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Fractional Integration and Structural Breaks in Italian Real House Prices

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Abstract

The primary goal of this paper is to analyse the Italian real house price index (RHPI). Classical approaches typically assume either stationarity or unit-root nonstationarity. Instead, this study adopts a fractional integration framework, where the differencing parameter is allowed to take any real value, including fractional ones. Using updated quarterly OECD data for Italy covering the period 1970Q1–2024Q2, the paper investigates the persistence properties of real house prices under alternative specifications for the short-memory component, including white-noise, Bloomfield, and seasonal autoregressive disturbances. The contribution of the paper lies in combining fractional integration methods, growth-rate dynamics, and structural-break analysis within a unified framework focused on the Italian housing market. The empirical results indicate a high degree of persistence in the series, although the estimated persistence parameter is sensitive to the specification adopted for the disturbance term. In particular, the specification based on Bloomfield disturbances produces weaker evidence of long-memory behaviour than the alternative models considered. To complement the persistence analysis, we also investigate the presence of structural breaks from a macroeconomic and historical perspective. The results suggest the presence of six structural breaks that are broadly consistent with the main cycles of the Italian real estate market identified in the existing literature.

Keywords: real house prices; Italy; fractional integration; persistence

MSC: 62M09; 62M15; 91B84

1. Introduction

Indices that measure housing price market trends play an essential role in financial stability and in the conduct of monetary and economic policies. In addition, housing price statistics provide important information for both short- and long-term economic analysis, making the study of housing price dynamics a relevant topic from practical perspectives. For these reasons, the literature on modelling and analysing housing prices is extensive and encompasses different approaches and objectives. A first strand of the literature examines the relationship between housing prices and macroeconomic fundamentals. An example is the paper [1] in which real house prices are modelled as an inverse function of housing demand using data for the United Kingdom. In a similar vein, ref. [2] identifies the main macroeconomic factors underlying house price fluctuations in France, Germany, Italy, Spain, Sweden, and the United Kingdom. A second strand of the literature focuses on spatial

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diffusion and autocorrelation in order to analyse differences in housing price dynamics across geographic areas, such as different states or simply different regions or cities in the same state. In this context, ref. [3] finds evidence of spatial autocorrelation across 21 housing submarkets in Philadelphia. Following a similar approach, ref. [4] incorporates a temporal dimension into the analysis by developing a spatio-temporal autoregressive model for the Singapore condominium market. More recently, ref. [5] investigates the long-run spatial correlation of house prices across 8 major Indian cities.

In this paper, we analyse the Italian real house price index (RHPI) as a univariate time series using a fractional integration approach. Similar lines of research can be found in [6] for the United Kingdom and in [7] for a panel of OECD countries. Most of the existing literature models housing prices which use either stationary $\mathrm{I}(0)$ or unit-root $\mathrm{I}(1)$ specifications, which may impose restrictive assumptions on the persistence properties of the series. Fractional integration provides a more flexible framework because the differencing parameter d is allowed to take any real value, thereby permitting intermediate degrees of persistence and long memory. This approach has already been employed in several studies on housing markets and macroeconomic time series. For example, ref. [8] analyses comovement in Euro-area house prices using fractional cointegration techniques, while [9] studies persistence and cyclical dynamics in US and UK house prices over very long historical samples. More recently, ref. [10] investigates persistence in real house prices for 47 OECD countries using quarterly OECD data and fractional integration methods. Recent studies have also emphasised the importance of jointly accounting for persistence and structural changes when modelling macroeconomic and financial time series. In particular, ref. [11] discusses the close relationship between long-memory behaviour and structural breaks, showing that neglecting structural changes may affect the identification of persistence dynamics.

Against this background, the contribution of the present paper is threefold. First, we provide an updated analysis of the Italian RHPI using quarterly OECD data covering the period 1970Q1–2024Q2. Second, we compare alternative short-memory specifications for the disturbance term, including white-noise, Bloomfield, and seasonal autoregressive errors, and assess how these assumptions affect the estimated persistence parameter. Third, we combine fractional integration methods with growth-rate analysis, nonlinear deterministic components, and multiple structural-break detection techniques to interpret the historical and institutional context of the Italian housing market. To the best of our knowledge, this is the first study combining fractional integration, alternative short-memory specifications, growth-rate persistence and structural-break analysis for the Italian RHPI within a unified framework.

To place our contribution in context, we now briefly review the literature focusing on the Italian housing market. The existing literature includes both qualitative discussions of house price indices and empirical studies based on the approaches discussed above. The first area includes the regression model proposed in [12], where house prices are studied as a function of tourism and other variables. A Vector AutoRegressive (VAR) model is analysed in [13] to investigate covariation and interpret the mutual influence between apartment and house rental prices and other macroeconomic variables. Ref. [14] follows the same line (although not directly applied to Italy) and add an analysis of structural breaks from an economic perspective as well. In the second area, however, we find the paper of [15], who develop a Spatial Autoregressive and Spatial Error (SAR-SE) model to study the dynamics of several house price indices in Italy. Another important contribution in the Italian context, not strictly belonging to the aforementioned approaches, is that of [16], in which an analysis of both short- and long-run house prices are considered: in the short-run, real house price appreciation is examined to determine the existence of a common national impact, city-specific fixed effects and persistence parameters, with the aim of studying co-movements between regional time series. For the long-run, a preliminary analysis is proposed on the existence of a 'convergence' process between Italian regional house price series using a principal components technique and a panel data model. The rest of the paper is structured as follows. Section 2 describes the dataset and its source. Section 3 presents the methodology employed in the analysis. Section 4 discusses the empirical results, while Section 5 focuses on the structural-break analysis and its historical interpretation. Finally, Section 6 concludes and discusses the implications of the findings.

2. Data

The considered dataset was selected from the Organisation for Economic Co-operation and Development (OECD) Analytical House Price Indicators database. In detail, the dataset includes OECD member (and some non-member) countries and contains both quarterly and annual statistics for each country. The main indicator in this dataset is the nominal house price index (HPI) identified by the OECD for each country, that covers the prices for the sale of newly built and existing dwellings (of all types). Moreover, the full OECD dataset also contains information on real house prices, rental prices, and the ratios of nominal prices to rents and to disposable household income per capita. As mentioned above, only the real HPI (RHPI) for Italy has been considered. This index is obtained as the ratio of the nominal HPI to the consumers' expenditure deflator (both seasonally adjusted): this provides information on how nominal house prices change over time to overall consumer prices in the economy. The data, represented in Figure 1, are collected quarterly and the sample period examined ranges from 1970 (first quarter) to 2024 (second quarter).

