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To cite this article: Alberto García Martín-Caro *et al* 2026 *Class. Quantum Grav.* **43** 155001

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PAPER

OPEN ACCESS

RECEIVED
13 May 2026

REVISED
9 July 2026

ACCEPTED FOR PUBLICATION
15 July 2026

PUBLISHED
29 July 2026

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Robustness of analogue Hawking radiation in cavities with moving boundaries

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Keywords: dynamical Casimir effect, numerical analysis, thermality, quantum field theory

Abstract

In this work we explore the limitations and robustness of thermal radiation in dynamical Casimir systems serving as analogues for Hawking radiation. Through detailed numerical analysis, we characterize particle production spectra in cavities with moving boundaries under various configurations. We establish a thermality criterion based on the detailed balance condition, and we find that thermal signatures emerge in the spectrum of specific expanding cavity configurations, although they are highly dependent on frequency bands and acceleration parameters. Furthermore, our framework successfully verifies previous theoretical predictions, demonstrating that non-standard statistical distributions (such as Fermi–Dirac) can be robustly reproduced in finite confined settings despite the underlying field being purely bosonic. Further, we derive fitting expressions that quantify deviations from idealized thermal spectra through grey-body factors, revealing oscillatory behaviours tied to acceleration duration. Our results identify experimental setups that can reliably reproduce the thermal production akin to that from gravitationally-induced phenomena and quantify how finite-size effects and transient dynamics modify the expected thermal distributions, providing a comprehensive framework for distinguishing genuine Hawking-like radiation from experimental artefacts.

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1. Introduction

Hawking radiation stands as one of the most studied theoretical predictions of quantum field theory in curved spacetimes. It arises from quantum fluctuations in the vacuum, where pairs of particle-antiparticle are generated in the vicinity of the horizon [1]. The resulting radiation is found to be in a thermal state described entirely by the macroscopic parameters of the black hole. This phenomenon was also shown to occur for observers following accelerated trajectories measuring quantum fields with Unruh-de Witt detectors [2], providing fundamental connections between gravity, thermodynamics, and quantum theory.

These studies kick-started research on potential experimental proposals in analogue systems. A well-known setting is an accelerating mirror (boundary condition), which can drive a quantum field, initially in the vacuum, into a thermal state at late times [3, 4]. While its experimental realization remained elusive for years, more recent examples of analogue systems dedicated to Hawking-like radiation with a solid experimental basis include Bose-Einstein condensates [5–14], surface waves in water flows [15–23], nonlinear optics [24–28], and quantum fluids of light [29–31].

In this work, we focus on analogue systems based on flat spacetimes with moving boundaries, following the lines of Davies and Fulling [3]. These systems, which also serve to study the dynamical Casimir effect [32, 33], have gained renewed interest due to their implementation in optical cavities [34] and superconducting circuits [35–37]. The quantization of fields in these non-stationary settings relies on foundational Hamiltonian formalisms developed by Law [38, 39] and Dodonov *et al* [40], which provided the first rigorous methods to derive Bogoliubov coefficients in dynamic boundaries nearly thirty years ago.

In superconducting realizations the perfectly reflecting (Dirichlet) boundary is an idealization of the condition actually imposed by the SQUID terminating the transmission line. In the linearized regime the SQUID enforces a Robin-type (inductive) boundary condition, $\phi + \lambda_R \partial_x \phi = 0$, with an effective length λ_R set by the Josephson inductance; modulating the external flux modulates λ_R and is equivalent to a Dirichlet mirror at a flux-dependent effective position [35, 36, 41], so that the moving-Dirichlet model retains the dominant physics. A Robin condition contributes a frequency-dependent reflection phase that reduces to the Dirichlet value as $k\lambda_R \rightarrow 0$; it shifts the effective frequencies and modifies the spectrum in the ultraviolet, while the infrared sector, which controls the thermal behaviour, is essentially unaffected as long as $\lambda_R \ll L$, since long-wavelength modes do not resolve the microscopic structure of the boundary. The fitting expressions are therefore expected to hold in the infrared under the replacement of Dirichlet by Robin conditions, with corrections confined to the ultraviolet; a quantitative treatment of the Robin case is left for future work.

