Quarterly Review of Economics and Finance 88 (2023) 8–20

Contents lists available at ScienceDirect

Quarterly Review of Economics and Finance

journal homepage: www.elsevier.com/locate/qref

Recessions and flattening of the yield curve (1960–2021): A two-way road under a regime switching approach

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A R T I C L E I N F O

Article history:
Received 15 November 2021
Received in revised form 31 October 2022
Accepted 28 December 2022
Available online 30 December 2022

JEL Classifications:
C32
E32
E43

Keywords:
Term spread
Inverted yield curve
Markov switching model
U. S. business cycle
COVID-19 recession

A B S T R A C T

The one-way relationship that goes from the term spread to recessions has been widely studied. However, the relationship between term spread and the business cycle, in addition to being bidirectional, is conditioned by the cyclical phase itself. To demonstrate this, we have modelled the bidirectional relationship between term spread and the business cycle by extracting two interrelated latent Markov variables: the first, drawn from four activity indicators, replicates the phases of the US business cycle; the second, from the term spread of the yield curve, identifies two regimes: an ordinary regime (positive slope) and a flattening regime. By analyzing both the transition between these regimes and forecasted probabilities, we find that this bidirectional relationship is not symmetrical. That is, the term spread signals a change in the business cycle regime while the cyclical factor only signals the beginning of the ordinary regime of the term spread, not its ending. To illustrate the model, we confirm the beginning of the COVID-19 recession in March of 2020, and the corresponding start of the ordinary regime in the term spread.

1. Introduction

The predictive ability of the term spread, especially to predict recessions, has been well established. Notable research includes that by Harvey (1988) who analyzes the predictive ability of the expected real term structure on consumption growth based on the consumption-based asset pricing model, in line with Kessel (1965) and Fama (1986) regarding the correlation between term structure and the business cycle. Harvey (1989) shows how the bond market (yield curve measures) provides more accurate predictions of GNP growth than stock market variables. Stock and Watson (1989) confirm the importance of spreads between interest rates of private and public debt instruments, and the slope of the public debt yield curve, as variables to be included in the coincident and leading indexes. Estrella and Hardouvelis (1991) emphasize the link between the positive slope of the yield curve and a future increase in real economic activity. This is because the slope includes information on factors independent of monetary policy which can be useful for private investors and policymakers. Dueker (1997), using probit models, shows the robustness of the yield curve compared to other variables as a predictor of recessions, result theoretically supported by the expectations theory for the term structure of interest rates. Estrella and Mishkin (1998) examine the predictive ability of the slope of the yield curve in US recessions with a horizon beyond two quarters. Dotsey (1998) also finds the spread to be a leading indicator of economic activity, although its usefulness as a predictor diminished in the last period analyzed. Hamilton and Kim (2002) decompose the contribution of the spread to predict GDP into the effect of expected changes in short-term rates and the effect of the term premium, and show how the cyclical behavior of interest rate volatility could account for these effects. Chauvet and Potter (2005) compare forecasts of recessions provided by four specifications of probit models. These models consider break points that affect the predictive ability of the term spread on recessions. Diebold, Rudebusch, and Aruoba (2006) find evidence of mutual influence of latent variables obtained from the yield curve (level, slope, and curvature) and observed macroeconomic variables (real activity, inflation, and a monetary policy instrument). Giacomini and Rossi (2006) analyze the stability of the predictive relationship between the yield curve and output growth, a relationship which break down during the Burns-Miller and Volker periods, but becomes more reliable during the early Greenspan era. Wright (2006) estimates a number of probit models to forecast recessions. Models that use

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https://doi.org/10.1016/j.qref.2022.12.007

both the level of the Federal Funds rate and the term spread provide better in-sample fit and out-of-sample predictive performance. Ang, Piazzesi, and Wei (2006), using a dynamic model for GDP and yields, find that the short rate surpasses the slope of the yield curve in forecasting GDP growth.

These studies generally use probit models where the term spread is an input variable with a dummy variable to signal recessions as the output. However, among the various modeling alternatives, Markov switching models might also prove effective in identifying the relationship between term spread and the business cycle. Regime switching models explicitly consider cyclical dynamics as a succession of expansion and recession phases. Unlike probit models, by means of a Markov switching model, a dating of the business cycle phases with a dummy variable is not imposed, but rather Markov switching model endogenously produces the probabilities of the underlying regimes of recession or expansion. Although *a priori* business cycle dating is not assumed here, dates established by the NBER are used as a convenient reference to validate the model. Another contribution of this paper is the consideration of a bidirectional relationship linking term spread and business cycle. With the exception of Ang et al. (2006), Chauvet and Senyuz (2016) and Diebold et al. (2006), the works cited above only consider a one-way relationship between term spread and real economic activity.

In fact, a bidirectional relationship between term spread and the business cycle can be expected from the following rationale. The slope of the yield curve, or term spread, is usually positive since, for fixed-income assets with similar risk, long-term interest rates are higher than short-term rates given the time preference and risk aversion of buyers. With the anticipation of a recession, yield curve flattens and term spread diminishes, and may even become inverted, with short-term rates exceeding long-term rates. This inversion is generally explained by the expected monetary policy, which in response to inflationary pressures, has the effect of increasing short-term interest rates. Once a recession begins, monetary policy often turns from contractive to expansive. Subsequent reduction of short-term interest rates causes the yield curve to recover its positive slope. For its part, long-term interest rates reflect expectations about a sequence of short-term interest rates covering a given timeframe and, therefore, incorporate expectations of future monetary policy, among other fundamental variables (Haubrich, 2020). With the expectation of a recession, long-term rates anticipate an expansionary monetary response and fall, contributing to the flattening of the yield curve before recessive periods. Contrarily, during expansionary phases, procyclical inflationary pressures contribute to creating the expectation of a restrictive monetary policy response, thus leading to a rise in long-term interest rates and widening term spread. This stylized process is subject to other variables that limit the role of the term spread as a predictor of the business cycle although the dynamics described are generally considered accurate.

Chauvet and Senyuz (2016) takes into account the bidirectional relationship between term spread and business cycle. Using a multivariate joint bi-factor model, they extract two latent Markov switching variables: one from monthly industrial production and another from empirical proxies for the level, slope, and curvature of the yield curve. Both latent factors follow an unobservable autoregressive process, the intercepts of which are functions of two distinct Markov switching variables. This model anticipates all business cycle peaks and troughs both in-sample and out-of-sample. In line with Chauvet and Senyuz (2016), we use a multivariate Markov switching model with two monthly latent factors: a first factor for real activity drawn from four coincident indicators widely used in the literature,[1] and a second factor which only considers the term spread.[2] Two distinct latent Markov variables were extracted: the first replicates the phases of the business cycle, according to the NBER dating committee, while the latent factor from the term spread permits the identification of two regimes, that is, an "ordinary regime" (positively sloping yield curve) and a "flattening regime" where the yield curve flattens or reverses its slope. It is assumed that the transition probabilities of both Markov processes are time-varying, that is, the business cycle factor depend on the term spread while the transition probabilities between term spread regimes depend on the business cycle factor.[3] Thus, regime changes in both factors are mutually dependent without forcing regime changes to occur at the same time. Thanks to this, we can test the existence of the four possible interactions between regime changes. For this, in both sets of transition probabilities, a Markov switching parametric change is assumed according the respective regime in business cycle and term spread.[4] We tested for asymmetries in the bidirectional relationship between the two latent variables. We estimated the proposed model using a Bayesian approach, employing a Metropolis algorithm to estimate the posterior marginal densities of the parameters.[5]

By analyzing transition and forecasted probabilities, we found a systematic alternation of regimes in both factors. Business cycle recessions initiate a regime of increasing term spread after a flattening phase. Once an ordinary phase of the term spread has come to an end, the flattening regime anticipates a phase change in the business cycle. However, according to the estimated time-varying transition probabilities, during the expansion phase, the cyclical factor does not anticipate the beginning of a flattening regime of the yield curve. The flattening regime precedes a recessive period, which is consistent with the empirical findings. Once the recession has begun, the business cycle factor significantly anticipates a change in the regime of the term spread, in line with the habitual expansionary monetary policy response to recessions and the consequent changes in long-term expectations. Based on this bidirectional dynamics, our model confirmed the beginning of a recession in March of 2020, which can be exclusively attributed to the economic measures adopted in response to the COVID-19 pandemic. Although not the primary goal of this work, we carried out an out-of-sample prediction exercise that served to confirm the beginning of the COVID-19 recession in March of 2020, and the corresponding start of the ordinary regime of the term spread.

This paper proceeds as follows: in Section 2 we will present the two-factor Markov switching model with time-varying transition probabilities; in Section 3 we will explain and interpret the results of the estimation of the model and assess its empirical accuracy; in Section 4 we will verify the predictive ability of the model and conduct the forecasting exercise mentioned. Finally, we offer our conclusions. In the Appendix, the likelihood function of the proposed model is exposed.

[1] For example in Chauvet (1998), Kim and Nelson (1998) and Stock and Watson (1991).

[2] In this paper, the term spread refers to the difference in return between long-term and short-term government bonds. Specifically the spread between 10 year and 3 month Treasury Constant Maturity rates (code T10Y3MM in the Federal Reserve Bank of St. Louis Economic Data).

[3] Chauvet and Senyuz (2016) establish the lead-lag relationship between the yield factor and the economic factor in the transition matrix that relates both factors, while, for our part, said relationship occurs through the respective time-varying transition probabilities of each factor.

[4] These parametric changes could fit some structural breaks mentioned in literature and the consequent predictive failures. As will be shown, our model has correctly signaled all recessions.

[5] In Markov switching models, a Gibbs sampling approach has been more usual (Kim & Nelson, 1999).