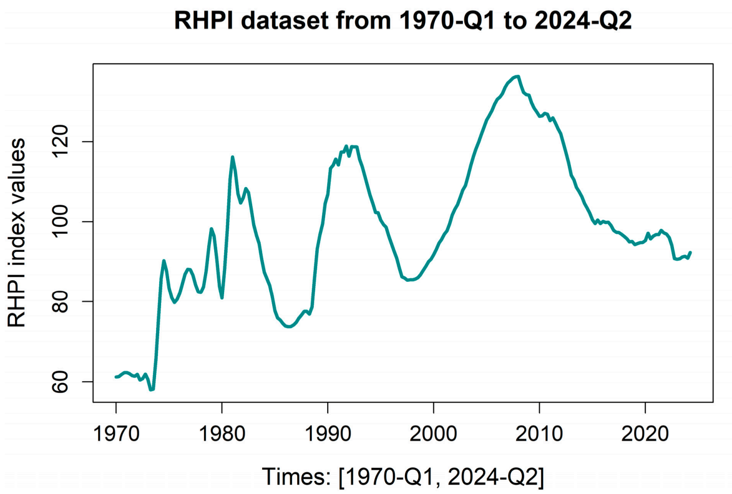

Figure 1. The RHPI dataset for Italy from 1970 (first quarter) to 2024 (second quarter).

In the empirical analysis, we consider the original RHPI series, its logarithmic transformation, and the first difference in the logarithm, which is interpreted as an approximation of the growth rate of real house prices. The empirical analysis proceeds in four steps. First, we perform preliminary unit-root and autoregressive analyses of the series. Second, we estimate fractional integration models under alternative assumptions for the short-memory component. Third, we analyse the persistence properties of the growth-rate series and consider nonlinear deterministic specifications. Finally, we investigate the presence of structural breaks and estimate the persistence parameter across the resulting sub-samples. The analysis was carried out in R, using version 4.2.0. The main routines rely on base R functions and on the packages employed for unit-root testing, fractional integration estimation, residual diagnostics, and structural-break detection. In particular, the unit-root tests include the ADF, KPSS, and ERS tests, while the fractional integration analysis is based on the Robinson's testing framework [17] with a grid search over values of d using increments of 0.01. For model comparison and diagnostics, we compute information criteria, residual autocorrelation tests, ARCH tests, and normality tests. For the structural-break analysis, we use the multiple-break framework [18] together with the fractional-break approach given in [19]. The resulting break dates are subsequently used to define the sub-samples analysed in the empirical section. Replication details are available from the authors upon request.

In Table 1 selected descriptive statistics of the dataset are summarised, while Figure 2 displays the corresponding Q-Q plot. From these results, the hypothesis of Gaussianity appears to be rejected.

Table 1. Descriptive Statistics for the RHPI dataset.

Mean	Median	Standard Deviation	Interquartile Range	Variation Coefficient	Skewness	Kurtosis
98.2769	96.8362	18.8758	25.2958	19.2068	0.0211	2.5552

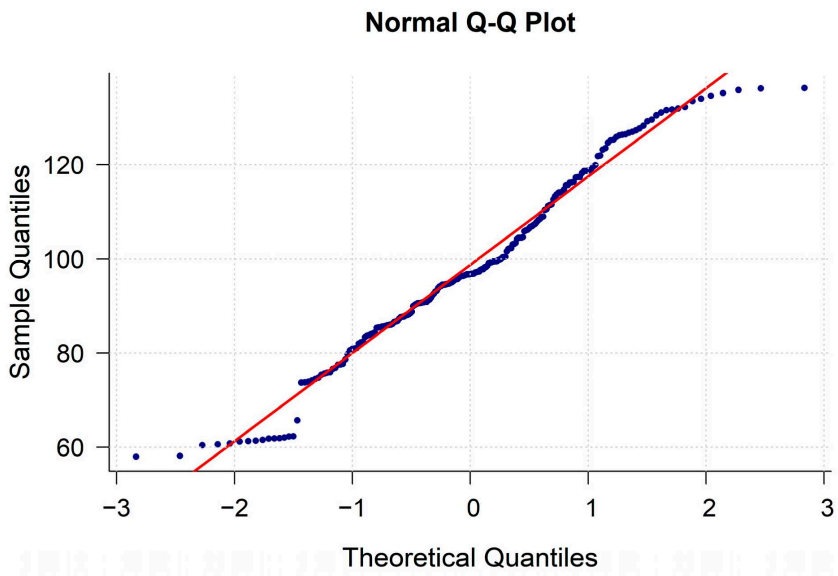

Figure 2. The Q-Q Plot for the RHPI dataset.

3. Methodology

The approach employed in this paper is based on the concept of long memory. In the time domain, a time series exhibits long-memory behaviour when the sum of all autocorrelations is not finite. In detail, given $\{x(t),\ t = 0,\ \pm1,\ \ldots\}$ a covariance stationary process with autocovariance function $\gamma(u) = \mathbb{E}[(x(t) - \mathbb{E}x(t))\ (x(t+u) - \mathbb{E}x(t+u))]$, where $\mathbb{E}[\,]$ denotes the expectation operator, we say the process displays the property of long memory if:

$$\sum_{u=-\infty}^{\infty} \gamma(u) = +\infty. \tag{1}$$

Alternatively, this property can be expressed in the frequency domain, where a process is said to display long memory if its spectral density function $f(\lambda)$ (which is the Fourier transform of the autocovariances) has a pole (or singularity) in the spectrum, i.e.,

$$f(\lambda) \to +\infty \text{ for at least at one } \lambda \text{ in the interval } [0,\ \pi). \tag{2}$$

There are many statistical models that satisfy the above two properties and, among them, one that is widely used by econometricians is based on the concept of fractional

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integration with positive d. A process is said to be integrated of order d, or I(d), if it can be expressed as

$$(1-L)^{d}\,x(t) = u(t), \quad t = 1,\,2,\,\ldots, \tag{3}$$

where L indicates the lag operator, i.e., $L^{k}x(t) = x(t-k)$ and $u(t)$ is $\mathrm{I}(0)$. This is equivalent to saying that the process $u(t)$ is a short-memory process, which means that the sum of the autocovariances in (1) is finite or—similarly in the frequency domain—that the spectral density is positive and bounded at all frequencies in the spectrum. In fact, it can be easily proved that the spectral density function of $x(t)$ in (3) is given by:

$$f(\lambda) = \frac{\sigma^2}{2\pi}\left|\frac{1}{1-e^{i\lambda}}\right|^{2d},$$

and, thus, $f(\lambda)$ tends to $+\infty$ as $\lambda \to 0$.

An appealing feature of this model is its flexibility, since it encompasses cases of stationarity I(0) (i.e., $d = 0$) as well as unit roots ($d = 1$), but also other cases corresponding to fractional values of d, namely:

i. anti-persistent processes, if $d < 0$;
ii. short memory, or $\mathrm{I}(0)$ processes, if $d = 0$;
iii. long-memory covariance stationarity processes, if $0 < d < 0.5$;
iv. nonstationary and mean reverting processes, if $0.5 \leq d < 1$;
v. unit roots, or $\mathrm{I}(1)$ processes, if $d = 1$;
vi. long memory processes after taking first differences, i.e., I(d) with $d > 1$.

