed	0.17 (−0.01, 0.43)	0.02814 (3.36)	—
Venture Capital	0.31 (0.16, 0.51)	—	—

Note: the values in brackets in column 2 are the 95% confidence intervals for the estimates of d. Those in columns 3 and 4 are the t-values of the estimated coefficients.

Table 6
Estimates of d based on autocorrelated Bloomfield errors.

Area: TOTAL			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.30 (0.01, 0.72)	**0.28 (0.01, 0.71)**	0.30 (0.04, 0.71)
Buyout & Growth Equity	0.24 (−0.08, 0.65)	**0.22 (−0.08, 0.65)**	0.25 (−0.04, 0.62)
Fund of Funds & Secondary Funds	**0.44 (0.05, 0.94)**	0.41 (0.05, 0.94)	0.46 (0.08, 0.94)
Infrastructure	0.00 (−0.17, 0.25)	0.01 (−0.21, 0.28)	**-0.06 (−0.30, 0.25)**
Natural Resources	0.24 (0.04, 0.55)	**0.23 (0.03, 0.53)**	0.20 (0.00, 0.53)
Real Estate	**0.65 (0.33, 1.11)**	0.65 (0.29, 1.08)	0.65 (0.31, 1.07)
Subordinated Capital & Distressed	-0.16 (−0.25, 0.05)	**-0.24 (−0.44, 0.05)**	-0.24 (−0.44, 0.05)
Venture Capital	**0.46 (0.17, 0.96)**	0.46 (0.17, 0.96)	0.47 (0.17, 0.96)
Area: USA			

(*continued on next page*)

Table 6 (*continued*)

Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.33 (0.04, 0.73)	**0.30 (0.02, 0.72)**	0.32 (0.05, 0.75)
Buyout & Growth Equity	0.27 (−0.10, 0.65)	**0.22 (−0.09, 0.60)**	0.25 (−0.04, 0.61)
Fund of Funds & Secondary Funds	**0.47 (0.12, 0.97)**	0.46 (0.09, 0.98)	0.51 (0.13, 0.98)
Infrastructure	0.42 (0.26, 0.60)	**0.37 (0.22, 0.57)**	0.27 (0.09, 0.53)
Natural Resources	0.21 (0.00, 0.52)	**0.18 (−0.01, 0.47)**	0.14 (−0.04, 0.47)
Real Estate	**0.79 (0.38, 1.25)**	0.78 (0.39, 1.25)	0.78 (0.41, 1.25)
Subordinated Capital & Distressed	-0.19 (−0.28, −0.03)	**-0.33 (−0.54, −0.02)**	-0.32 (−0.53, 0.04)
Venture Capital	**0.49 (0.15, 0.97)**	0.47 (0.14, 0.97)	0.49 (0.16, 0.97)
Area: EUROPA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	-0.30 (−0.40, −0.02)	-0.31 (−0.50, −0.02)	**-0.42 (−0.66, −0.04)**
Buyout & Growth Equity	0.07 (−0.10, 0.38)	**0.07 (−0.11, 0.37)**	0.14 (−0.09, 0.59)
Fund of Funds & Secondary Funds	0.10 (−0.22, 0.69)	**0.13 (−0.28, 0.70)**	0.13 (−0.26, 0.70)
Infrastructure	-0.04 (−0.34, 0.34)	**-0.04 (−0.30, 0.32)**	0.06 (−0.23, 0.91)
Natural Resources	0.06 (−0.16, 0.35)	0.06 (−0.14, 0.29)	**-0.07 (−0.31, 0.23)**
Real Estate	**0.36 (0.04, 0.80)**	0.38 (0.04, 1.04)	0.61 (0.06, 1.03)
Subordinated Capital & Distressed	-0.18 (−0.42, 0.29)	-0.08 (−0.27, 0.22)	**-0.22 (−0.56, 0.17)**
Venture Capital	-0.05 (−0.24, 0.20)	**-0.05 (−0.21, 0.19)**	-0.02 (−0.19, 0.23)
Area: ASIA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.08 (−0.06, 0.38)	0.09 (−0.09, 0.40)	**-0.08 (−0.36, 0.32)**
Buyout & Growth Equity	0.04 (−0.11, 0.30)	0.04 (−0.14, 0.32)	**-0.07 (−0.31, 0.31)**
Fund of Funds & Secondary Funds	0.17 (−0.13, 0.56)	**0.16 (−0.10, 1.81)**	0.24 (−0.07, 1.82)
Infrastructure	**-0.01 (−0.22, 0.27)**	-0.01 (−0.21, 0.27)	-0.01 (−0.23, 0.31)
Natural Resources			
Real Estate	-0.20 (−0.88, 0.37)	-0.11 (−0.40, 0.31)	**-0.17 (−0.48, 0.29)**
Subordinated Capital & Distressed	-0.62 (−0.70, 0.16)	-0.17 (−0.37, 0.10)	**-0.28 (−0.51, 0.08)**
Venture Capital	0.10 (−0.05, 0.40)	0.12 (−0.05, 0.44)	**-0.11 (−0.38, 0.35)**
Area: REST OF THE WORLD			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.02 (−0.20, 0.48)	**0.02 (−0.29, 0.47)**	0.04 (−0.27, 0.48)
Buyout & Growth Equity	0.13 (−0.14, 0.49)	**0.12 (−0.18, 0.51)**	0.11 (−0.18, 0.51)
Fund of Funds & Secondary Funds	0.05 (−0.26, 0.57)	0.06 (−0.39, 0.52)	**0.06 (−0.47, 0.52)**
Infrastructure	-0.27 (−0.46, 0.05)	-0.42 (−0.74, 0.06)	**-0.51 (−0.93, 0.04)**
Natural Resources	0.14 (−0.06, 0.44)	**0.13 (−0.05, 0.44)**	0.06 (−0.17, 0.43)
Real Estate	**0.36 (0.05, 0.79)**	0.34 (0.04, 0.81)	0.35 (−0.03, 0.81)
Subordinated Capital & Distressed	-0.26 (−0.40, 0.20)	-0.14 (−0.33, 0.14)	**-0.46 (−0.71, 0.02)**
Venture Capital	0.12 (−0.20, 0.53)	**0.11 (−0.19, 0.53)**	0.09 (−0.21, 0.53)