2. A two-factor Markov switching model with time-varying transition probabilities

As mentioned, we applied a two-factor Markov switching model with time-varying transition probabilities in both factors. The factor for the entire economy (first submodel) is estimated from the four coincident indicators by Stock and Watson (1991); the factor from the term spread (second submodel) only considers the spread between 10 year and 3 month Treasury constant maturities.[6]

The Markov switching cyclical factor model was taken from Kim (1994) and Kim and Nelson (1998) who modified the Stock and Watson (1991) dynamic common factor model by embodying the Markov regime change by Hamilton (1989) in the common factor. Stock and Watson (1991) elaborate a coincident economic indicator model using four monthly series, similar to that of the U. S. Department of Commerce (DOC Index). The series that compose the index are the Index of Industrial Production, Personal Income less Transfer Payments, Manufacturing and Trade Industries Sales, and the Number of Nonfarm Employees.[7] Stock and Watson (1991) assume the cyclical component is unobservable and common to the four series, in that each series is the result of the sum of this common component and another specific or idiosyncratic. An autoregressive structure was used to model both these common and specific factors.

According Hamilton (1989), the stationary log-transformed, $\Delta \log Y_t$, of a trended series Y_t is assumed to follow a two regime process, such as $\phi(L)(\Delta \log Y_t - \mu(S_t)) = \eta_t$, with $\phi(L)$ an autoregressive polynomial and $\eta_t \sim iidN(0, \sigma_\eta^2)$. The changing mean growth of Y_t, $\mu(S_t)$, depends on a two-state non-observed variable $S_t = \{0, 1\}$, which follows a first order Markov process. In conventional business cycle analysis, $S_t = 0$ denotes a recessive state and $S_t = 1$ an expansionary state. In this case, in $\mu(S_t) = \mu_0(1 - S_t) + \mu_1 S_t$, $\mu_1 > \mu_0$ is expected. Kim and Nelson (1998) assume the non-observed common cyclical factor of Stock and Watson (1991) is affected by Markov regime changes. Consequently, every demeaned growth series,[8] $\Delta \log y_{i,t} = \Delta \log Y_{i,t} - \overline{\Delta \log Y_i}$ (with $\overline{\Delta \log Y_i}$ the observed mean growth of $Y_{i,t}$) depends linearly on a common and a specific component:

$$\Delta \log y_{i,t} = \gamma_i C_t + C_{i,t} + \eta_{i,t} \tag{1a}$$

where γ_i is a factor loading and $\eta_{i,t} \sim iidN(0, \sigma_{\eta,i}^2)$. The specific component of $\Delta \log y_{i,t}$, $C_{i,t}$, can be assumed to follow an autoregressive process:

$$\phi_i(L) C_{i,t} = \varepsilon_{i,t} \tag{1b}$$

where $\varepsilon_{i,t} \sim iidN(0, \sigma_{\varepsilon,i}^2)$. The correlation between series is captured exclusively by the common cyclical component C_t and, accordingly, $E[\eta_{i,t}\eta_{j,s}] = 0$, $E[\varepsilon_{i,t}\varepsilon_{j,s}] = 0$ for $i \neq j$ and $t \neq s$, and $E[\eta_{i,t}\varepsilon_{j,s}] = 0$ for every i, j, t and s. The Markov regime changes affect the intercept of C_t which can be modelled as.[9]

$$\phi_C(L) C_t = \delta_C(S_t) + \varepsilon_{C,t} \tag{1c}$$

where $\delta_C(S_t) = \delta_{C,0}(1 - S_t) + \delta_{C,1} S_t$, and $\varepsilon_{C,t} \sim iidN(0, \sigma_C^2)$ is uncorrelated with $\eta_{i,t}$ and $\varepsilon_{i,t}$ in all leads and lags.

The probabilities of being at states 0 or 1 are assumed to follow a first-order Markov process, so the probability of being in regime S_t at period t depends only on the previous regime S_{t-1}. For example, $p(S_t = 0/S_{t-1} = 1, Y_{t-1}, \theta) = p_{10,t}$ denotes the probability of entering a recession at t after a period of expansion at $t - 1$. Transition probabilities may be time-invariant or, alternatively, depend on certain economic fundamentals as assumed here when conditioning to Y_{t-1}. In the transition matrix $p_t = \begin{bmatrix} p_{00,t} & p_{10,t} \\ p_{01,t} & p_{11,t} \end{bmatrix}$, the columns sum up the unity: under two regimes, from a given state at $t - 1$, at period t the system necessarily continues in the same regime or switches to the alternative. Apart from the transition probabilities, filtered and forecasted probabilities are also considered. Filtered probabilities, $p(S_t = i/Y_t, \theta)$ with $i = \{0, 1\}$, depend on sample information until period t, Y_t, and on the complete model whose parameters are denoted by θ (see Appendix). Forecasted probabilities conditioned by information Y_t are obtained from $\begin{bmatrix} p(S_{t+h} = 0/Y_t, \theta) \\ p(S_{t+h} = 1/Y_t, \theta) \end{bmatrix} = \begin{bmatrix} p_{00,t+h} & p_{10,t+h} \\ p_{01,t+h} & p_{11,t+h} \end{bmatrix} \begin{bmatrix} p(S_{t+h-1} = 0/Y_t, \theta) \\ p(S_{t+h-1} = 1/Y_t, \theta) \end{bmatrix}$, with $h = 1, 2, .,$ the forecasting horizon.

Time-varying transition probabilities depending on economic fundamentals can be modelled following a logistic form (Diebold, Lee, & Weinbach, 1994; Filardo, 1994) or a probit form (Filardo & Gordon, 1998; Kim & Yoo, 1995) as assumed here. In the latter case, a latent variable S_t^* is defined as $p(S_t = 0) = p(S_t^* < 0)$ and $p(S_t = 1) = p(S_t^* \geq 0)$, where

$$S_t^* = \alpha_{C,1}(1 - S_{t-1}) + \alpha_{C,2} S_{t-1} + (\alpha_{C,3}(1 - S_{t-1}) + \alpha_{C,4} S_{t-1}) x_{t-1} + \xi_t \tag{2}$$

for $S_{t-1} = \{0, 1\}$ and $\xi_t \sim iidN(0, 1)$. In expression (2) a constant and only one predetermined variable, x_{t-1}, are considered. The transition probabilities of remaining in the same regime at t as at $t - 1$ are given by

$$p_{00,t} = p(S_t = 0/S_{t-1} = 0, x_{t-1}) = p(\xi_t < -(\alpha_{C,1} + \alpha_{C,3} x_{t-1})) = \Phi(-(\alpha_{C,1} + \alpha_{C,3} x_{t-} \tag{3a}$$

$$p_{11,t} = p(S_t = 1/S_{t-1} = 1, x_{t-1}) = p(\xi_t \geq -(\alpha_{C,2} + \alpha_{C,4} x_{t-1})) = 1 - \Phi(-(\alpha_{C,2} + \alpha_{C,} \tag{3b}$$

where $\Phi(.)$ denotes the cumulative density function of the standard normal distribution. The corresponding transition probabilities of regime change are $p_{01,t} = 1 - p_{00,t}$ and $p_{10,t} = 1 - p_{11,t}$. In the submodel corresponding to the common cyclical factor, x_{t-1} is the lagged term spread sp_{t-1}.

This refers to the common cyclical factor. For the submodel of the term spread this series alone is considered; thus, Eq. (1a) lacks a specific component and its factor loading is identically equal to 1. Hence,

$$sp_t = \beta_{sp,t} + \eta_{sp,t} \tag{4a}$$

and the analogous to Eq. (1c) is

$$\phi_{sp}(L) \beta_{sp,t} = \delta_{sp}(S_{sp,t}) + \varepsilon_{sp,t} \tag{4b}$$

with $\delta_{sp}(S_{sp,t}) = \delta_{sp,0}(1 - S_{sp,t}) + \delta_{sp,1} S_{sp,t}$, and $S_{sp,t} = \{0, 1\}$ a latent variable corresponding to the flattening and the ordinary regimes of the yield curve respectively. Error terms are distributed according to $\eta_{sp,t} \sim iidN(0, \sigma_{\eta,sp}^2)$ and $\varepsilon_{sp,t} \sim iidN(0, \sigma_{sp}^2)$. For the term spread, in equations analogous to (2) and (3), the predetermined variable x_{t-1} corresponds to the common cyclical factor, C_{t-1}, estimated from the first submodel. Both submodels, for the common cyclical factor and the term spread, are expressed in state-space form.

The likelihood function of the complete model, $f(Y_t/\theta)$, is presented in the Appendix. In $f(Y_t/\theta)$, Y_t consists of sample information

[6] Thus, apart from observation noise, the term spread and its estimated factor are practically similar.

[7] The monthly series used here are the Index of Industrial Production (seasonally adjusted, code INDPRO), Real Personal Income excluding current transfer receipts (billions of chained 2012 dollars, seasonally adjusted, code W875RX1), Real Manufacturing and Trade Industries Sales (millions of chained 2012 dollars, seasonally adjusted, code CMRMTSPL), and the series of All Employees (total non-farm, thousands of persons, seasonally adjusted, code PAYEMS). Source: Federal Reserve of St. Louis Economic Data, https://fred.stlouisfed.org.

[8] Demeaning $\Delta \log Y_{i,t}$ in estimation avoids the problem of overidentification in determining the specific component $C_{i,t}$ (see Kim & Nelson, 1999, p. 50).