Notice that, if a series is nonstationary, i.e., $d \geq 0.5$, but $d < 1$, it will still be mean reverting, with shocks having only temporary effects.

A process is said to be integrated of order d, or I(d), if it can be expressed as

$$(1-L)^{d}x(t) = u(t)$$

where L denotes the lag operator ($L\,x(t) = x(t-1)$) and $u(t)$ is an $\mathrm{I}(0)$ process. In this context, $\mathrm{I}(0)$ means that the process is covariance stationary with a finite sum of autocovariances and a spectral density function that is positive and bounded at all frequencies. For any real value of d, the fractional differencing operator can be defined through the binomial expansion

$$(1-L)^{d} = \sum_{j=0}^{\infty}\pi_j(d)L^{j},$$

where

$$\pi_j(d) = (-1)^{j}\binom{d}{j} = \frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)},\ j = 0,1,2,\ldots$$

and $\Gamma(\cdot)$ denotes the Gamma function defined for $\Re(\mathrm{z}) > 0$ as

$$\Gamma(z) = \int_{0}^{\infty} x^{z-1}e^{-x}dx,$$

and is extended to the remaining complex plane by analytic continuation, except at the non-positive integers where it has singularities. Equivalently, the coefficients satisfy the recursive relation

$$\pi_0(d) = 1, \pi_j(d) = \frac{j-1-d}{j}\pi_{j-1}(d),\ j \geq 1.$$

Under this specification, the spectral density function of x_t is given by

$$f_x(\lambda) = |\,1-e^{-i\lambda}\,|^{-2d}\,f_u(\lambda),$$

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where $f_u(\lambda)$ is the spectral density of the $\mathrm{I}(0)$ process $u(t)$. Therefore, when $d > 0$, the spectral density diverges as the frequency approaches zero, which generates long-memory behaviour. Note that in empirical applications with finite samples, the infinite expansion is implemented through a finite truncation of the lag polynomial, using the available observations in the sample. The coefficients $\pi_j(d)$ decay hyperbolically as j increases, so higher-order terms become progressively smaller.

The value of d measures the degree of dependence between observations. The higher the value of d, the stronger the persistence in the series is and the slower the decay of the autocorrelation function is. Fractional integration models therefore provide a flexible framework that includes stationary $\mathrm{I}(0)$ processes, unit-root $\mathrm{I}(1)$ processes, and intermediate cases characterised by long memory and mean reversion.

Higher values of d imply a stronger degree of dependence between observations, which implies that shocks will have highly persistent effects and that autocorrelations will decay at a hyperbolically slow rate. These processes were originally studied in the 1960s by Adelman in [20] and Granger in [21], who pointed out that for many aggregate economic time series the spectral density increases sharply as the frequency approaches zero. Robinson in [22] and Granger in [23] subsequently showed that fractional integration can result from the aggregation of heterogeneous AR processes.

The testing procedure used in this paper was developed in [17] and it allows testing for any real value of d, using I(d) models, and iterating for different values for d_0 (in our case, using increments of 0.01). Specifically, it tests the null hypothesis, $H_0: \ d = d_0$, for any real value d_0 in Equation (3), where $x(t)$ can be the errors in a regression model of form:

$$y\ (t) = \alpha + \beta t + x(t), \quad t = 1,\ 2,\ \ldots. \tag{4}$$

This method uses a test statistic that follows a standard normal distribution; therefore, it allows confidence intervals to be constructed for all values of d_0 for which the null hypothesis cannot be rejected (see [24] for the implementation of the test statistic in this simple version of Robinson's tests [17]).

4. Empirical Results

In this section, we present the results of our analysis. We start with a very simple AR(2) model,

$$y(t) = a_1 y(t-1) + a_2 y(t-2) + \varepsilon(t), \quad \varepsilon(t) \ \sim \ WN(0, \sigma^2), \quad t = 1,\ 2,\ \ldots \tag{5}$$

which is motivated by a preliminary inspection of the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the RHPI dataset, as shown in Figures 3 and 4. In Equation (5) $\varepsilon(t)$ denotes a white noise (WN) process with zero mean, constant variance, and no serial correlation. In order to reduce the possible presence of confounding exponential trends, the ACF and PACF of the logged data are also shown in Figure 3.

Table 2 shows the estimates of the coefficients a_1 and a_2 in (5) for the original and the logged series, using both the Yule–Walker equations (Y–W) and the maximum likelihood method (MLE). To compute these estimations, the **R** function *ar* of *stats* package has been employed (**R** version 4.2.0). These coefficients are all different from 0 at the 95% level of significance. However, even under the condition of both series being causal, the sum of the two estimated coefficients with both methods is very close to 1, suggesting that the series may be nonstationary and may display unit-root behaviour.

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Table 2. Estimation of the AR(2) model.

Time Series	a_1 Coefficient		a_2 Coefficient	
Original	Y–W	1.3470	Y–W	−0.3729
	MLE	1.6879	MLE	−0.7036
Logged values	Y–W	1.3168	Y–W	−0.3475
	MLE	1.6964	MLE	−0.7132

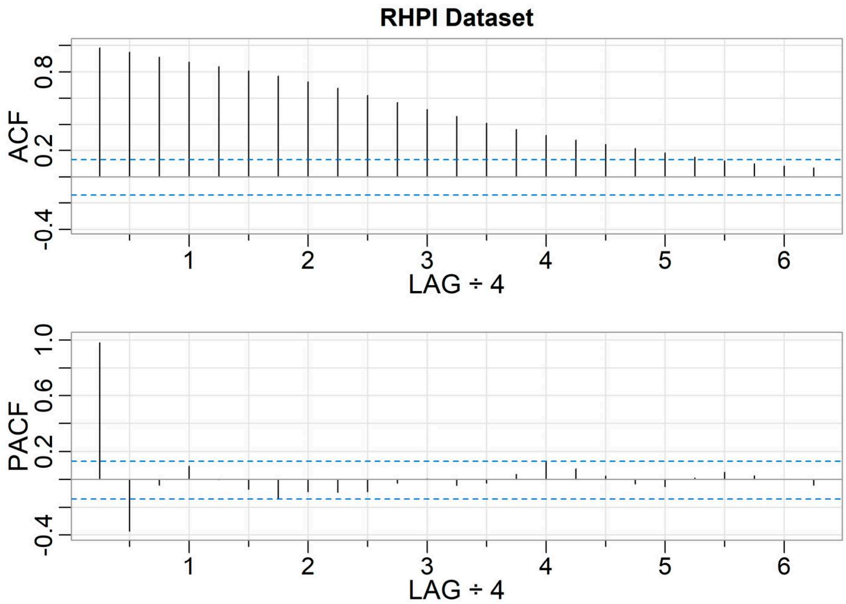

Figure 3. Autocorrelation (ACF) and Partial Autocorrelation (PACF) functions for the RHPI dataset.