Note: the values in bold correspond to the selected specification on the basis of the statistical significance of the deterministic terms. The confidence intervals at the 95% level are reported in brackets.

Table 7
Estimates of d based on autocorrelated Bloomfield errors.

Area: TOTAL			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.28 (0.01, 0.71)	0.0293 (2.17)	—
Buyout & Growth Equity	0.22 (−0.08, 0.65)	0.03527 (3.00)	—
Fund of Funds & Secondary Funds	0.44 (0.05, 0.94)	—	—
Infrastructure	-0.06 (−0.30, 0.25)	-0.00470 (−1.64)	0.00041 (2.67)
Natural Resources	0.23 (0.03, 0.53)	0.02657 (2.46)	—
Real Estate	0.65 (0.33, 1.11)	—	—
Subordinated Capital & Distressed	-0.24 (−0.44, 0.05)	0.02667 (26.14)	—
Venture Capital	0.46 (0.17, 0.96)	—	—
Area: USA			
Series	No deterministic terms	An intercept	A linear time trend
All Types	0.30 (0.02, 0.72)	0.02983 (1.97)	—
Buyout & Growth Equity	0.22 (−0.09, 0.60)	0.03633 (3.16)	—
Fund of Funds & Secondary Funds	0.47 (0.12, 0.97)	—	—
Infrastructure	0.37 (0.22, 0.57)	0.01524 (2.23)	—
Natural Resources	0.18 (−0.01, 0.47)	0.02722 (3.12)	—
Real Estate	0.79 (0.38, 1.25)	—	—
Subordinated Capital & Distressed	-0.33 (−0.54, −0.02)	0.02654 (36.01)	—
Venture Capital	0.49 (0.15, 0.97)	—	—
Area: EUROPA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	-0.42 (−0.66, −0.04)	0.04688 (11.38)	-0.00013 (−2.14)
Buyout & Growth Equity	0.07 (−0.11, 0.37)	0.03491 (3.78)	
Fund of Funds & Secondary Funds	0.13 (−0.28, 0.70)	0.02759 (2.70)	

(*continued on next page*)

Table 7 (*continued*)