The relevance of moving boundaries transcends the study of the dynamical Casimir effect, as they provide a powerful dictionary for gravitational physics. By a suitable choice of the mirror's trajectory, one can establish a one-to-one correspondence between the Bogoliubov coefficients of the moving boundary and those of a field in a collapsing stellar model forming a black hole [42]. For instance, a mirror following a particular exponential trajectory mimics the late-time behaviour of a Schwarzschild horizon, producing a purely thermal flux of particles. In this context, our systematic analysis of different trajectories—expanding, collapsing, and vibrating—is not merely a collection of case studies, but a probe into how departures from these idealized ‘horizon-mimicking’ trajectories, due to finite-size effects or non-adiabatic transients, affect the validity of the gravitational analogy. In [43] the authors noted that there are configurations of the boundaries for which it is possible to obtain a nearly thermal spectrum, with superposed oscillations attributed to a greybody factor consequence of the finiteness of the motion of the trajectories of the mirrors. Building on previous findings, the work presented in our manuscript examines additional configurations to test the robustness of thermal particle production from accelerating boundaries. Here, robustness refers to the stability of the thermal particle-production spectrum under changes of the boundary trajectory—finite-size effects, non-adiabatic transients, return-to-origin motions and repeated cycles—all analysed for a closed system evolving unitarily. Furthermore, we investigate whether a thermal spectrum emerges for configurations where cavity collapses to its original length after expansion, even if the process is repeated several few times.

As shown in [43], moving boundaries can produce thermal spectra; however, realistic experimental configurations introduce significant challenges. The scarcity of closed-form expressions for most physical

paths necessitates a more systematic approach beyond standard approximations. This paper provides a rigorous numerical-analytical framework to quantify the robustness of thermal signatures. We provide simple fitting expressions of our numerical results that also parameterize the deviations from the Planckian spectrum. This allows us to characterize the transition between genuine Hawking-like radiation and spurious creation effects attributable to the mirror dynamics, offering a predictive toolkit for future experimental validation [44, 45].

This manuscript is organized as follows. Section 2 is devoted to the description of the formalism to quantize a field in flat spacetime with moving boundaries. In section 3 we study an expanding cavity for several values of the acceleration. We also propose a set of fitting functions that approximate the numerical results and that are useful for discerning the spurious effects on the thermal spectra. In section 4 we present a detailed numerical study of different scenarios where the cavity expands and returns to its original position, the thermal properties of the spectrum, and a comparison with previous results found in the recent literature. We also discuss the effects of concatenating these expanding and collapsing trajectories in the resulting spectrum. Finally, in section 5 we present the main conclusions of this work. For the sake of completeness, we add two Appendices.

2. Quantum fields with nonstationary boundaries

We will start this section motivating our study of Hawking radiation with the analogue model provided originally by Davies and Fulling [3] for a massless scalar field in a 1+1 spacetime with one Dirichlet boundary condition $\phi(t, x = f(t)) = 0$. Similar models, with more realistic trajectories, where the mirror becomes inertial at late times, yield similar results regarding the production of a thermal spectrum. Here, one typically assumes that $f(t) = 0$ for $t < 0$, $0 \neq |\dot{f}(t)| < 1$ for $t > 0$ and $t < t_i$, and finally the mirror remains inertial with speed $\text{const} = |\dot{f}(t)| \simeq 1$ for $t > t_i$. For concreteness, the trajectories typically take the form

$$f(t) = -t - \frac{1}{2\kappa} e^{-2\kappa t} + \frac{1}{2\kappa}, \quad (1)$$

for $0 < t < t_i$, with κ some positive constant, and

$$f(t) = -\left(1 - e^{-2\kappa t_i}\right)(t - t_i) - t_i - \frac{1}{2\kappa} e^{-2\kappa t_i} + \frac{1}{2\kappa} \quad (2)$$

for $t > t_i$. These scenarios have been studied in detail, for example in [46, 47]. After some approximations, in the limit $\omega' \gg \omega$, with ω' and ω the *in* and *out* frequencies, and for $t_i \gg \kappa^{-1}$, the (moduli of the) Bogoliubov coefficients typically satisfy the celebrated *detailed balance condition*:

$$|\alpha_{\omega', \omega}|^2 = e^{2\pi\omega/\kappa} |\beta_{\omega', \omega}|^2 \quad \text{with} \quad |\beta_{\omega', \omega}|^2 \simeq \frac{1}{2\pi\kappa\omega'} \frac{1}{e^{2\pi\omega/\kappa} - 1}. \quad (3)$$

This condition is usually considered a hallmark of thermal equilibrium and guarantees that the rate of particle emission and absorption are related as in a thermal ensemble, ensuring that the created particle distribution is consistent with a thermal state in equilibrium at a temperature $T = \kappa/2\pi$ ⁵.