[9] This specification differs from Hamilton's, where $\phi(L)(\Delta \log Y_t - \mu(S_t)) = \eta_t$. In Eq. (1c), we have followed Chauvet (1998) and Kim and Yoo (1995). Note that $\mu(S_t) = \phi_C^{-1}(1)\delta_C(S_t)$.

until period t, and θ is the vector of parameters. The posterior density function of θ, according to Bayes rule, is

$$f(\theta/Y_t) = \frac{f(Y_t/\theta)f(\theta)}{f(Y_t)} \propto f(Y_t/\theta)f(\theta) \quad (5)$$

where $f(\theta)$ is the prior density function of the parameters, and $f(Y_t) = \int f(Y_t/\theta)f(\theta)d\theta$ is the marginal density of the data, which, not being dependent on θ, does not affect maximization of (5) when the posterior mode is obtained. The posterior density $f(\theta/Y_t)$ is a non-linear function of the parameters in which a closed form is not available. To estimate this density, a sampling method such as the Metropolis algorithm[10] can be used. This algorithm approximates the posterior density (or any target density) by generating a Markov chain $\{\theta^j\}$ which, under certain regularity conditions (Chib & Greenberg, 1996), converges to $f(\theta/Y_t)$. The procedure implemented is as follows:

i) a starting point θ^0 is chosen, in our case, the posterior mode;
ii) a proposal θ^* from a symmetric jumping distribution, $J(\theta^*/\theta^{j-1})$, is drawn.[11] A Gaussian jumping distribution is employed here:

$$J(\theta^*/\theta^{j-1}) = N(\theta^{j-1}, c^2\Sigma_m) \quad (6)$$

where Σ_m is the inverse of the Hessian matrix computed at the posterior mode of $f(\theta/Y_t)$. The parameter c is a scale factor or tuning parameter;
iii) the acceptance ratio, $r = \frac{f(\theta^*/Y_t)}{f(\theta^{j-1}/Y_t)}$, is computed;
iv) the proposal θ^* is accepted or rejected according to the following:

$$\theta^j = \begin{cases} \theta^* & \text{withprobability } \min(r, 1) \\ \theta^{j-1} & \text{otherwise} \end{cases}$$

Note that, if $r > 1$, θ^* is clearly accepted because θ^* increases the posterior density. But if $r < 1$, θ^* is accepted with probability equal to r, and therefore r must be compared with a realization of a random variable, typically $U(0, 1)$.
v) accepted or rejected θ^*, return to step ii) until convergence.

The Markov chain $\{\theta^j\}$ approximates the posterior density $f(\theta/Y_t)$ for a sufficiently large number of realizations. The scale factor c in (6) must be selected by balancing two opposing tendencies. If c is too small, acceptance rates will be high and θ^* convergence will slow down. On the contrary, if c is too large, r will be low and θ^* will fall into regions with low probability density, remaining excessively in them. Under these considerations, the judgment on convergence must be based on simultaneously considering the mean value of r throughout the iterations and visualizing the graphs and histograms of the marginal distributions of θ. When appropriate, it will be necessary to adjust the value of the scale factor accordingly.

3. Estimation results

The results of the estimated model are presented in Table 1. The model was estimated using two sample periods. The first period, 1960m02 to 2020m03, ends the month when the economic impact of COVID-19 begins, causing a significant reduction in the values of the four activity indicators. The second period is until 2020m06. In the months of April, May and June, the four activity indicators showed extremely atypical growth rates, so we proceeded to estimate dummy variables whose values are shown in the following Section (Table 4). The first and third columns of Table 1 show the estimated values of the posterior modes and their corresponding standard errors resulting from the maximization of the posterior density (see Appendix). The second and fourth columns present the results of the posterior means and standard errors of the parameters resulting from 10,000 iterations of the Metropolis algorithm for the posterior density. We have used a scale factor (see Eq. (6)) of 0.30 in

Table 1
Posterior modes, posterior means and standard errors (se) of the parameters of the regime switching model of equations (1) to (4).

Estimation period:	1960m02-2020m03		1960m02-2020m06	
	posterior mode (se)	posterior mean (se) 10,000 draws	posterior mode (se)	posterior mean (se) 10,000 draws
γ_2	0.4167	0.4195	0.4169	0.4261
	(0.0389)	(0.0392)	(0.0388)	(0.0388)
γ_3	0.8957	0.8954	0.8947	0.9101
	(0.0614)	(0.0608)	(0.0613)	(0.0655)
γ_4	0.2640	0.2680	0.2651	0.2689
	(0.0160)	(0.0167)	(0.0162)	(0.0177)
$\sigma^2_{\eta,1}$	0.1120	0.1170	0.1118	0.1133
	(0.0223)	(0.0213)	(0.0223)	(0.0216)
$\sigma^2_{\eta,2}$	0.1371	0.1362	0.1366	0.1366
	(0.0283)	(0.0259)	(0.0281)	(0.0253)
$\sigma^2_{\eta,3}$	0.3141	0.3229	0.3133	0.3205
	(0.1076)	(0.0870)	(0.1072)	(0.0887)
$\sigma^2_{\eta,4}$	0.0156	0.0158	0.0156	0.0156
	(0.0014)	(0.0014)	(0.0014)	(0.0013)
$\sigma^2_{\eta,sp}$	0.0245	0.0250	0.0245	0.0253
	(0.0028)	(0.0028)	(0.0028)	(0.0029)
ϕ_C	0.4084	0.4196	0.4093	0.4254
	(0.0653)	(0.0686)	(0.0653)	(0.0665)
ϕ_{sp}	0.8682	0.8703	0.8706	0.8717
	(0.0161)	(0.0169)	(0.0159)	(0.0168)
σ^2_C	0.1845	0.1817	0.1831	0.1818
	(0.0245)	(0.0244)	(0.0244)	(0.0253)
$\sigma^2_{\varepsilon,1}$	0.1120	0.1132	0.1118	0.1184
	(0.0223)	(0.0220)	(0.0223)	(0.0203)
$\sigma^2_{\varepsilon,2}$	0.1371	0.1404	0.1366	0.1395
	(0.0283)	(0.0254)	(0.0281)	(0.0285)
$\sigma^2_{\varepsilon,3}$	0.3141	0.3198	0.3133	0.3247
	(0.1076)	(0.0884)	(0.1072)	(0.0883)
$\sigma^2_{\varepsilon,4}$	0.0156	0.0159	0.0156	0.0159
	(0.0014)	(0.0014)	(0.0014)	(0.0015)
σ^2_{sp}	0.0904	0.0925	0.0901	0.0918
	(0.0066)	(0.0063)	(0.0065)	(0.0070)
$\delta_{C,0}$	-0.5386	-0.5116	-0.5273	-0.4878
	(0.0851)	(0.0868)	(0.0842)	(0.0868)
$\delta_{C,1}$	0.1136	0.1105	0.1240	0.1220
	(0.0223)	(0.0239)	(0.0228)	(0.0221)
$\delta_{sp,0}$	-0.1670	-0.1627	-0.1651	-0.1658
	(0.0263)	(0.0266)	(0.0263)	(0.0284)
$\delta_{sp,1}$	0.1871	0.1868	0.1833	0.1813
	(0.0250)	(0.0253)	(0.0248)	(0.0249)
$\alpha_{C,1}$	1.7058	1.7711	1.7135	1.8030
	(0.2811)	(0.3085)	(0.2801)	(0.2922)
$\alpha_{C,3}$	-0.7331	-0.8244	-0.7385	-0.8344
	(0.2625)	(0.2698)	(0.2616)	(0.2653)
$\alpha_{C,2}$	-3.2733	-3.6699	-3.2668	-3.6428
	(0.5182)	(0.7226)	(0.5163)	(0.8929)
$\alpha_{C,4}$	-1.0414	-1.2708	-1.0391	-1.2983
	(0.3320)	(0.4493)	(0.3315)	(0.5812)
$\alpha_{sp,1}$	2.1175	2.2489	2.0718	2.1835
	(0.2837)	(0.3572)	(0.2786)	(0.2866)
$\alpha_{sp,3}$	2.0000	2.2947	2.0041	2.3395
	(0.5033)	(0.6315)	(0.5079)	(0.6181)
$\alpha_{sp,2}$	-1.7209	-1.7458	-1.7100	-1.7477
	(0.1738)	(0.1875)	(0.1736)	(0.1671)
$\alpha_{sp,4}$	-0.4003	-0.4729	-0.4132	-0.5068
	(0.1974)	(0.2019)	(0.2019)	(0.2030)

[10] Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953) and Metropolis and Ulam (1949).
[11] The symmetry of the jumping distribution implies that $J(\theta^i/\theta^j) = J(\theta^j/\theta^i)$ for all i and j. Hastings (1970) generalizes Metropolis algorithm to include asymmetric jumping distributions. In this case the acceptance ratio is calculated as $r = \frac{f(\theta^*/Y_t)J(\theta^{j-1}/\theta^*)}{f(\theta^{j-1}/Y_t)J(\theta^*/\theta^{j-1})}$.

the first sample period, and 0.25 in the second, after testing other values. Visual convergence towards the posterior marginal densities of each of the parameters can be admitted.[12] The acceptance rates for both sample periods were 42.7 % and 43.6 % respectively.

Unlike the Gibbs algorithm, the Metropolis algorithm does not require natural conjugate priors to obtain the conditional posterior distributions. As a consequence we can choose prior densities without this limitation. For the prior densities of the parameters we opted for zero-avoiding prior distributions and vague priors, imposing only *a priori* knowledge of the signs of the parameters. Except for the autoregressive parameters, for the rest of the parameters we assumed a Gamma distribution $\Gamma\left(\frac{\nu}{2}, \frac{\delta}{2}\right)$ with $\nu = 6$ and $\delta = 2$, on which we have imposed sign restrictions. This distribution, defined for positive values, has an expectation of $\frac{\nu}{\delta} = 3$ and a variance of $\frac{2\nu}{\delta^2} = 3$, values that hardly restrict the search region. For the factor loadings, given the procyclical nature of the activity indicators, we assumed that $\gamma_i \sim \Gamma(3, 1)$ for $i = 2, 3, 4$. The factor loading corresponding to the Index of Industrial Production, γ_1, was standardized to the unit (so, it was not estimated) in order to allow the identification of the model. For all variances in Table 1 it was assumed that $\frac{1}{\sigma_k^2} \sim \Gamma(3, 1)$. On the other hand, for the autoregressive parameters of order 1 of the common cyclical factor and of the term spread, it was assumed that $\phi_C, \phi_{sp} \sim B(4, 4)$. This is a symmetric Beta distribution bounded between 0 and 1 that excludes these extreme values, and thus imposes stationarity and persistence in C_t and $\beta_{sp,t}$. In the parameters of $\delta_C(S_t)$ and $\delta_{sp}(S_{sp,t})$, depending on the regimes, negative signs were assumed in the recessive regime and in the flattening regime of the term spread, so that $\delta_{C,0}, \delta_{sp,0} \sim -\Gamma(3, 1)$; and positive in the respective alternative regimes, $\delta_{C,1}, \delta_{sp,1} \sim \Gamma(3, 1)$.