Infrastructure	-0.04 (−0.30, 0.32)	0.02799 (3.35)	
Natural Resources	-0.07 (−0.31, 0.23)	0.32511 (3.40)	0.00741 (−3.16)
Real Estate	0.36 (0.04, 0.80)	—	—
Subordinated Capital & Distressed	-0.22 (−0.56, 0.17)	0.04427 (6.84)	-0.00027 (−2.40)
Venture Capital	-0.05 (−0.21, 0.19)	0.08004 (1.76)	
Area: ASIA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	-0.08 (−0.36, 0.32)	-0.00147 (−1.22)	0.00033 (3.65)
Buyout & Growth Equity	-0.07 (−0.31, 0.31)	0.00365 (1.49)	0.00030 (2.96)
Fund of Funds & Secondary Funds	0.16 (−0.10, 1.81)	0.03654 (2.86)	—
Infrastructure	-0.01 (−0.22, 0.27)	—	—
Natural Resources	—	—	—
Real Estate	-0.17 (−0.48, 0.29)	0.02982 (4.19)	-0.00033 (−1.84)
Subordinated Capital & Distressed	-0.28 (−0.51, 0.08)	0.03876 (7.25)	-0.00044 (−3.27)
Venture Capital	-0.11 (−0.38, 0.35)	-0.00843 (−1.06)	0.00055 (4.98)
Area: REST OF THE WORLD			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	0.02 (−0.29, 0.47)	0.02329 (5.02)	—
Buyout & Growth Equity	0.12 (−0.18, 0.51)	0.02509 (2.75)	—
Fund of Funds & Secondary Funds	0.06 (−0.47, 0.52)	0.01044 (0.87)	0.00038 (1.95)
Infrastructure	-0.51 (−0.93, 0.04)	0.01627 (6.37)	0.00015 (2.04)
Natural Resources	0.13 (−0.05, 0.44)	0.01938 (1.83)	
Real Estate	0.36 (0.05, 0.79)	—	—
Subordinated Capital & Distressed	-0.46 (−0.71, 0.02)	0.04282 (19.16)	-0.00032 (−6.50)
Venture Capital	0.11 (−0.19, 0.53)	0.02396 (1.75)	—

Note: the values in brackets in column 2 are the 95% confidence intervals for the estimates of d. Those in columns 3 and 4 are the t-values of the estimated coefficients.

Table 8
Estimates of d based on seasonal autoregressive errors.

Area: TOTAL			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	**0.41 (0.29, 0.56)**	0.41 (0.29, 0.56)	0.41 (0.29, 0.56)
Buyout & Growth Equity	0.29 (0.16, 0.46)	**0.29 (0.16, 0.46)**	0.29 (0.16, 0.46)
Fund of Funds & Secondary Funds	**0.48 (0.35, 0.63)**	0.47 (0.33, 0.63)	0.48 (0.35, 0.64)
Infrastructure	0.13 (0.00, 0.33)	0.14 (0.00, 0.34)	**0.11 (−0.04, 0.33)**
Natural Resources	0.29 (0.17, 0.43)	**0.28 (0.16, 0.42)**	0.27 (0.15, 0.42)
Real Estate	**0.47 (0.34, 0.63)**	0.46 (0.33, 0.63)	0.46 (0.33, 0.63)
Subordinated Capital & Distressed	0.21 (0.04, 0.43)	**0.20 (0.04, 0.42)**	0.21 (0.04, 0.43)
Venture Capital	**0.55 (0.44, 0.69)**	0.55 (0.44, 0.69)	0.55 (0.44, 0.69)
Area: USA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	**0.44 (0.32, 0.58)**	0.44 (0.32, 0.58)	0.44 (0.32, 0.58)
Buyout & Growth Equity	0.30 (0.17, 0.45)	**0.29 (0.17, 0.45)**	0.30 (0.17, 0.45)
Fund of Funds & Secondary Funds	**0.49 (0.37, 0.64)**	0.49 (0.36, 0.64)	0.50 (0.36, 0.64)
Infrastructure	-0.04 (−0.14, 0.11)	-0.06 (−0.23, 0.16)	**-0.18 (−0.37, 0.10)**
Natural Resources	0.28 (0.16, 0.43)	**0.27 (0.14, 0.42)**	0.26 (0.14, 0.42)
Real Estate	**0.43 (0.30, 0.58)**	0.42 (0.29, 0.57)	0.42 (0.30, 0.57)
Subordinated Capital & Distressed	0.20 (0.03, 0.43)	**0.19 (0.03, 0.42)**	0.20 (0.03, 0.42)
Venture Capital	**0.57 (0.45, 0.70)**	0.56 (0.45, 0.70)	0.57 (0.45, 0.70)
Area: EUROPA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	-0.08 (−0.22, 0.13)	**-0.08 (−0.22, 0.13)**	-0.09 (−0.25, 0.13)
Buyout & Growth Equity	0.10 (−0.01, 0.25)	**0.11 (−0.01, 0.26)**	0.12 (0.00, 0.28)
Fund of Funds & Secondary Funds	0.21 (0.05, 0.40)	**0.21 (0.05, 0.41)**	0.22 (0.06, 0.41)
Infrastructure	0.09 (−0.09, 0.33)	**0.09 (−0.09, 0.32)**	0.17 (−0.04, 0.80)
Natural Resources	**0.00 (−0.33, 0.23)**	0.00 (−0.27, 0.20)	-0.10 (−0.36, 0.11)
Real Estate	**0.32 (0.16, 0.33)**	0.32 (0.16, 0.53)	0.35 (0.16, 0.56)
Subordinated Capital & Distressed	-0.06 (−0.23, 0.13)	-0.05 (−0.17, 0.12)	**-0.09 (−0.24, 0.09)**
Venture Capital	-0.04 (−0.22, 0.11)	**-0.03 (−0.19, 0.10)**	-0.04 (−0.17, 0.10)
Area: ASIA			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	**0.29 (0.17, 0.44)**	0.29 (0.18, 0.44)	0.29 (0.18, 0.45)
Buyout & Growth Equity	**0.26 (0.14, 0.43)**	0.27 (0.15, 0.43)	0.27 (0.14, 0.43)
Fund of Funds & Secondary Funds	0.15 (0.00, 0.37)	**0.15 (0.00, 0.37)**	0.18 (0.01, 0.48)
Infrastructure	**0.00 (−0.13, 0.17)**	0.00 (−0.13, 0.17)	0.00 (−0.14, 0.17)
Natural Resources	—	—	—
Real Estate	**0.36 (0.14, 0.67)**	0.33 (0.12, 0.67)	0.32 (0.10, 0.67)
Subordinated Capital & Distressed	0.07 (−0.16, 0.37)	**0.06 (−0.12, 0.36)**	0.06 (−0.14, 0.47)