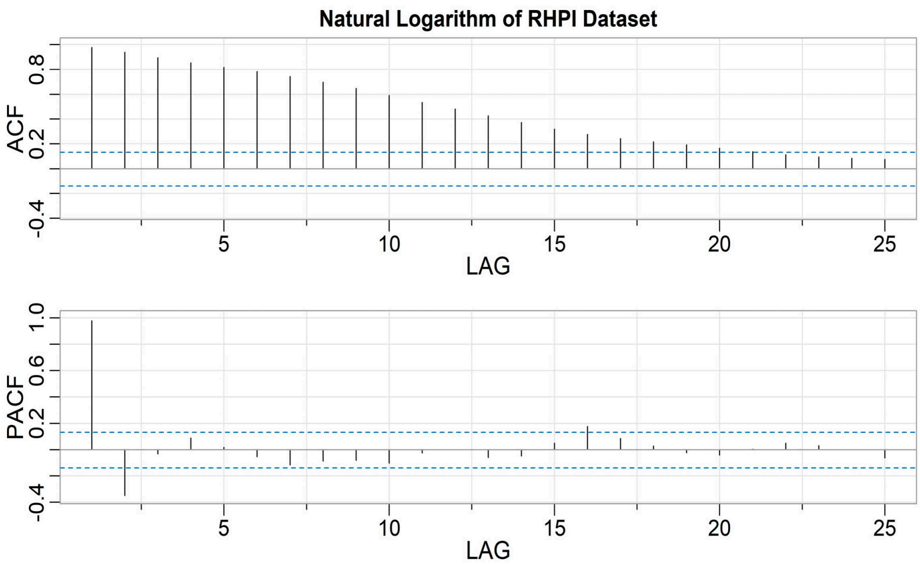

Figure 4. ACF and PACF of the log-transformed RHPI dataset.

In fact, by running the classic unit root tests (Augmented Dickey–Fuller (ADF, [25]), Kwiatkowski–Phillips–Schmidt–Shin tests (KPSS, [26]) and Elliot, Rothenberg, and Stock tests (ERS, [27]) tests) (The **R** function employed to compute these tests are: *adf.test* and *kpss.test* of *tseries* package for the ADF and KPSS tests respectively; *ur.ers* of *urca* package for the ERS test.), the results (reproduced in Table 3) provide evidence which is consistent with nonstationarity.

Table 3. Results of unit-root tests.

Series	Test	Statistic	Nb. Lag/Trunc.	*p*-Value
Original	ADF (1979)	−2.8827	6	0.2056
	KPSS (1992)	0.3870	4	0.01
	ERS (1996) (DF-GLS)	−1.8207	4	---
Logged values	ADF (1979)	−2.9562	6	0.1748
	KPSS (1992)	0.3950	4	0.01
	ERS (1996) (DF-GLS)	−1.6825	4	---

--- indicates that the coefficient is found to be insignificantly different from zero.

Based on the evidence of unit roots, we investigate whether the results in Table 3 may be biassed, since the series may be fractionally integrated. As the papers [28–30] show that the above (unit root) tests might be biassed if the series were in fact integrated of a fractional order.

As a result of that, we examine the model given by Equations (3) and (4), i.e.:

$$y(t) = \alpha + \beta t + x(t), \quad (1-L)^d x(t) = u(t), \quad t = 1, 2, \ldots, \tag{6}$$

where $y(t)$ refers to the time series under investigation, in our case the RHPI data (original and logged values), α and β are unknown coefficients determined within the model and corresponding to an intercept and a linear time trend, and $(1-L)^d$ is the fractional integration polynomial in L (the lag or backshift operator, $Lx(t) = x(t-1)$) with fractional integration degree d, which makes the residuals $u(t)$ integrated of order $\mathrm{I}(0)$ under different assumptions. First, we suppose that $u(t)$ is a white noise process (i.e., with zero mean, constant variance, and no serial correlation); then, we allow weak autocorrelation by using the exponential spectral model of Bloomfield [31], which is an approximation to AR structures; finally, based on the quarterly structure of the data, we allow $u(t)$ to follow a seasonal AR(1) process of the following form:

$$u(t) = \rho\, u(t-4) + \varepsilon(t), \tag{7}$$

where $\varepsilon(t)$ here is a white noise process. In Table 4 we display the estimates of d and their associated 95% confidence bands in three different scenarios:

i. with no deterministic terms (i.e., with $\alpha = \beta = 0$ a priori in Equation (6)),
ii. including an intercept (i.e., with $\beta = 0$),
iii. with an intercept and a linear trend (i.e., α and $\beta \neq 0$).

The selected specification for each series is highlighted in bold in Table 4. This selection is made based on the t-values of the estimated coefficients, while Table 5 reports the estimated coefficients and corresponding t-values associated with the selected specifications.

The results indicate that the time trend is insignificant across all models, while the intercept (constant) is significant. Focusing on the estimates of d obtained from the selected specifications reported in Table 4, we observe that the results based on white-noise and seasonal AR(1) disturbances are relatively similar, with estimated values of d substantially higher than 1 for both the original and the logged series. By contrast, when the disturbance term follows the exponential spectral model of Bloomfield [31], the estimated value of d becomes considerably smaller and the unit-root null hypothesis ($d = 1$) cannot be rejected.

The substantial differences in the estimated values of the differencing parameter d reported in Tables 4 and 5 across the alternative disturbance specifications suggest that model selection plays an important role in the interpretation of persistence. In particular, the estimates obtained under white-noise and seasonal autoregressive disturbances are substantially larger than those obtained under the Bloomfield specification. This indicates that the estimated degree of persistence is sensitive to the assumptions adopted for the short-memory component.

Table 4. Estimates of d. Model given by Equation (3). Original and logged data.

Series	No terms	With an intercept	With an intercept and a linear time trend
i. White noise errors			
Original data	1.14 (1.04, 1.27)	**1.82 (1.58, 2.12)**	1.82 (1.58, 2.12)
Logged data	1.01 (0.90, 1.10)	**1.91 (1.63, 2.25)**	1.91 (1.63, 2.25)
ii. Bloomfield autocorrelated errors			
Series	No terms	With an intercept	With an intercept and a linear time trend
Original data	1.02 (0.87, 1.21)	**1.09 (0.94, 1.31)**	1.08 (0.95, 1.30)
Logged data	0.96 (0.82, 1.15)	**1.00 (0.87, 1.21)**	1.00 (0.87, 1.21)
iii. Seasonal AR errors			
Series	No terms	With an intercept	With an intercept and a linear time trend
Original data	1.15 (1.06, 1.27)	**1.68 (1.55, 1.87)**	1.67 (1.54, 1.87)
Logged data	0.99 (0.76, 1.11)	**1.72 (1.57, 1.95)**	1.72 (1.57, 1.95)

In parentheses, the 95% confidence bands for the estimates of d. In bold, the selected model across the deterministic terms for each series.