Posterior distributions of the factor loadings are sufficiently far from the null value: all factor loadings were statistically significant. None of the specific components of the four indicators has a significant autoregressive autocorrelation. The common cyclical factor, C_t, shows the expected statistical characteristics. The values of the intercept of C_t, $\delta_C(S_t)$, in both regimes, recessive, $\delta_{C,0}$, and expansive, $\delta_{C,1}$, are statistically significant. Minimal differences were found in the estimated values of both sample periods. The classification that $\delta_C(S_t)$ induces in the sample period, recessive and expansive, satisfactorily coincides with the dating of the business cycle of the NBER[13] (see Fig. 1 and Table 2). Of the 96 months the NBER classifies as recessive in the period 1960m02–2020m06, the filtered probabilities (posterior means) identify 86, some 90 %, when we consider a period as recessive if $p(S_t = 0/Y_t) > 0.5$. The 10 discrepancies occur immediately before or after recessions.[14] The Quadratic Probability Score[15] is 0.07 applying the same cut-off probability criterion.

The term spread had an average value of 1.43 % with a standard deviation of 1.20 % throughout the sample period 1960m02 to 2020m06. This series is represented in Fig. 2, showing oscillations within maximum and minimum values of 4.15 % and −2.65 %, together with the filtered probabilities of the flattening regime. The

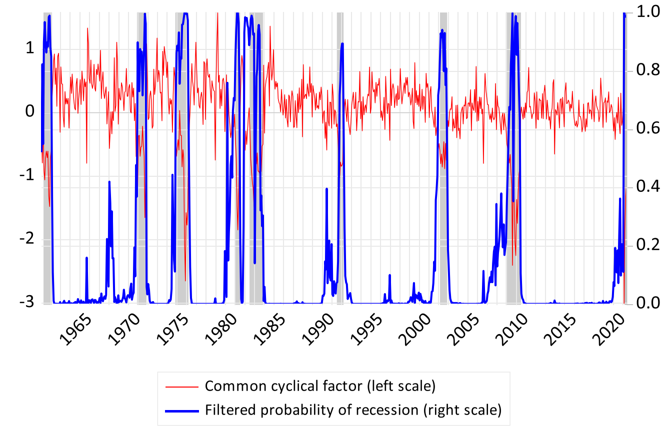

Fig. 1. Common cyclical factor C_t (posterior means) and filtered probabilities of recession (posterior means). Period: 1960m02–2020m06.

two regimes detected in the intercept of the demeaned term spread, $\delta_{sp}(S_{sp,t})$, show negative or close to zero values ($\delta_{sp,0} = -0.16$) for the flattening regime, and positive ($\delta_{sp,1} = 0.18$) for the ordinary regime (see Table 1). Fig. 2 shows how, after a flattening phase of significant duration, the economy enters into recession. Once the recession started, the term spread jumps to its ordinary regime. The ordinary regime overlaps a stretch of the expansionary phase of the business cycle, until, at a certain time, the term spread changes to a flattening regime. With the exception of the second half of the 1960 s, with few periods of high term spread, this pattern is repeated throughout the sample.

Table 2 shows the dating corresponding to Figs. 1 and 2, and supports the empirical adequacy of the regime switching model estimated in Table 1. In order to date the periods of recession and flattening of the yield curve (and of their complementary regimes) a cut-off probability of 0.5 was used in the filtered probabilities (represented in Figs. 1 and 2). In addition, Table 2 shows the average term spread under each regime and its average duration. For the business cycle factor, C_t, the recessive and expansionary periods are also dated, indicating their average duration and the annualized growth rates of the activity indicators used for estimation. In the last column, the business cycle dating used by the NBER is given as a reference. From this dating, we find that the filtered probabilities of the recessive regime adjust acceptably well to the NBER dating, although discrepancies of a few months sometimes occur. In accordance with Fig. 1, the activity indicators decreased during recessive periods and increased in expansive periods. The values of their rates show the greater severity of the recessions of the mid-1970′s and early 1980′s, the Global Financial crisis of 2008, and the COVID-19 crisis since March 2020. A reduction in the magnitude of these rates during expansionary phases since 2001 are also observed. The average duration of the recessive periods, according to the cut-off probability criterion, was 12.2 months with a standard deviation of 6.2 months; expansionary periods had a duration of 76.5 months with a high standard deviation of 42.7 months. Regarding the dating of the flattening and ordinary periods of the term spread, as a general rule, recessions are always preceded by a flattening period and followed by a period of positive term spread. The estimated model correctly discriminates between both regimes: in the flattening regime periods, the average level of the term spread is close to 0 % or even negative, while in ordinary periods it always exceeds 1.5 %. The mean duration of the flattening periods was 34.7 months with a standard deviation of 26.0 months while the figures for the ordinary regime are 41.7 and 31.4, respectively.

Notwithstanding the mentioned time pattern between term spread regimes and business cycle phases, Fig. 2 and the dating of

[12] The histograms and graphs of the parameters for the 10,000 iterations for the first sample period (for the second, few changes were observed) are available upon request.

[13] https://www.nber.org/cycles.html.

[14] The cyclical factor C_t also shows a high contemporary correlation of 76 % with the growth rate of the Coincident Economic Activity Index from the Federal Reserve Bank of Philadelphia from 1979m01 to the present (code USPHCI, Federal Reserve of St. Louis Economic Data). This index is not the DOC Index originally considered by Stock and Watson (1991).

[15] The Quadratic Probability Score, $QPS = \frac{2}{T}\sum_{t=1}^{T}(p_{t-s} - N_t)^2$, measures the proximity of the probability with the realization of the event. The prediction of the probability, p_{t-s}, is made s periods ahead, in our case $s = 0$ since filtered probabilities are used. The dummy variable, $N_t = \{0, 1\}$, is 1 when the period t is recessive, according to the NBER. The QPS ranges between 0 and 2, where 0 represents maximum accuracy.

Table 2
Empirical adequacy of the estimated regime switching model (1) to (4): dating of the regimes of recession/expansion and of the flattening/ordinary regimes of the yield curve. In parenthesis, duration in months. Estimation period: 1960m02–2020m06.

Term spread periods (prob > 0.5) and duration (months)	Average term spread	Business cycle periods (prob > 0.5) and duration (months)	Annualized mean growth rate of activity indicators				Recessions according to NBER (peaks and troughs)
			IPI	Income	Sales	Employment	
1960m02–1960m05 (5)	0.79	1960m05–1961m02 (10)	-7.6	0.3	-5.7	-2.8	1960m04–1961m02
1960m06–1962m01 (20)	1.53	1961m03–1970m03 (109)	5.9	4.9	4.7	3.2	
1962m02–1970m01 (96)	0.50	1970m04–1970m11 (8)	-5.9	-0.3	-2.8	-2.2	1969m12–1970m11
1970m02–1972m10 (33)	1.71	1970m12–1973m11 (36)	7.8	5.1	8.8	3.4	
1972m11–1974m08 (22)	-0.17	1973m12–1975m04 (17)	-10.0	-3.8	-8.6	-1.5	1973m11–1975m03
1974m09–1977m08 (36)	2.19	1975m05–1979m10 (54)	5.6	3.8	5.1	3.7	
1977m09–1980m04 (32)	0.23	1979m11–1980m07 (9)	-8.6	-1.5	-6.8	-0.9	1980m01–1980m07
1980m05–1980m09 (5)	1.94	1980m08–1980m12 (5)	13.4	6.7	12.5	2.9	
1980m10–1982m03 (18)	-0.14	1981m01–1982m12 (24)	-4.5	1.0	-3.9	1.2	1981m07–1982m11
1982m04–1985m11 (77)	2.31	1983m01–1990m09 (93)	3.8	3.6	4.1	2.7	
1988m09–1990m08 (24)	0.44	1990m10–1991m05 (8)	-4.4	-2.2	-0.8	-1.7	1990m07–1991m03
1990m09–1995m01 (53)	2.69	1991m06–2000m12 (115)	4.3	3.9	4.3	2.1	
1995m02–1996m03 (14)	0.83						
1996m04–1996m09 (6)	1.55						
1996m10–2001m02 (52)	0.61	2001m01–2002m01 (13)	-4.1	-0.9	-1.2	-1.3	2001m03–2001m11
2001m03–2004m112 (46)	2.71	2002m02–2008m01 (72)	2.4	2.4	2.8	0.9	
2005m01–2008m01 (37)	0.41	2008m02–2009m06 (17)	-13.2	-4.8	-11.1	-3.9	2007m12–2009m06
2008m02–2016m04 (99)	2.46	2009m07–2020m02 (128)	1.7	2.8	2.6	1.4	
2016m05–2020m03 (47)	0.87	2020m03–2020m06 (4)	-31.3	-14.6	-3.7	-30.0	2020m02–2020m04
Full sample average values	1.43		2.2	2.9	2.7	1.5	

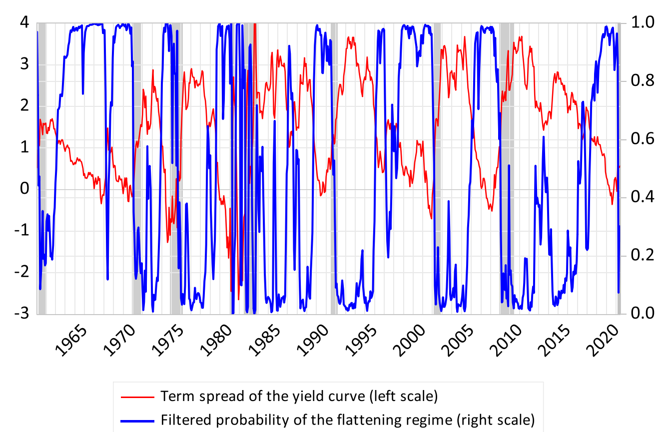

Fig. 2. Term spread of the yield curve and filtered probabilities of the flattening regime (posterior means). Period: 1960m02–2020m06.