(*continued on next page*)

Table 8 (*continued*)

Venture Capital	**0.30 (0.16, 0.49)**	0.31 (0.18 0.50)	0.26 (0.09, 0.48)
Area: REST OF THE WORLD			
Series	No deterministic terms	An intercept	An intercept and a linear time trend
All Types	**0.35 (0.19, 0.55)**	0.35 (0.19, 0.53)	0.35 (0.19, 0.55)
Buyout & Growth Equity	**0.25 (0.11, 0.43)**	0.25 (0.11, 0.43)	0.25 (0.10, 0.43)
Fund of Funds & Secondary Funds	0.33 (0.18, 0.53)	**0.33 (0.18, 0.53)**	0.32 (0.16, 0.52)
Infrastructure	-0.12 (−0.26, 0.09)	**-0.14 (−0.31, 0.09)**	-0.15 (−0.33, 0.09)
Natural Resources	**0.23 (0.10, 0.41)**	0.22 (0.09, 0.40)	0.20 (0.05, 0.39)
Real Estate	**0.69 (0.49, 0.97)**	0.69 (0.49, 0.98)	0.70 (0.49, 0.98)
Subordinated Capital & Distressed	0.20 (0.01, 0.45)	**0.17 (0.01, 0.42)**	0.13 (−0.08, 0.42)
Venture Capital	**0.31 (0.16, 0.51)**	0.31 (0.16, 0.51)	0.31 (0.16, 0.51)

Note: the values in bold correspond to the selected specification on the basis of the statistical significance of the deterministic terms. The confidence intervals at the 95% level are reported in brackets.

Table 9
Estimates of d based on seasonal autoregressive errors.