Table 5. Estimated coefficients in the selected models in Table 4. Original and logged data.

Series	d (95% band)	Intercept (t-value)	Time trend (t-value)	
i White noise errors				
Original data	1.82 (1.58, 2.12)	60.931 (38.81)	---	
Logged data	1.91 (1.63, 2.25)	4.109 (246.53)	---	
ii Bloomfield autocorrelated errors				
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)	
Original data	1.09 (0.94, 1.31)	60.889 (33.55)	---	
Logged data	1.00 (0.87, 1.21)	4.110 (211.64)	---	
iii Seasonal AR errors				
Series	d (95% band)	Intercept (t-value)	Trend (t-value)	Seasonal
Original data	1.68 (1.55, 1.87)	60.917 (38.81)	---	−0.358
Logged data	1.72 (1.57, 1.95)	4.109 (245.55)	---	−0.395

--- indicates that the coefficient is found to be insignificantly different from zero.

To further assess the adequacy of the competing specifications, we complement the estimation results with model-comparison criteria and residual diagnostic tests. Specifically, we consider likelihood-based information criteria whenever applicable, including the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). For the Bloomfield specification, the comparison is based on the corresponding Whittle pseudo-likelihood approximation [17,32] rather than on a standard Gaussian likelihood. In addition, we perform Ljung–Box tests for residual autocorrelation, ARCH tests for conditional heteroskedasticity, and Jarque–Bera tests for residual normality.

Table 6 reports the full set of model-comparison criteria under alternative deterministic and disturbance specifications. The results confirm that the estimated degree of persistence is highly sensitive to the assumptions adopted for the short-memory component. In particular, the Bloomfield specification generally produces lower estimates of d than the white-noise and seasonal autoregressive specifications. To summarise the evidence, Table 7 reports the preferred specifications selected according to the minimum AIC and BIC values for each series.

Table 6. Model-comparison criteria under alternative deterministic and disturbance specifications.

Series	Det.	Error	*d*	LogLik	AIC	BIC	Crit.
Original	None	WN	1.14	−1313.4	2628.8	2632.18	GL
Original	Int.	WN	1.82	−949.29	1900.57	1903.96	GL
Original	Trend	WN	1.82	−913.81	1829.62	1833.01	GL
Original	None	Bloom	1.02	−470.1	946.19	956.35	WP
Original	Int.	Bloom	1.09	−468.33	942.66	952.81	WP
Original	Trend	Bloom	1.08	−467.55	941.11	951.26	WP
Original	None	SAR(1)	1.15	−772.76	1549.53	1556.3	GL
Original	Int.	SAR(1)	1.68	−763.91	1531.82	1538.59	GL
Original	Trend	SAR(1)	1.67	−760.64	1525.28	1532.05	GL
Log	None	WN	1.01	−640.72	1283.44	1286.82	GL
Log	Int.	WN	1.91	42.1	−82.2	−78.82	GL
Log	Trend	WN	1.91	82.68	−163.36	−159.97	GL
Log	None	Bloom	0.96	474.79	−943.57	−933.42	WP
Log	Int.	Bloom	1	518.77	−1031.54	−1021.38	WP
Log	Trend	Bloom	1	519.8	−1033.59	−1023.44	WP
Log	None	SAR(1)	0.99	202.45	−400.9	−394.13	GL
Log	Int.	SAR(1)	1.72	217.56	−431.13	−424.36	GL
Log	Trend	SAR(1)	1.72	221.95	−439.9	−433.14	GL
Growth	None	WN	0.9	457.96	−913.92	−910.55	GL
Growth	Int.	WN	0.91	458.42	−914.83	−911.45	GL
Growth	Trend	WN	0.91	460.45	−918.91	−915.53	GL
Growth	None	Bloom	0	544.23	−1082.47	−1072.33	WP
Growth	Int.	Bloom	0	544.35	−1082.71	−1072.57	WP
Growth	Trend	Bloom	−0.02	544.73	−1083.47	−1073.33	WP
Growth	None	SAR(1)	0.72	461.33	−918.65	−911.89	GL
Growth	Int.	SAR(1)	0.72	461.96	−919.92	−913.16	GL
Growth	Trend	SAR(1)	0.72	465.01	−926.02	−919.26	GL

Det. = Deterministic specification; Int. = Intercept; WN = White-noise disturbances; Bloom = Bloomfield disturbances; SAR(1) = Seasonal autoregressive disturbances; GL = Gaussian likelihood; WP = Whittle pseudo-likelihood.

Table 7. Preferred model specifications selected according to the minimum AIC and BIC values.

Metric	Original	Log	Growth
d selected by BIC	1.08	1	−0.02
BIC selected Det.	Trend	Trend	Trend
BIC selected Error	Bloom	Bloom	Bloom
Minimum BIC	951.26	−1023.44	−1073.33
d selected by AIC	1.08	1	−0.02
AIC selected Det.	Trend	Trend	Trend
AIC selected Error	Bloom	Bloom	Bloom
Minimum AIC	941.11	−1033.59	−1083.47

The selected specifications under both criteria generally correspond to the Bloomfield disturbance model with a linear deterministic trend. However, the residual diagnostics do not provide unequivocal evidence in favour of a single specification, since the diagnostic tests frequently reject the null hypotheses of residual independence, homoskedasticity, and normality at conventional significance levels. Therefore, the Bloomfield specification should be interpreted as one admissible representation of short-memory dynamics rather than as the uniquely most realistic model. For brevity, individual p-values are not reported. Overall, the results indicate that the persistence properties of the RHPI series are robustly high in levels, although the precise magnitude of the estimated differencing parameter depends on the specification adopted for the disturbance term. This finding highlights the sensitivity of persistence estimates to the specification of the error structure. For this reason, we next analyse the first differences in the logarithmically transformed series, which approximately correspond to the growth rate of real house prices and may display weaker dependence properties than the original series.

Based on the high degree of persistence observed in the data, we next work with the first differences in the log-transformed prices, noting that this approximates the growth rate of the series, i.e.,

$$(1-L)\log p(t) = \log p(t) - \log p(t-1) \ \sim \ \frac{p(t)-p(t-1)}{p(t-1)} \ \sim \text{Growth Rate of } p(t).$$

We now report the results for the growth rate series $(1-L)\log p(t)$. We consider the same model as in Tables 2 and 3 and the same type of disturbances. We expect that even the growth rate of the data can be modelled by an AR(2) time series: once again, a preliminary inspection of the ACF and the PACF confirms this expectation, as shown in Figure 5.