Table 2 show short periods during which a jump to the alternative regime of the term spread occurred without being followed by a regime change in the business cycle. Of note is the variability in the sign of the term spread observed from 1980m10 to 1982m04,[16] and some transitory reductions in the spread in 1984m08–1984m10, 1985m12–1986m08 and 1986m12–1987m03. Within the long expansionary phase of the 1990 s[17] the prolonged period of flattening regime prior to the recession of 2000 stands out, interrupted from 1995m02–1996m03. Finally, the filtered probability shows a certain interruption of the ordinary regime period from 2001 to 2016 in the year 2012.

Given that the filtered probabilities show the expected succession of phases in both the term spread and the business cycle factor, we must now verify whether these variables determine the values of the transition probabilities, causing either the permanence or the regime changes. For this, the parameters $\alpha_{C,i}$ and $\alpha_{sp,i}$ are relevant. Given the negative signs that appear in Eqs. (3a) and (3b), in order to facilitate the interpretation, Table 1 presents the results incorporating such sign in the value of the estimation (the same was done with the prior densities $\Gamma(3, 1)$), then, $a_{C,i} = -\alpha_{C,i}$, $a_{sp,i} = -\alpha_{sp,i}$. For the transition probabilities of C_t, we have $p^C_{00,t} = \Phi(a_{C,1} + a_{C,3}sp_{t-1})$ and $p^C_{11,t} = 1 - \Phi(a_{C,2} + a_{C,4}sp_{t-1})$, and those of sp_t, $p^{sp}_{00,t} = \Phi(a_{sp,1} + a_{sp,3}C_{t-1})$ and $p^{sp}_{11,t} = 1 - \Phi(a_{sp,2} + a_{sp,4}C_{t-1})$. Thus, negative $a_{C,3}$ is interpreted as a reduction in the probability of remaining in recession (or, in other words, an increase in the probability of leaving it, since$p^C_{01,t} = 1 - p^C_{00,t}$) when the term spread increases. From an expansive phase, negative $a_{C,4}$ implies an increase in the probability of remaining in expansion as the term spread increases, and a consequent decrease in the probability of entering into recession. For the term spread, positive $a_{sp,3}$ indicates an increase in the probability of remaining in the flattening regime when C_t improves (since the flattening regime is left in recessions); while negative $a_{sp,4}$ indicates an increase in the probability of remaining in the ordinary regime of the term spread when C_t improves.

Although the filtered probabilities in Figs. 1 and 2 are highly informative, it is interesting to test the ability of the model to predict moments of regime change. For example, in the forecasted probability of recession for period t with information up to $t-1$, $p(S_t = 0/Y_{t-1}) = p^C_{00,t}p(S_{t-1} = 0/Y_{t-1}) + p^C_{10,t}p(S_{t-1} = 1/Y_{t-1})$, the first addend reflects the forecasted probability of remaining in recession, while the second, $p^C_{10,t}p(S_{t-1} = 1/Y_{t-1})$, that of entering into recession from expansion. Both contain information provided by the term spread. In Fig. 3, we have plotted these together with the transition probabilities, $p^C_{10,t}$.[18]

The values of $p^C_{10,t}p(S_{t-1} = 1/Y_{t-1})$ are low, some not exceeding 10 %, although it should be noted that the probability of entering into recession is conditioned by the fact we are in an expansive period. Do not forget that the greatest weight corresponds to the complementary probability, that of remaining in expansion, $p^C_{11,t}p(S_{t-1} = 1/Y_{t-1})$. The interesting thing is to interpret $p^C_{10,t}$ and

[16] In the first years of Volcker's term as Chairman of the Federal Reserve (August, 1979- August, 1987), drastic anti-inflationary measures were adopted, then softened once the inflation rate was substantially reduced around 1982. For a historical overview of the main features of monetary policy since the end of the 19th century, see Bordo and Schwartz (1999) and, for a more recent period, Benati and Goodhart (2010).
[17] The so-called Great Moderation years, Stock and Watson (2003).

[18] Specifically, and also hereinafter, we represent the posterior means obtained by averaging the set of probabilities obtained from the 10,000 draws of the parameter vector.

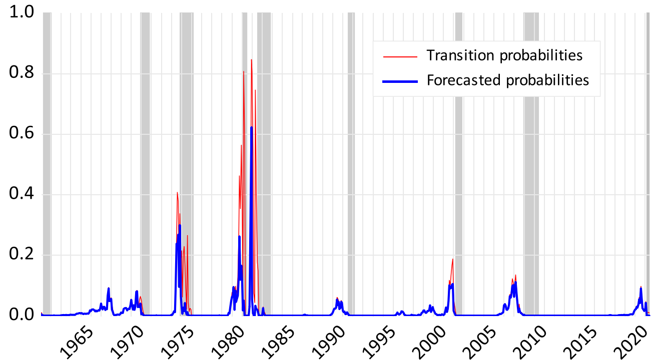

Fig. 3. Transition and forecasted probabilities (posterior means) of entering recession from expansion. Period: 1960m02–2020m06.

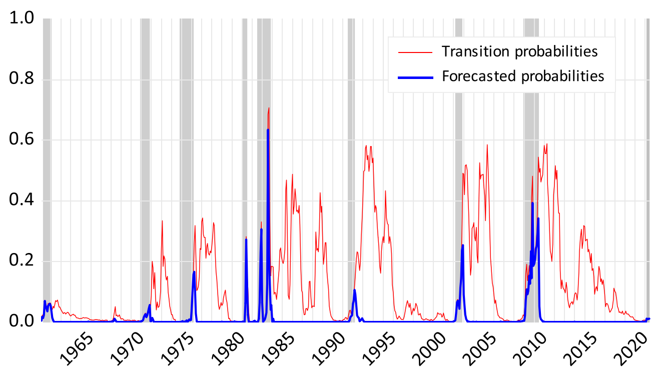

Fig. 4. Transition and forecasted probabilities (posterior means) of leaving a recession. Period: 1960m02–2020m06.

$p^C_{10,t}p(S_{t-1} = 1/Y_{t-1})$ as signals occurring just months before recessive periods begin (with the exception of a false signal around 1967). When recessions begin, these probabilities fall to zero because the conditioning has changed. In this case, we are interested in the forecasted probability of emerging from recession, $p^C_{01,t}p(S_{t-1} = 0/Y_t)$ (and to a lesser extent in the forecasted probability of remaining in recession, $p^C_{00,t}p(S_{t-1} = 0/Y_{t-1})$) which is represented in Fig. 4.

Fig. 4 shows how the term spread signals an exit from recessions as well as the consolidation of expansionary phases. Transition probabilities, $p^C_{01,t}$, show high values beginning in recessive periods and continue during much of the subsequent expansion. They largely coincide with the ordinary phase of the term spread. The values of $p^C_{01,t}p(S_{t-1} = 0/Y_{t-1})$ show the contribution of the increases of the term spread (caused mainly by monetary expansions that reduce short-term interest rates) in emerging from recessions. Once the expansion has started, the favorable contribution of the increase in the term spread continues by inducing high values in $p^C_{11,t}p(S_{t-1} = 1/Y_{t-1})$, that is to say, the forecasted probability of continuing in expansion.

While the term spread is clearly relevant in signaling the change of cyclical phase, when analyzing the opposite relationship, we can see that the cyclical component is only relevant in detecting the ending of the flattening regime; in other words, the variation of the term spread as a consequence of monetary expansions characteristic of recessions. Fig. 5 shows the transition probabilities of leaving the flattening regime and entering into an ordinary regime in the term spread, $p^{sp}_{01,t}$, as well as $p^{sp}_{01,t}p(S^{sp}_{t-1} = 0/Y_{t-1})$. It is evident that the end of the flattening regime coincides with the recessive phases. There are also some signals of exit from a flattening regime outside of recessive periods, coinciding with temporary increases of the term spread. On the contrary, the probability of a transition from an

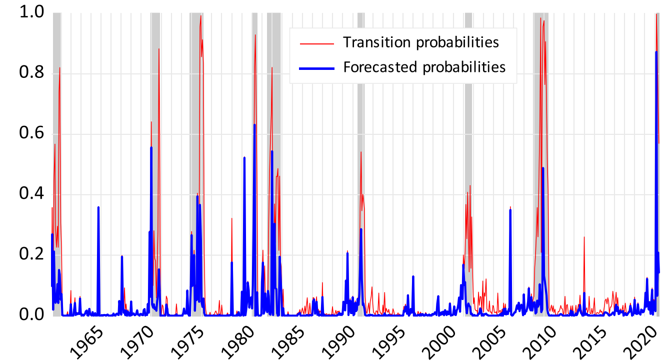

Fig. 5. Transition and forecasted probabilities (posterior means) of emerging from the flattening regime of the term spread (=beginning of the ordinary regime). Period: 1960m02–2020m06.