Area: TOTAL				
Series	No deterministic terms	An intercept	An intercept and a linear time trend	—
			—	—
			—	—
			—	—
			0.00044 (1.78)	
All Types	0.41 (0.29, 0.56)	—	—	.0.082
Buyout & Growth Equity	0.29 (0.16, 0.46)	0.03478 (2.51)	—	0.051
Fund of Funds & Secondary Funds	0.48 (0.35, 0.63)	—	—	-0.083
Infrastructure	0.11 (−0.04, 0.33)	0.00736 (−0.15)	0.00044 (1.78)	0.130
Natural Resources	0.28 (0.16, 0.42)	0.02619 (1.80)	—	-0.007
Real Estate	0.47 (0.34, 0.63)	—	—	0.308
Subordinated Capital & Distressed	0.20 (0.04, 0.42)	0.02551 (3.19)	—	-0.049
Venture Capital	0.55 (0.44, 0.69)	—	—	-0.194
Area: USA				
Series	No deterministic terms	An intercept	An intercept and a linear time trend	
All Types	0.44 (0.32, 0.58)	—	—	-0.133
Buyout & Growth Equity	0.29 (0.17, 0.45)	0.03601 (2.53)	—	0.010
Fund of Funds & Secondary Funds	0.49 (0.37, 0.64)	—	—	-0.112
Infrastructure	-0.18 (−0.37, 0.10)	0.,01538 (3.45)	0.00036 (2.62)	0.067
Natural Resources	0.27 (0.14, 0.42)	0.02644 (1.71)	—	-0.019
Real State	0.43 (0.30, 0.58)	—	—	0.320
Subordinated Capital & Distressed	0.19 (0.03, 0.42)	0.02614 (3.30)	——	-0.059
Venture Capital	0.57 (0.45, 0.70)	—	—	-0.216
Area: EUROPA				
Series	No deterministic terms	An intercept	An intercept and a linear time trend	
All Types	-0.08 (−0.22, 0.13)	0.38511 (6.06)	—	0.040
Buyout & Growth Equity	0.11 (−0.01, 0.26)	0.03392 (2.92)	—	0.059
Fund of Funds & Secondary Funds	0.21 (0.05, 0.41)	0.02701 (1.85)	—	-0.061
Infrastructure	0.09 (−0.09, 0.32)	0.03744 (2.08)	—	-0.029
Natural Resources	0.00 (−0.33, 0.23)	—	—	0.232
Real Estate	0.32 (0.16, 0.33)	—	—	0.204
Subordinated Capital & Distressed	-0.09 (−0.24, 0.09)	0.04397 (4.39)	-0.00027 (−1.64)	-0.019
Venture Capital	-0.03 (−0.19, 0.10)	0.08081 (1.69)	—	0.054
Area: ASIA				
Series	No deterministic terms	An intercept	An intercept and a linear time trend	
All Types	0.29 (0.17, 0.44)	—	—	-0.068
Buyout & Growth Equity	0.26 (0.14, 0.43)	—	—	-0.138
Fund of Funds & Secondary Funds	0.15 (0.00, 0.37)	0.03738 (2.73)	—	0.066
Infrastructure	0.00 (−0.13, 0.17)	—	—	0.053
Natural Resources	—	—	—	—
Real Estate	0.36 (0.14, 0.67)	—	—	0.013
Subordinated Capital & Distressed	0.06 (−0.12, 0.36)	0.02427 (3.15)	—	0.223
Venture Capital	0.30 (0.16, 0.49)	—	—	-0.064
Area: REST OF THE WORLD				
Series	No deterministic terms	An intercept	An intercept and a linear time trend	
All Types	0.35 (0.19, 0.55)	—	—	-0.008

(*continued on next page*)

Table 9 (*continued*)

Buyout & Growth Equity	0.25 (0.11, 0.43)	—	—	-0.018
Fund of Funds & Secondary Funds	0.33 (0.18, 0.53)	0.02893 (1.69)	—	-0.112
Infrastructure	-0.14 (−0.31, 0.09)	0.02168 (6.67)	—	0.208
Natural Resources	0.23 (0.10, 0.41)	—	—	-0.047
Real Estate	0.69 (0.49, 0.97)	—	—	0.166
Subordinated Capital & Distressed	0.17 (0.01, 0.42)	0.02815 (2.86)	—	-0058
Venture Capital	0.31 (0.16, 0.51)	—	—	0.011

Note: the values in brackets in column 2 are the 95% confidence intervals for the estimates of d. Those in columns 3 and 4 are the t-values of the estimated coefficients.

Table 10
Semi-parametric estimates of the differencing parameter.

Area (All Types of Assets)	bandwidth = 0.65		
	GPH Estimate	Local Whittle	
		Estimate	Standard error
USA	0.40	0.19	0.09
EUROPE	-0.12	-0.27	0.10
ASIA/PACIFIC	0.20	0.09	0.10
REST OF THE WORLD	0.06	0.07	0.10
TOTAL	0.29	0.15	0.09

Table 11
Estimation of d based on ARFIMA.

Area	**Type**	**d**
USA	VCAP	0.4634 *
	B&GE	0.2898 *
	SC&D	0.2042
	NRES	0.2543 *
	RSTATE	0.3964 *
	INFR	0.0001
	FoF	0.4343 *
	All Types	0.4036 *
EUROPE	VCAP	0.0001
	B&GE	0.1505
	SC&D	0.0001
	NRES	0.0474
	RSTATE	0.3450 *
	INFR	0.2055
	FoF	0.2057
	All Types	0.0001
ASIA/PACIFIC	VCAP	0.2978 *
	B&GE	0.2403
	SC&D	0.0584
	RSTATE	0.2940 *
	INFR	0.0123
	FoF	0.1976
	All Types	0.2529 *
REST OF THE WORLD	VCAP	0.3042 *
	B&GE	0.2449
	SC&D	0.1577
	NRES	0.2037
	RSTATE	0.4722 *
	INFR	0.0001
	FoF	0.3157 *
	All Types	0.3283 *
TOTAL	VCAP	0.4624 *
	B&GE	0.2860 *
	SC&D	0.2036
	NRES	0.2637 *
	RSTATE	0.4162 *
	INFR	0.1636
	FoF	0.4317 *
	All Types	0.3868 *

Note: * stands for evidence of long memory at the 5% level – see Haslett and Raftery (1989).