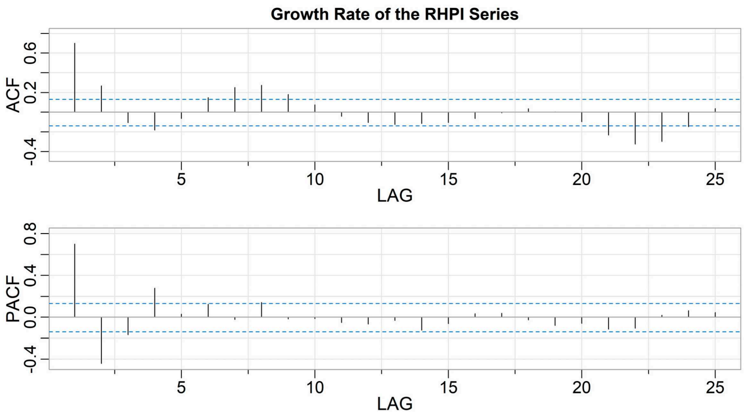

Figure 5. Autocorrelation (ACF) and Partial Autocorrelation (PACF) functions for the growth rate of the RHPI dataset.

Table 8 reports the estimates of the coefficients a_1 and a_2 in (5) for the growth rate series, again using both the Yule–Walker equations (Y–W) and the maximum likelihood method (MLE). To compute these estimations, the **R** function *ar* of *stats* package has been employed. Once again, these coefficients are all different from 0 at the 95% level of significance. Table 9 then reports the results of the classic unit root tests, ADF, KPSS, and ERS, applied to the growth rate series.

Then, we repeat the analysis conducted in Tables 4 and 5, based on fractional integration in the growth rates series, and report the results in Tables 10 and 11 below.

The results on the growth rates indicate that they are very heterogeneous depending on how we specify the error term. Thus, if $u(t)$ is white noise, the estimate of the differencing parameter d is $d = 0.90$ and the unit root null hypothesis cannot be rejected. However, if $u(t)$ follows the model of Bloomfield, the estimate of d is very close to 0; finally, if $u(t)$ is seasonal, $d = 0.72$ and the hypothesis $d = 1$ are rejected in favour of mean reversion ($d < 1$). Once again, the specification based on Bloomfield disturbances produces weaker evidence of long-memory behaviour than the alternative specifications considered, a result that is also consistent with Lo's statistic [33].

Table 8. AR(2) model on the growth rate series.

Series	a_1 Coefficient		a_2 Coefficient	
Growth rates	Y–W	1.0119	Y–W	−0.4419
	MLE	1.0116	MLE	−0.4422

Table 9. Unit root tests on the growth rate series.

Series	Test	Statistic	Nb. Lag/Trunc.	*p*-Value
Growth rates	ADF (1979)	−4.3684	5	0.01
	KPSS (1992)	0.0450	4	0.1
	ERS (1996) (DF-GLS)	−5.1717	4	---

--- indicates that the coefficient is found to be insignificantly different from zero.

Table 10. Estimates of d. Model given by Equation (3). Growth rate series.

Type of Errors	No Terms	With an Intercept	With an Intercept and a Linear Time Trend
White noise	**0.90 (0.63, 1.26)**	0.91 (0.63, 1.26)	0.91 (0.63, 1.26)
Bloomfield	0.00 (−0.13, 0.21)	0.00 (−0.13, 0.21)	**−0.02 (−0.24, 0.21)**
Seasonal AR(1)	**0.72 (0.57, 0.95)**	0.72 (0.57, 0.95)	0.72 (0.57, 0.95)

In parenthesis, the 95% confidence bands for the estimates of d. In bold, the selected model across the deterministic terms for each series.

Table 11. Estimated coefficients in the selected models in Table 10. Growth rate series.

Type of Error	d (95% Band)	Intercept (t-Value)		Time Trend (t-Value)
White noise	0.90 (0.63, 1.26)	---		---
Bloomfield	−0.02 (−0.24, 0.21)	$0.9 \cdot 10^{-2}$ (3.67)		$-0.65 \cdot 10^{-4}$ (−3.31)
Seasonal AR(1)	0.72 (0.57, 0.95)	---	---	Seasonal = −0.395

--- indicates that the coefficient is found to be insignificantly different from zero.

Nevertheless, we also conducted additional analyses. First, we consider a nonlinear deterministic specification based on Chebyshev polynomial approximations in time, following [34]:

$$y_t = \sum_{i=0}^{m} \theta_i T_i(z_t) + x_t,$$

where $T_i(\cdot)$ denotes the Chebyshev polynomial of the first kind which admits the trigonometric representation

$$T_i(z) = \cos(i \arccos z), \quad z \in [-1, 1].$$

Following the standard approach in the literature, the time index is rescaled into the interval $[-1,\ 1]$. In this context, if $m = 0$, the model contains an intercept and, if $m > 0$, it becomes non-linear: the higher m is, the less linear the approximated deterministic component becomes. Hamming in [35] and Smyth in [36] provided a detailed description of these polynomials, whilst Bierens in [37] and Tomasevic et al. in [38] argued that it is possible to approximate highly non-linear trends with polynomials of a relatively low order.

Looking at the results in Table 12, we see that only some estimates of θ_0 appear statistically significant, implying no evidence of nonlinear structures in the data. This happens both for the original data and for the logged transformed values. By contrast, using the growth rate series, the estimate of d is equal to 0.91, so that the unit root cannot be rejected, but none of the coefficients are found to be significant.

Table 12. Estimated coefficients in a non-linear model with an I(d) component.

Series	d (95% Band)	θ_0	θ_1	θ_2	θ_3
Original	1.31 (1.28, 1.36)	**72.017 (1.91)**	-1.332 (-0.03)	-3.004 (-0.20)	6.822 (0.77)
Logged	1.25 (1.19, 1.32)	**4.314 (4.10)**	0.280 (0.41)	-0.174 (-0.68)	0.077 (0.50)
Growth rate	0.91 (0.62, 1.25)	$-0.52 \cdot 10^{-2}$ (-0.04)	$0.28 \cdot 10^{-2}$ (0.04)	$-0.4 \cdot 10^{-3}$ (-0.01)	$-0.26 \cdot 10^{-2}$ (0.10)

In bold, the significant coefficients according to the t-values.

5. Structural Breaks

In this section, we investigate the presence of structural breaks in the data. The break analysis is based on the multiple structural-break framework given in [18], together with the fractional integration approach proposed in [19]. The latter is an extension of the former permitting the analysis over a grid of differencing parameter values. The analysis allows for multiple break dates and considers alternative specifications for the persistence structure of the series. Long-memory behaviour and structural breaks are not mutually exclusive phenomena [39–42]. Structural changes may coexist with persistent dynamics, and ignoring breaks may bias the estimation of the differencing parameter. Therefore, the break analysis is interpreted as complementary to the fractional integration framework rather than as an alternative to it.