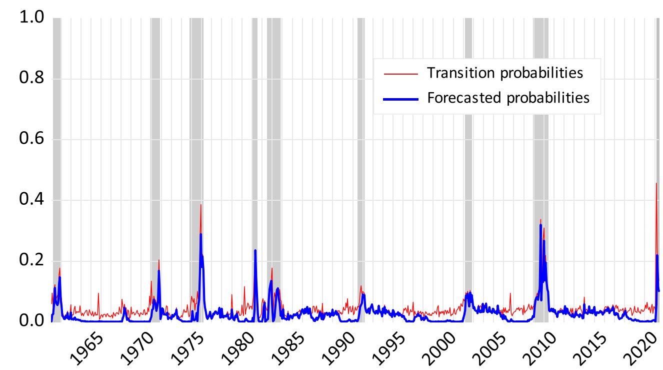

Fig. 6. Transition and forecasted probabilities (posterior means) of emerging from the ordinary regime of the term spread (=beginning of the flattening regime). Period: 1960m02–2020m06.

ordinary regime to a flattening regime of the term spread, $p^{sp}_{10,t}$ (Fig. 6), remains close to zero throughout the cyclical expansion phases, regardless of the regime change in the term spread that occurs during these periods (compare Figs. 1 and 2).

Summarizing, in this Section we have proceeded to relate business cycle and term spread through a model that admits a bidirectional relationship between both variables (Ang et al., 2006; Chauvet & Senyuz, 2016; Diebold et al., 2006) compared to the approach that only considers a one-way relationship that goes from the term spread to real economic activity. Unlike the usual modeling based on probit models that use the dating elaborated by the NBER as a dummy variable, the estimation of a regime switching model does not presuppose said dating but allows its estimation from sample information. The dating here obtained from activity indicators satisfactorily coincides with that of the NBER. This is in accordance with Chauvet (1998), Chauvet and Senyuz (2016), Kim and Yoo (1995) and Kim and Nelson (1998, 1999). Unlike Chauvet and Senyuz (2016), the bidirectional relationship between term spread and economic activity has been modeled using the transition probabilities of the regimes considered in each of the variables. The fact of supposing parameter changes in this relationship has made it possible to find an asymmetric behavior in the time sequence of the term spread and business cycle regimes.

4. In-sample and out-of-sample forecasting ability

In this Section we will analyze the forecasting ability of the regime switching model both in-sample and out-of-sample. Although in the previous section we verified the empirical adequacy of the model with regard to the dating of the business cycle phases and of

Table 3
Estimation of alternative probit models for the NBER recession periods. Significance levels: 10 % (*), 5 % (**), 1 % (***). Period: 1960m02–2020m06.

	Model #1	Model #2	Model #3
Constant	-0.6139***	0.7366***	-0.2611
	(0.1040)	(0.2253)	(0.2996)
C_{t-1}	-	-	-2.6846***
			(0.3299)
sp_{t-1}	0.5475***	-0.0929	-0.4141**
	(0.1261)	(0.1480)	(0.1865)
sp_{t-3}	-0.2176	-0.1063	-0.0280
	(0.1355)	(0.1319)	(0.1604)
sp_{t-6}	-0.3500***	-0.3650***	-0.3746***
	(0.1047)	(0.1019)	(0.1248)
sp_{t-12}	-0.6797***	-0.5622***	-0.2894**
	(0.0906)	(0.0903)	(0.1148)
Flattening regime: $S_{sp,t-1} = 0$	-	-1.7292***	-1.1722***
		(0.2602)	(0.3390)
McFadden R^2	0.33	0.43	0.68

the flattening periods of the yield curve, here we will first estimate a probit model in which the variable to explain is the dating of the recessive periods according to the NBER. Secondly, we will assess the out-of-sample forecasting ability by focusing on the recessionary period of 2020 caused by the COVID-19 pandemic.

Starting with the estimation of a probit model, in Table 3 the endogenous variable is a dummy variable that takes a value equal to 1 in the case of a recessionary period according the NBER. The results of the estimation of the three models allow us to assess the explanatory ability of the variables obtained by the regime switching model of the preceding Section. In addition to a constant, the term spread with different lags was included as an explanatory variable. The first column of Table 3 shows that exclusively considering the term spread to predict recessive periods offers an explanatory ability of 33 % (McFadden R^2). Note the positive sign of the one-month lagged term spread, while longer lags, 6 and 12 months, anticipate recessive periods with a negative sign. In the second column of Table 3, a lagged dummy variable that takes the value 1 when the term spread is in its flattening regime is added as an explanatory variable. A cut off probability of 0.5 was used to obtain it from the estimated filtered probabilities. Finally, in the third column, the lagged common cyclical factor, C_{t-1}, also obtained from our regime switching model, is added as an explanatory variable. This variable is also significant. Regarding its negative sign, remember that C_t is procyclical (positive factor loads in Table 1) while the endogenous variable of these probit models takes the value 1 in the event of a recession and 0 otherwise. In probit model #3, note that the term spread lagged one month has a negative sign while the significance and sign for lags of 6 and 12 months remains the same with respect to models #1 and #2. This is consistent with studies that analyze the forecasting ability of the term spread on recessions at horizons longer than one quarter (Dotsey, 1998; Dueker, 1997; Estrella & Mishkin, 1998; Wright, 2006). Furthermore, using information in addition to the term spread to forecast recessions is in line with Stock and Watson (1989). Finally, Fig. 7 shows the goodness of fit of probit models #2 and #3 of Table 3. It was observed that including the information of the cyclical indicators embodied in C_t reduces the brief (and false) recession signals produced by model #2 (also by model #1, not shown). Greater precision in the locating of recessive periods is also achieved, especially for the Global Financial crisis of 2008–09.

We now turn to the out-of-sample forecasting exercise. Note that, unlike single-equation probit models, the regime switching model estimated in Section 3 provides predictions of real activity, term spread and the respective probabilities of the regimes affecting both variables. These will be taken into consideration below. In 2020, the collapse of activity indicators as a result of the lockdowns caused

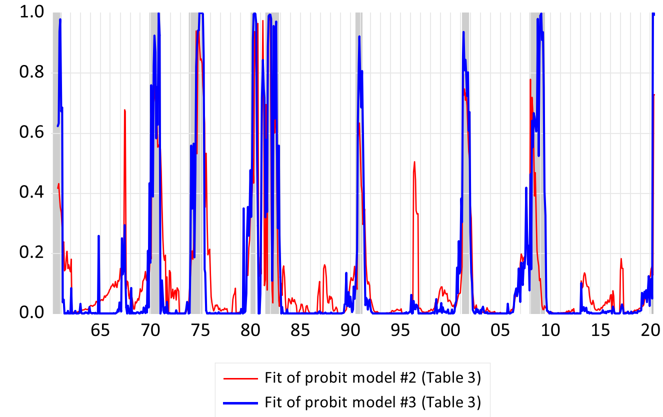

Fig. 7. Fit of probit models #2 and #3 of Table 3. Period: 1960m02–2020m06.

the NBER to date the end of the previous expansionary phase (a peak) in February 2020, declaring the start of a recession.[19] Using the estimations of Table 1, we take April 2020 and July 2020 as the beginning of two predictive horizons (left and right columns in Fig. 8 respectively), and December 2021 as the end.

According to the Bayesian methodology, predictive density is the distribution of the observed series y_{t+h} conditioned on sample information until period t and marginalized over the parameters θ

$$f(y_{t+h}/Y_t) = \int f(y_{t+h}/Y_t, \theta) f(\theta/Y_t) d\theta \tag{7}$$

where $f(\theta/Y_t)$ is the posterior density of the parameters according to (5). For each of the draws of θ from $f(\theta/Y_t)$, the prediction equations of the Kalman filter provide predictions of both the observation vector $y_{t/t-1}$ and the state vector $\beta_{t/t-1}$. When forecasting, lacking new sample information, the updating equations of the Kalman filter are not applied and the prediction equations must be iterated. Vector of observations, vector of states and probabilities converge to their corresponding steady state values. For each draw of the parameters, the forecasted probabilities of each regime depend on the transition probabilities as seen above. In turn, transition probabilities depend on $x_{t+h-1/t} = \{sp_{t+h-1/t}, C_{t+h-1/t}\}$, that is, the respective predictions of the term spread and of the cyclical factor obtained from the corresponding submodels, conditioned on information up to period t. Consequently, the number of draws of the prediction density $f(y_{t+h}/Y_t)$ is equal to that of the parameter vector θ. The corresponding fan plots of the predictions of the cyclical factor, term spread, and probabilities of recession and of flattening regime are shown in Fig. 8. To interpret Fig. 8, keep in mind that before the end of both sample periods (2020m03 and 2020m06) the dots correspond to the posterior means of either the estimated cyclical factor, C_t, or the probabilities indicated in each case. From these dates, the confidence bands of the predictions, whose central values correspond to their median value, are drawn. The term spread is an observed variable, so there is a tranche (Figures 8.3) of actual observations that has been drawn alongside its predictions.

In the four activity indicators (Industrial Production Index, Personal income, Sales, and Employment) for the estimation period ending in June 2020, it was necessary to model the three monthly rates of growth of April, May and June using dummy variables (see Table 4). Note that the three numerical magnitudes, both in April,

[19] "In the case of the February 2020 peak in economic activity, the committee concluded that the subsequent drop in activity had been so great and so widely diffused throughout the economy that, even if it proved to be quite brief, the downturn should be classified as a recession". The NBER dated the end of the recession in April 2020 being the shortest recession on record. https://www.nber.org/research/business-cycle-dating.