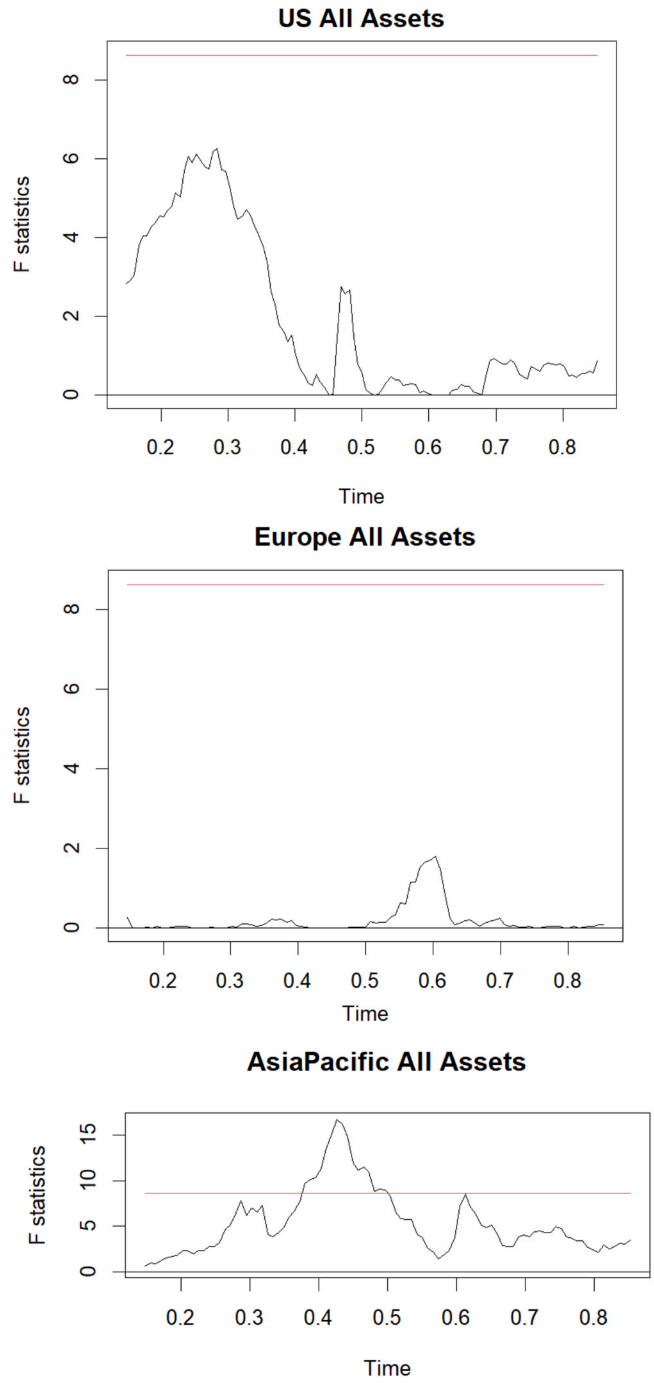

Fig. 1. F and CUMSUM tests for detecting structural breaks.

followed by a rebound. Next, we carry out F and CUSUM tests to detect structural breaks in the total asset series (see Fig. 1). Evidence of breaks is only found in the case of Asia-Pacific over the 2003Q1 to 2007Q3 period.

Finally, we compute half-lives under the assumption of an AR(1) structure to quantify the time it takes to recover from shocks. Note that these cannot be calculated without specifying a model for the autocorrelated errors. The results are displayed in Table 12 and are consistent with the previous ones results since the half-lives are estimated to be smaller in the geographical areas characterised by a lower degree of persistence (0.17 quarters in Europe compared to 0.99 in the US).