Using these approaches, we identify six structural breaks in the data whose dates are reported in Table 13 and illustrated in Figure 6. The same break dates are obtained for both the original and the logged series. The resulting break dates are subsequently used to define the sub-samples reported in Table 14 and to compare the empirical evidence with the major historical cycles of the Italian housing market discussed in [43]. It should be stressed that this comparison is interpretative rather than causal. The historical and macroeconomic events discussed below are intended to provide an economic interpretation of the estimated break dates, rather than definitive statistical evidence of causal relationships between specific events and the structural changes identified in the series.

Table 13. Number of break dates for each series.

Series	N. of Breaks	Break Dates
Original	6	1973Q1; 1981Q1; 1986Q1; 1992Q4; 1997Q4; 2008Q1
Logged	6	1973Q1; 1981Q1; 1986Q1; 1992Q4; 1997Q4; 2008Q1

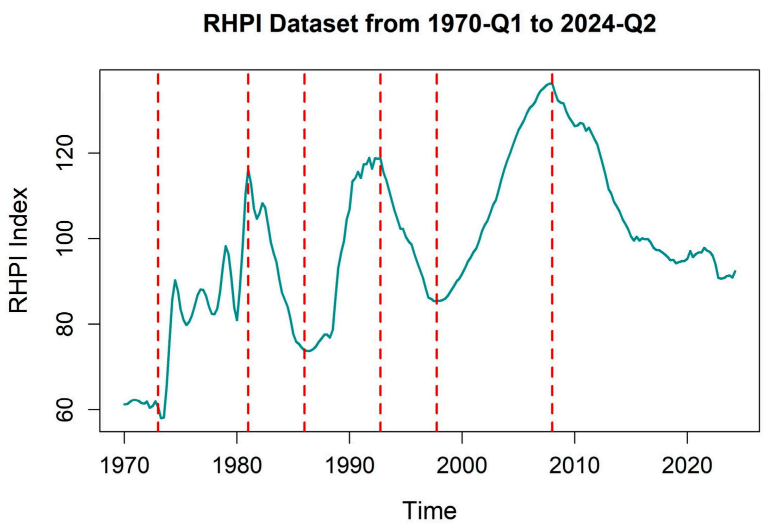

Figure 6. RHPI series for Italy from 1970Q1 to 2024Q2. The dashed vertical lines indicate the estimated structural break dates.

Table 14. Estimated coefficient for each sub-sample.

i. Original data			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.63 (0.01, 1.34)	61.667 (71.14)	−0.2002 (−1.78)
2nd sub-sample	2.62 (1.66, 3.63)	54.965 (18.27)	10.8948 (2.17)
3rd sub-sample	1.24 (0.36, 1.87)	114.908 (64.36)	−2.0751 (−2.73)
4th sub-sample	1.32 (1.07, 1.63)	73.232 (34.31)	---
5th sub-sample	1.24 (0.62, 1.77)	116.948 (161.93)	−1.5762 (−5.13)
6th sub-sample	1.64 (1.48, 1.88)	84.844 (204.58)	0.6014 (1.44)
7th-sub-sample	1.31 (1.18, 1.51)	184.221 (146.88)	−0.7443 (−2.01)
ii. Logged data			
Series	d (95% band)	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.63 (0.02, 1.34)	4.1221 (285.17)	−0.0033 (−1.79)
2nd sub-sample	2.85 (1.92, 3.24)	4.027 (127.13)	0.0219 (2.98)
3rd sub-sample	1.45 (0.65, 1.94)	4.751 (288.65)	−0.0243 (−2.01)
4th sub-sample	1.40 (1.16, 1.76)	---	---
5th sub-sample	1.22 (0.07, 1.82)	4.763 (645.24)	−0.0156 (−5.26)
6th sub-sample	1.62 (1.45, 1.85)	4.441 (143.67)	0.0064 (1.71)
7th-sub-sample	1.30 (1.16, 1.52)	4.905 (558.00)	−0.0063 (−1.86)

--- indicates that the coefficient is found to be insignificantly different from zero.

According to ISTAT [43], there are four main cycles in the Italian housing market for the evolution of real prices from 1969 to 2015. These cycles are listed below, together with the dates that correspond most closely to the estimated break dates reported above.

I. Growth phase: 1969–1974 (+86%); Contraction phase: 1975–1977 (−8%);
II. Growth phase: 1978–1982 (+86%); Contraction phase: 1983–1986 (−20%);
III. Growth phase: 1987–1992 (+83%); Contraction phase: 1993–1999 (−56%);
IV. Growth phase: 2000–2007 (+81%); Contraction phase: **2008**–2015 (−78%).

Figure 7 shows the plot of RHPI data with the cycles listed above marked by blue lines.

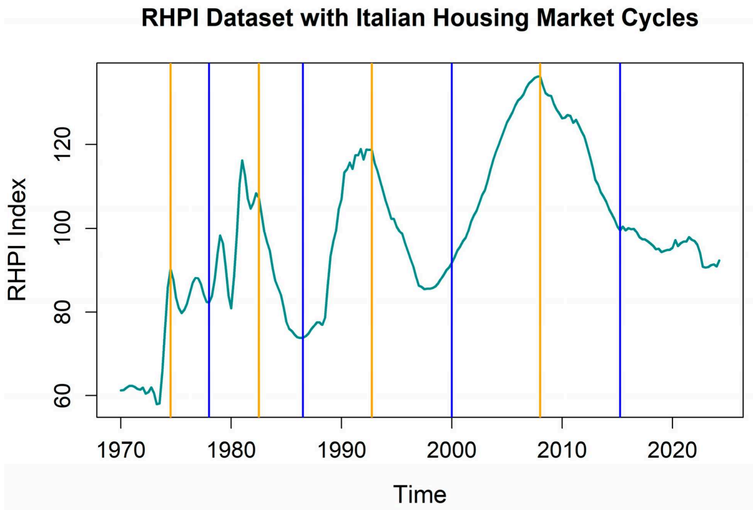

Figure 7. The RHPI dataset with the four cycles of the Italian housing (from 1969 to 2015). The growth and the contraction phases are separated with an orange line.

Notice that the estimated breaks are broadly compatible with the historical cycles (shown in Figure 6) identified by Breglia in [43]. We now briefly discuss the historical and macroeconomic factors associated with these cycles.

I. Thanks to the so-called "economic miracle" of the 1950s–1960s, Italy's massive export growth consolidated a production system centred on small and medium-sized manufacturing enterprises. These firms required low labour costs and cheap natural and energy resources to survive in a competitive international environment. This process, together with post-war reconstruction, led to a rapid growth that began to slow down in the late 1960s, mainly for two reasons. The first was market saturation and the second was a misbehaviour toward workers. The latter led to a series of long factory strikes and labour conflicts in the fall of 1969 (known as the "hot autumn") that reduced profit margins in Italy. Moreover, the energy crisis triggered by the Kippur War (October 1973) led the Organization of Petroleum Exporting Countries (OPEC) to drastically increase the price of oil. In response to this crisis, the Italian government adopted an "austerity" policy ('73–'74) to reduce energy consumption and counter the devaluation of the lira, without considering public budget constraints [44]. All this increased inflation and led to a surge in prices that also affected the housing market (Notice that real prices (such as RHPI) take also inflation into account.), which in 1975 led Italy to have its first recession in three decades.