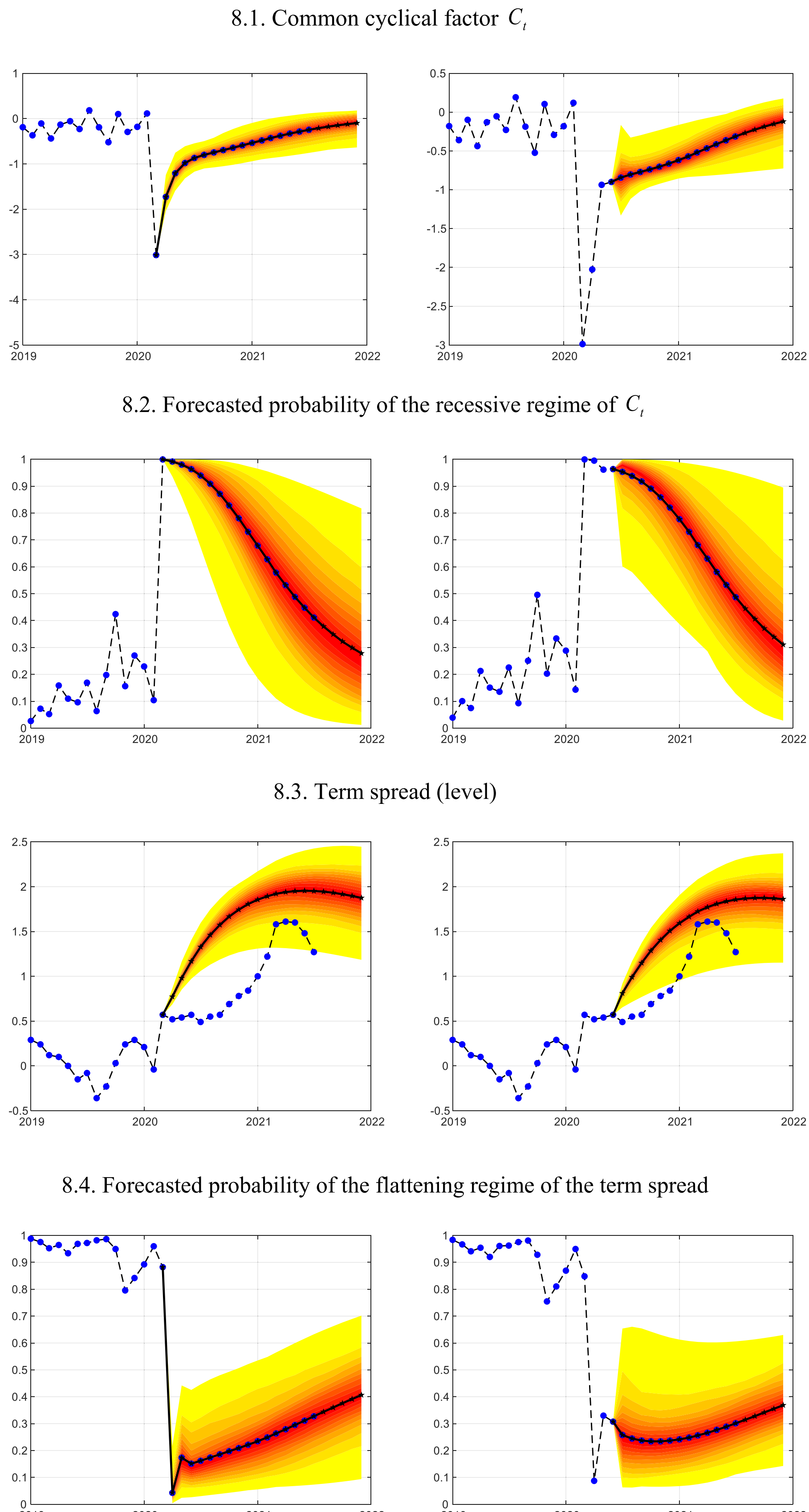

Fig. 8. Posterior predictions (fan plots). Estimation periods: 1960m02–2020m03 (left column) and 1960m02–2020m06 (right column).

Table 4
Outliers values (posterior means) for the activity indicators during the COVID-19 crisis. Estimation period: 1960m02–2020m06.

	IPI	Income	Sales	Employment
2020m04	-12.2846	-5.6033	-10.6943	-14.4586
	(0.644)	(0.5430)	(0.9402)	(0.2324)
2020m05	2.0594	1.7935	8.2028	2.2888
	(0.6168)	(0.5463)	(0.9108)	(0.2160)
2020m06	6.7048	1.5638	7.3085	3.6774
	(0.6483)	(0.5460)	(0.9375)	(0.2242)

with a pronounced decrease, and in May and June, with large increases, have no precedents in the entire sample period. As they respond to information impossible to predict with data until March, the predictions made with information up to that month anticipated a recovery in growth rates reaching non atypical positive values around the third quarter of 2020.

The predictions of the common cyclical component, C_t, with information up to March and June differ only slightly given the effect of modeling with dummy variables. The filtered probability of recession (with information up to March) jumped abruptly in March to unity from the previous low values (Fig. 8): although in October 2019 the probability of recession reached 40 %, in November and December it fell below 30 %, reaching approximately 10 % in February 2020. This is consistent with activity indicators which showed improvements in the months of January and February. In other words, with information up to February 2020, the possibility of entering into recession in 2020 seemed to be receding, a process that abruptly reversed in March with the crisis and lockdowns caused by COVID-19.

The term spread until March 2020 was in its flattening regime (see Fig. 2 and the corresponding fan plots in Figures 8.4) with even negative levels (inverted yield curve) in the months of June to September 2019, which seemed to anticipate a near entry into recession. Activity indicators that worsened in October, as said, began to improve in November. On average, the term spread seemed to move away from negative values. In March, the value of the term spread increased to 0.57 % and the probability of a flattening regime for that month was slightly reduced. However, with information up to March, when a recession was already observed in C_t, a regime change in the term spread was correctly predicted for April, bringing the flattening regime to an end. From April to September 2020, the term spread remained around 0.5 %, rising to highs of approximately 1.5 % (Figures 8.3). The fan plot corresponding to the predictions made with information up to June 2020 includes in its confidence bands the values actually observed in 2021, although predictions throughout 2020 grew faster than the observed term spread. In any case, the model predicts the consequent continuity of an ordinary regime of the term spread that follows a recession.

5. Conclusions

In this work we have modelled the bidirectional relationship between the term spread and the business cycle by extracting two interrelated latent Markov variables: the first, from four activity indicators, replicates the phases of the US business cycle; the second, from the term spread of the yield curve, reveals an ordinary regime and a flattening regime. The succession of term spread regimes and recessionary and expansionary phases of the business cycle are systematic but asymmetric. Recessions initiate an ordinary regime of term spread that continue throughout much of the following expansionary phase. When the ordinary regime of the term spread comes to an end, the subsequent flattening regime anticipates a cyclical phase change. In accordance with the estimated transition and forecasted probabilities, while throughout the cyclical expansion phase the cyclical component, C_t, does not anticipate a start of the flattening regime of the term spread, this regime is always present before a recession. Once the recession has started, C_t significantly anticipates a change in the regime in the term spread, which would be consistent with an expansionary monetary policy response and the consequent expectations of financial markets about long-term interest rates. The empirical adequacy of the model and its in-sample and out-of-sample forecasting ability have been sufficiently tested.

Based on these bidirectional dynamics, our model served to confirm the beginning of a recession in March of 2020 which can be exclusively attributed to the economic measures adopted in response to the COVID-19 pandemic. With information available until March 2020, the recession in 2020 was not evident, despite the increased probability of recession throughout 2019, which, however, decreased in January and February of 2020. The term spread showed negative values or close to zero throughout 2019, thus continuing the flattening regime of the yield curve pending confirmation or refutation of a recession. The exceptional nature of the crisis beginning in March 2020 abruptly and negatively dispelled any uncertainties, since, as verified in this work, the forecasted probabilities estimated using information up to March for both the cyclical factor and the term spread, suddenly jump to the opposite values to those they had been registering.

In this work, we have not analyzed monetary policy of a period as long as the one considered. For a more complete understanding of the interactions between business cycle and interest rates, it is necessary to consider inflation given that inflation, along with economic growth, fall within the mandate of the Federal Reserve. As we have shown, the start of a flattening regime is not signaled by the estimated business cycle factor. It is plausible that this beginning is explained by anti-inflationary policy responses, or the expectation these will take place. This is an avenue for further research.

Declaration of Competing Interest

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Appendix. Likelihood function of the model (1) to (4)

The present model consists of two submodels expressed in state-space form: one for the four activity indicators from which the common cyclical factor C_t is obtained, and the other for the term spread. Both submodels are affected by Markov regime switching. The connection between both submodels is in the time-varying transition probabilities, as these depend on a variable provided by the other submodel.

Kim's algorithm (Kim, 1994; Kim & Nelson, 1999, pp. 99–106) is applied in each submodel. We briefly explain how the likelihood function of a state-space model is affected by Markov regime switching. Using a general notation and admitting the possibility of regime change in all parameters, a state-space system could be expressed as:

$$\left.\begin{aligned} y_t &= H_j\beta_t^{(i,j)} + \eta_t^{(i,j)} \\ \beta_t^{(i,j)} &= \delta_j + F_j\beta_{t-1}^{(i,j)} + \varepsilon_t^{(i,j)} \end{aligned}\right\} \tag{8}$$

with $\eta_t^{(i,j)} \sim iidN(0, R_j)$ and $\varepsilon_t^{(i,j)} \sim iidN(0, Q_j)$. The superscript (i, j), with $i, j = \{0, 1\}$, expresses the state of the system under the regime j at t after having remained in the regime i in the previous period $t-1$. The Kalman filter shows the likelihood function of a system like (8) through the decomposition of the prediction error. Conditional on sample information until $t-1$, Y_t is distributed as

$$f(y_t/S_t = j, S_{t-1} = i, Y_{t-1}, \theta) = (2\pi)^{-\frac{N}{2}}\left|f_{t|t-1}^{(i,j)}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\eta_{t|t-1}^{(i,j)\prime}\left[f_{t|t-1}^{(i,j)}\right]^{-1}\eta_{t|t-1}^{(i,j)}\right\} \tag{9}$$

for $i, j = \{0, 1\}$, where $\eta_{t|t-1}^{(i,j)} = y_t - H_j\beta_{t|t-1}^{(i,j)}$ is the prediction error, $f_{t|t-1}^{(i,j)} = H_jP_{t|t-1}^{(i,j)}H_j' + R_j$ its covariance matrix, N the dimension of Y_t, and θ the vector of parameters.