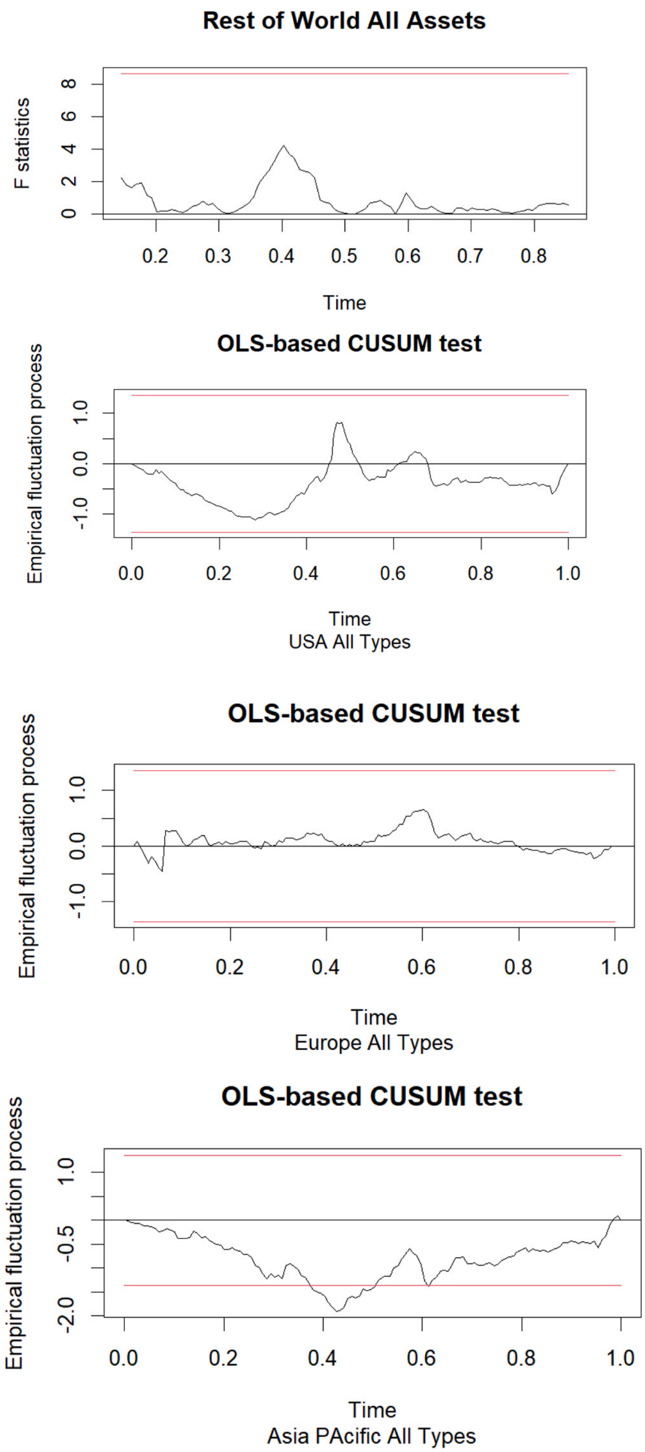

Fig. 1. (*continued*).

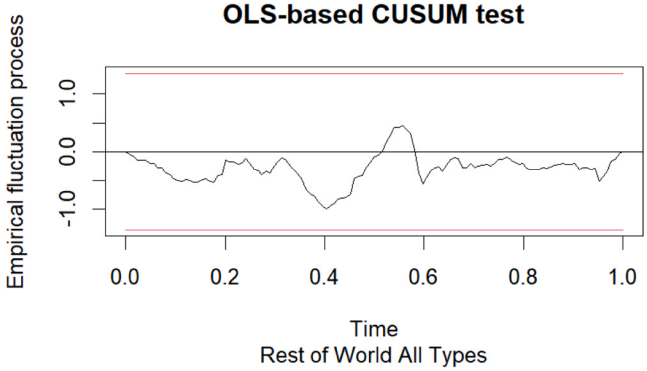

Fig. 1. (*continued*).

Table 12
Estimates of the Half-Lives assuming an AR(1) structure.

Area	Half-Life
United States	0.99
Europe	0.17
Asia/Pacific	0.64
Rest of World	0.79
Total	0.93

5. Conclusions

This paper has analysed the stochastic behaviour of returns on private equity, specifically 'Pooled IRR' as a measure of profitability, applying fractional integration methods to an extensive dataset including various geographical areas and investment types. The chosen modelling framework is more general than standard methods since it allows the differencing parameter d to take any real value, including fractional ones. Different model specifications, assumptions about the errors and estimation techniques have been considered as a robustness check.

The results can be summarised as follows. Under the assumption of white noise errors, there are some differences between geographical areas and asset types. Specifically, for the USA (and also for the Total), the estimates of d are significantly positive in all cases, which implies long memory, except for Infrastructures for which the short memory hypothesis cannot be rejected. Among the long memory cases, Venture Capital, Real State and Fund of Funds & Secondary Funds display the highest degrees of integration. In Asia and the Rest of the World most of the series also display long memory ($d > 0$) (All types, Buyout & Growth Equity, Fund of Funds & Secondary Funds. Real State, Natural Resources and Venture Capital), whilst short memory is found in Infrastructures and Subordinated Capital & Distressed. Finally, Europe displays the lowest orders of integration, and only Buyout & Growth Equity, Fund of Funds & Secondary Funds, and Real Estate exhibit long memory.