II. The beginning of the new phase of growth in the RHPI coincides with the promulgation of the Fair Rent Law in 1978, which introduced into the Italian legal system a conventional mechanism for determining the rental values and rents of urban real estate for residential use. Under the conditions set by this law, renting out new houses as an investment became less attractive, especially in the private rental market: this led to the sale rather than the rental of investment houses, so that demand quickly increased prices. This led to a virtual stagnation in new housing construction, and even to a reduction in the supply of new housing. Consequently, housing prices increased and became disjointed from construction costs, the cost of living, and other indicators, as opposed to the situation beforehand [45]. Then, from 1983 to 1986 there was a decline in prices, mainly because of high inflation. In the last few months of 1985, very few households chose housing as an investment, not only for the reasons explained above, but also because at that time other much less complex and more profitable investment options (e.g., BOTs and CCTs) appeared on the market. For this reason, this period has been remembered as "brick nausea" [43].

III. As previously discussed, during that period of contraction investments shifted to government bonds and stocks, but investors later lost confidence in the financial market after the stock market crash of October 1987 (Black Monday). Consequently, investments shifted back to the real estate market, which was considered safer than the much more volatile financial market [43]. A sign of this is the gradual rise in prices (driven by increased demand), which can also be seen in Figure 5. This continued until 1992, when Italy was forced out of the European Monetary System (EMS), having joined it in the 1980s. As a consequence, many important policy reforms were introduced, such as the partial abrogation of the residential real estate articles introduced in 1978, which were finally abolished in December 1998. However, these reforms did not produce any substantial results, even though the regular devaluation of the lira helped the Italian economy remain competitive for a few years, until Italy became a founding member of the Eurozone in 1999, see [44]).

IV. What happened in Italy during this last cycle is closely related to the global financial crisis that officially began in the United States in September 2008, but whose origins date back *to* the early 2000s. In fact, in response to the attack on the Twin Towers, interest rates on loans were lowered in the United States, leading to the proliferation of "subprime loans". Basically, banks began to lend to everyone (especially to borrowers at high risk of default) and to sell these debts to investors at discounted prices in order to generate additional liquidity through securitization. In the meantime, there was an increase in the size of large financial institutions (the so-called "too big to fail" banks), which gave investors and households the impression that these institutions would never fail. Due to these factors, the asset bubble began to expand [46] until interest rates on loans rose again around 2004. As a result, many households did not repay their loans, investors lost money because securitized assets lost their value, and so did the banks. This liquidity crisis became evident when the American company Lehman Brothers declared bankruptcy in September 2008. The problem had global repercussions because these credit "packages" had also been sold in Europe. In Italy, house prices tended to rise between 2000 and 2007, but after the spread of the crisis in 2008, Italy needed until 2015 to recover. Looking at the graph of the RHPI, we can see a slight increase in prices, which could be a symptom of recovery, although unfortunately prices declined again during the COVID-19 pandemic because of reduced demand. Once the emergency was over in 2023-24, prices seemed to rise again.

Table 14 indicates the estimates of the differencing parameter for each sub-sample based on the break dates reported in Table 13.

Whether working with the original or logged data, the results of Table 13 are very similar. For the first sub-sample, the estimate of d is 0.63 in both data sets and the null hypothesis of $d = 1$ cannot be rejected. However, for the second sub-sample d is much higher, 2.62 for the original data and 2.85 for the transformed data. For the third sub-sample, d is higher than 1 but again the null hypothesis of $d = 1$ cannot be rejected, while it is rejected in favour of $d > 1$ in the fourth sub-sample. For the fifth subsample, the hypothesis $d = 1$ is not rejected, while d is significantly greater than 1 in the sixth and seventh sub-samples.

Notice that according to these results the I(1) hypothesis cannot be rejected in the first, third, and fifth sub-samples, while this hypothesis is rejected in all the remaining sub-samples in favour of values of d much higher than 1. Thus, no evidence of mean reversion or transitory shocks ($d < 1$) are found in any single case.

6. Conclusions

In this study, we analysed the Italian real house price index using quarterly data from 1970 to 2024 within a fractional integration framework. The empirical evidence indicates a high degree of persistence in the series, although the estimated degree of dependence remains sensitive to the specification adopted for the short-memory component. Although the model-selection criteria tend to favour specifications with Bloomfield disturbances, no single specification uniformly dominates according to the residual diagnostics. For this reason, the Bloomfield specification is not treated as automatically superior, but rather as one admissible representation of the short-memory dynamics whose implications are assessed together with diagnostic tests, model-comparison criteria, and robustness checks.

Based on the high degree of persistence observed in the data, we next studied the first differences in log-transformed prices, as this provides a good approximation to the growth rate of the series. Applying the same methodology described above, we found that the growth rates are very heterogeneous, depending on how we specify the error term. However, once again, the specification based on Bloomfield disturbances produces weaker evidence of long-memory behaviour than the alternative specifications considered, a result that is also consistent with Lo's statistic [33]. To support these conclusions, we also tried to replace the linear deterministic component of the model with a nonlinear one, using Chebyshev polynomials in the temporal variable, but the analysis showed no nonlinear structures in either the original or logged data. In conclusion, using the growth rate series, under the white-noise specification the estimate of d is 0.91 so the unit root hypothesis cannot be rejected even though none of the coefficients are found to be significant.

In the second part of this work, we have investigated the presence of structural breaks in the data, using multiple breaks approach [18] and its extension to the fractional case [19]. The results indicate the presence of six structural breaks (in both the original and the logged data) that are broadly consistent with the main cycles in the Italian housing market for the evolution of real prices identified by ISTAT in [43]. After explaining the historical and macroeconomic reasons behind these cycles, we estimated the differencing parameter d for each subsample determined by the break dates, obtaining very similar results on both the original and logged data. The evidence suggests that the I(1) hypothesis cannot be rejected in the first, third, and fifth sub-samples, while this hypothesis is rejected in all the remaining sub-samples in favour of values of d much higher than 1. Consequently, no evidence of mean reversion or transitory shocks (i.e., $d < 1$) are found in any single case.

This study was limited to one indicator of house prices that appears correlated with other indices included in the original data set. The evidence reported here is specific to the Italian housing market and should not automatically be generalised to other countries characterised by different institutional and policy environments. It is reasonable to assume that these further indices also follow similar behaviour. Future research will investigate whether this assumption is reasonable and whether any functional relationships exist with the house price indicator considered here. Another interesting line of research concerns the application of fractional integration methods to study multidimensional dependence across different indices within the same country, as well as across different countries for the same index.

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