Under a two-state regime switching, we are unsure of the state at t or at $t-1$. At $t-1$, one of the two unobserved states, $S_{t-1} = \{0, 1\}$, could have been activated. Starting from each of these two realizations, S_{t-1} has been followed at t by one of the two states $S_t = \{0, 1\}$, so that, considering both periods, we have 2^2 possible trajectories. It has been possible to reach Y_t by any of those four paths, and thus the density function must consider all of them

$$f(y_t/Y_{t-1}, \theta) = \sum_{j=0,1}\sum_{i=0,1} f(y_t, S_t = j, S_{t-1} = i/Y_{t-1}, \theta) \tag{10}$$

By the definition of conditional probability we know that

$$f(y_t/S_t = j, S_{t-1} = i, Y_{t-1}, \theta) = \frac{f(y_t, S_t = j, S_{t-1} = i/Y_{t-1}, \theta)}{p(S_t = j, S_{t-1} = i/Y_{t-1}, \theta)} \tag{11}$$

Time-varying transition probabilities depend on predetermined sample information (see equation (3))

$$p_{ij,t} = p(S_t = j/S_{t-1} = i, Y_{t-1}, \theta) = \frac{p(S_t = j, S_{t-1} = i/Y_{t-1}, \theta)}{p(S_{t-1} = i/Y_{t-1}, \theta)} \tag{12}$$

Combining (11) and (12) and substituting into (10), we obtain:

$$f(y_t/Y_{t-1}, \theta) = \sum_{j=0,1}\sum_{i=0,1} f(y_t/S_t = j, S_{t-1} = i, Y_{t-1}, \theta)p(S_t = j/S_{t-1} = i, Y_{t-1}, \theta)p(S_{t-1} = i/Y_{t-1}, \theta) \tag{13}$$

Finally, substituting (9) in (13), the conditional density of Y_t is

$$f(y_t/Y_{t-1}, \theta) = \sum_{j=0,1}\sum_{i=0,1} (2\pi)^{-\frac{N}{2}}\left|f_{t|t-1}^{(i,j)}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\eta_{t|t-1}^{(i,j)\prime}\left[f_{t|t-1}^{(i,j)}\right]^{-1}\eta_{t|t-1}^{(i,j)}\right\}p(S_t = j/S_{t-1} = i, Y_{t-1}, \theta)p(S_{t-1} = i/Y_{t-1}, \theta) \tag{14}$$

This expression is valid for the 2^2 trajectories that arise when transitioning from $t-1$ to t. However, for T periods the number of possible trajectories rises to 2^T, and if rather than only two regimes, there were M regimes, the number of possible trajectories would be M^T. To deal with this curse of dimensionality, Kim (1994) designs an approximation that, proceeding to reduce the dimension in the transition from one period to the next, allows it to be kept constant (this is done when updating the state vector and its covariance matrix). Under Kim's algorithm for the entire sample the conditional likelihood function is

$$f(Y_T/\theta) = \prod_{t=2}^{T} f(y_t/Y_{t-1}, \theta) \tag{15}$$

For our model, the observation vector of the first submodel consists of the demeaned monthly growth rates of the four activity indicators $y_t^C = (\Delta\log y_{1,t}, \Delta\log y_{2,t}, \Delta\log y_{3,t}, \Delta\log y_{4,t})'$. According to the assumptions in equations (1)–(3), the measurement equation relating observed variables and unobserved components is

$$\begin{bmatrix} \Delta\log y_{1,t} \\ \Delta\log y_{2,t} \\ \Delta\log y_{3,t} \\ \Delta\log y_{4,t} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ \gamma_2 & 0 & 1 & 0 & 0 \\ \gamma_3 & 0 & 0 & 1 & 0 \\ \gamma_4 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} C_t \\ C_{1,t} \\ C_{2,t} \\ C_{3,t} \\ C_{4,t} \end{bmatrix} + \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{bmatrix} \tag{16}$$

where the first factor loading, that of $\Delta\log y_{1,t}$, the Index of Industrial Production, has been normalized to one to allow the identification of the model. According to Eqs. (1b) and (1c) and the estimations in Table 1, in which no autocorrelation in the specific factors $C_{i,t}$ was found, the transition equation representing the dynamics of the unobserved components C_t and $C_{i,t}$ is

$$\begin{bmatrix} C_t \\ C_{1,t} \\ C_{2,t} \\ C_{3,t} \\ C_{4,t} \end{bmatrix} = \begin{bmatrix} \delta_C(S_t) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} C_{t-1} \\ C_{1,t-1} \\ C_{2,t-1} \\ C_{3,t-1} \\ C_{4,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{C,t} \\ \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \end{bmatrix} \tag{17}$$

where the intercept, $\delta_C(S_t) = \delta_{C,0}(1 - S_t) + \delta_{C,1}S_t$ with $S_t = \{0, 1\}$, is affected by a two state regime switching process. Gaussianity and orthogonality assumptions of the error terms imply that $\eta_t^C \sim iidN(0_{4x1}, R_C)$ and $\varepsilon_t^C \sim iidN(0_{5x1}, Q_C)$, where $\eta_t^C = (\eta_{1,t}, \eta_{2,t}, \eta_{3,t}, \eta_{4,t})'$, $\varepsilon_t^C = (\varepsilon_{C,t}, \varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t}, \varepsilon_{4,t})'$, $R_C = \begin{bmatrix} \sigma_{\eta,1}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\eta,2}^2 & 0 & 0 \\ 0 & 0 & \sigma_{\eta,3}^2 & 0 \\ 0 & 0 & 0 & \sigma_{\eta,4}^2 \end{bmatrix}$ and $Q_C = \begin{bmatrix} \sigma_C^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\varepsilon,1}^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\varepsilon,2}^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\varepsilon,3}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\varepsilon,4}^2 \end{bmatrix}$. By doing $H_C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ \gamma_2 & 0 & 1 & 0 & 0 \\ \gamma_3 & 0 & 0 & 1 & 0 \\ \gamma_4 & 0 & 0 & 0 & 1 \end{bmatrix}$, $\beta_{C,t} = \begin{bmatrix} C_t \\ C_{1,t} \\ C_{2,t} \\ C_{3,t} \\ C_{4,t} \end{bmatrix}$, $\delta_j^C = \begin{bmatrix} \delta_{C,j} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ for $j = \{0, 1\}$, and $F_C = \begin{bmatrix} \phi_C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, the state-space representation of the system (16)-(17) particularizes the system (8) as

$$\left.\begin{aligned} y_t^C &= H_C \beta_{C,t}^{(i,j)} + \eta_{C,t}^{(i,j)} \\ \beta_{C,t}^{(i,j)} &= \delta_j^C + F_C \beta_{C,t-1}^{(i,j)} + \varepsilon_{C,t}^{(i,j)} \end{aligned}\right\} \tag{18}$$

In system (18) only the parameter $\delta_{C,j}$ is affected by regime changes. The conditional likelihood function of this submodel is

$$f_C\left(Y_T^C/\theta_1\right) = \prod_{t=2}^{T} f_C\left(y_t^C/Y_{t-1}, \theta_1\right) \tag{19}$$

where θ_1 is the vector of parameters of (18). The density $f_C(y_t^C/Y_{t-1}, \theta_1)$ obeys the logic that led to the Eq. (14).

In the second submodel, the only observed variable is the (demeaned) term spread, sp_t. According the model in equation (4) and the estimations in Table 1, we have:

$$\left.\begin{aligned} sp_t &= \beta_{sp,t}^{(i,j)} + \eta_{sp,t}^{(i,j)} \\ \beta_{sp,t}^{(i,j)} &= \delta_{sp,j} + \phi_{sp}\beta_{sp,t-1}^{(i,j)} + \varepsilon_{sp,t}^{(i,j)} \end{aligned}\right\} \tag{20}$$

where $\delta_{sp,j}$ represents the corresponding value in $\delta_{sp}(S_{sp,t}) = \delta_{sp,0}(1 - S_{sp,t}) + \delta_{sp,1}S_{sp,t}$, with $S_{sp,t} = \{0, 1\}$ a latent variable denoting the flattening and ordinary regimes of the yield curve respectively, $\eta_{sp,t} \sim iidN\left(0, \sigma_{\eta,sp}^2\right)$, and $\varepsilon_{sp,t} \sim iidN\left(0, \sigma_{sp}^2\right)$. The conditional likelihood function of submodel (20) is

$$f_{sp}(SP_T/\theta_2) = \prod_{t=2}^{T} f_{sp}(sp_t/Y_{t-1}, \theta_2) \tag{21}$$

where θ_2 is its vector of parameters. The density $f_{sp}(sp_t/Y_{t-1}, \theta_2)$ also follows the logic leading to the density function of Eq. (14). The cyclical factor C_t, estimated in submodel (18) from information of the vector y_t^C, intervenes lagged in submodel (21) through their transition probabilities. Finally, the posterior density $f(\theta/Y_T)$ of Eq. (5) for the complete set of parameters is

$$f(\theta/Y_T) \propto f(Y_T/\theta)f(\theta) = f_C(Y_T^C/\theta_1)f_{sp}(SP_T/\theta_2)\prod_{k=1}^{K} f(\theta_k) \tag{22}$$

where K is the total number of parameters and $f(\theta_k)$ their prior densities assumed to be independent.

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