When allowing for autocorrelation in the errors the values of d are generally smaller than in the previous case. For the Total and the USA the results are similar: short memory or evidence of I(0) behaviour for Buyout & Growth Equity, Infrastructure and Subordinated Capital & Distressed. For the rest of the series, d is found to be significantly higher than 0, the highest value corresponding to Real Estate in two cases (0.65 for the Total and 0.79 for the USA). For the other three geographical areas (Europe, Asia and the Rest of the World), the short memory (I(0)) hypothesis cannot be rejected in any case with the exception of Real Estate in Europe and the Rest of the World, in both cases with $d = 0.36$.

Finally, when allowing for seasonal AR disturbances the results are very similar to the white noise ones: for the Total and the USA, the estimates of d are strictly positive for all series except Infrastructure, and the same holds for the Rest of the World. For Europe, short memory is found in all series, except for Fund of Funds & Secondary Funds and Real Estate, for which $d = 0$ cannot be rejected. To end with Asia, short memory is found in Infrastructure and Subordinated Capital & Distressed, while in the other cases d is found to be higher than 0.

On the whole, our results confirm the stationarity of PE returns measured by the 'Pooled IRR' series, although they also provide evidence of long-memory behaviour in all series except for Europe. The USA displays the highest degree of persistence, followed by the Rest of the World and Asia, while the order of integration for Europe is found to be close to 0 regardless of the estimation method employed. These findings imply that the effects of shocks are long-lived in all regions except Europe. The presence of long- memory casts doubt on the independence of returns and supports the idea of smoothing profits, but it might also reflect better performance, as mentioned by Kaplan and Schoar (2005).[8] Further, the short memory of PE returns in Europe raises the issue of benchmarking their performance against the US ones; note that both the average IRR and the standard deviation are higher in Europe for all individual asset classes as well as for the aggregate one, and thus the lower autocorrelation counteracts the higher variance in European PE returns.[9]

Future research could analyse the factors driving PE returns (such as illiquidity premium, leverage, and risk adjustment). Moreover, further attention should be paid to the issue of structural breaks since, as shown by many authors, overlooking them as well as non-linearities may account for the long-memory property of the data (see Diebold and Inoue, 2001; Granger and Hyung, 1999; etc.). For this purpose, the model could be extended to allow for non-linear structures such as Chebyshev polynomials in time (Cuestas and Gil-Alana, 2016), Fourier transform functions (Gil-Alana and Yaya, 2021; Caporale et al., 2022) or neural networks (Yaya et al., 2021), all of them in the context of fractional integration. These issues will be addressed in subsequent papers.

CRediT authorship contribution statement

Guglielmo Maria Caporale contributed to the design of the research and the interpretation of the findings and edited the whole manuscript. Francisco Puertolas wrote the introduction, the literature review and conclusions. Luis Alberiko Gil-Alana participated in the empirical results, the interpretation of the results and conclusions.

Data availability

Data will be made available on request.

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[8] Note that, although private equity could be affected by monetary policy, it is not immediately apparent how this would happen. Brunnermeier and Pedersen (2009) and Robinson and Sensoy (2016) develop models characterized by a countercyclical liquidity premium, and Acharya and Pedersen (2005) suggest that one reason for the strong average performance of private equity is that it is especially illiquid during bear markets. These models predict that investors in funds with countercyclical capital calls should earn a liquidity premium to compensate for the opportunity cost of providing capital for illiquid investments made in bad times. One key reason for this is the fact that general partners choose endogenously when to call and distribute capital, which is deployed less aggressively in the wake of bullish markets; this has implications for the overall liquidity exposure that limited partners face as they commit capital across the business cycle. It is well known that stock prices of publicly quoted companies react quickly to changes in the economic environment. To see how this affects private equity one can regress the latter against the former. Following Steger (2017), we have used as a benchmark the Russell 2000 index. The estimates of the coefficients a and b are 0.03 and 0.27 respectively, and R2 is 0.20. This evidence suggests that private equity returns are not closely tied to stock markets (even when using an index including small-caps) and thus are not much influenced by macroeconomics.

[9] We have investigated this issue further to see whether this result is due to the presence of outliers in the first part of the sample. However, removing the early observations still produces a negative estimate of d (−0.12). We then examined the possible impact of breaks by considering two different subsamples (one from 1990Q1 to 2002Q4, and the other from 2003Q1 to 2020Q3), the corresponding estimates of d being respectively − 0.046 and 0.305. It would appear, therefore, that there is a statistical (rather than economic) explanation for the higher degree of persistence observed in the US, namely the presence of both outliers and breaks in the data.

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