To gauge the experimental relevance of this temperature we restore physical units. Following the Fulling–Davies convention in equation (3), κ is a frequency expressed in units of the cavity scale c/L_0 , with L_0 the (initial) length of the cavity introduced below, c the speed of light in vacuum, and the Hawking temperature $T_H = \hbar\kappa/(2\pi k_B)$ reads

$$T_H = \frac{\hbar c}{2\pi k_B} \frac{\kappa}{L_0} \approx 3.6 \times 10^{-4} \text{ Km} \times \frac{\kappa}{L_0}, \quad (4)$$

where κ takes the values used in our simulations ($\kappa = 33.3$ and $\kappa = 1200$) and L_0 is the cavity length. For L_0 in the few-cm range typical of superconducting resonators ($L_0 \sim 3\text{--}15$ cm, i.e. a fundamental in the few-GHz range), this gives $T_H \sim 0.4\text{--}0.08$ K for $\kappa = 33.3$ and $T_H \sim 15\text{--}3$ K for $\kappa = 1200$, in both cases well above the base temperature of a dilution refrigerator ($\sim 10\text{--}20$ mK). For the largest accelerations the boundary becomes ultra-relativistic, so that the main experimental challenge is the acceleration itself rather than the ambient thermal background. For more details, see [52, 53].

⁵ More technically, the detailed balance condition (3) is usually presented as an equivalent relation for the (Fourier transformed) Wightman function, which implies the Kubo–Martin–Schwinger [48, 49] condition of periodicity in imaginary time of the two point function for a thermal state [50], although the equivalence is lost in the case of non-thermal states, see [51].

Interestingly, there is a considerable number of works discussing particle production for different trajectories, including boundaries that accelerate for finite periods of time [54], and trajectories where the mirror returns to its original position [55]. In this manuscript we are interested in these later kind. On one hand, because they are more prone to experimental implementation. On the other hand, they are physically relevant regarding their analogy with black hole physics: they can mimic different scenarios of black hole formation and evaporation, helping unveiling possible issues on information loss paradox that could have gone under the radar [53, 56–59]. The trajectories considered in [55] can also be tuned to produce particle production with a multitude of spectra. Among them, we underline the possibility of create particle spectra following a Bose–Einstein thermal properties, and more interestingly, trajectories that result in Fermi–Dirac particle distribution (despite the underlying degrees of freedom are bosonic). We will come back to the study of such trajectories in section 4.1.

In our study, we will focus our attention on a Klein–Gordon (massless) scalar field ϕ in a one-dimensional cavity represented by two Dirichlet boundary conditions $\phi(t, x = f(t)) = \phi(t, x = g(t)) = 0$. A comprehensive and detailed analysis can be found in [43, 52]. The functions $f(t)$ and $g(t)$ represent the prescribed trajectories of the left and right boundaries, respectively. We will assume that the field propagates at the speed of light, which, as customary, we will set in what follows to $c = 1$. Then, the field $\phi(t)$ admits a natural decomposition in modes given by

$$\phi(t, x) = \sum_{n=1}^{\infty} \phi_n(t) \sin \left[\frac{n\pi}{g(t) - f(t)} (x - f(t)) \right]. \quad (5)$$

The dynamics of the Fourier modes of this Klein–Gordon field is determined by the Hamiltonian

$$H_T = \sum_{n=1}^{\infty} \frac{1}{4L} \left[\pi_n^2 + (n\pi)^2 \phi_n^2 \right] - \frac{\dot{L}}{4L} \pi_n \phi_n + \sum_m (1 - \delta_{nm}) \frac{nm}{n^2 - m^2} \left[\frac{\dot{f}}{L} \left((-1)^{n+m} - 1 \right) + \frac{\dot{L}}{L} (-1)^{n+m} \right] \pi_m \phi_n, \quad (6)$$

where $L(t) = g(t) - f(t)$ is the length of the cavity, and

$$\pi_n = L \dot{\phi}_n + \frac{\dot{L}}{2} \phi_n - 2 \sum_m (1 - \delta_{nm}) \frac{mn}{m^2 - n^2} \left[\dot{f} \left((-1)^{m+n} - 1 \right) + \dot{L} (-1)^{m+n} \right] \phi_m, \quad (7)$$

are the conjugate momenta of the Fourier modes ϕ_n . They satisfy the Poisson algebra

$$\{\phi_n, \pi_m\} = 2\delta_{nm}. \quad (8)$$

The equations of motion can be computed via $\dot{\phi}_n = \{\phi_n, H_T\}$ and $\dot{\pi}_n = \{\pi_n, H_T\}$. We easily obtain

$$\dot{\phi}_n = \frac{1}{L} \pi_n - \frac{\dot{L}}{2L} \phi_n + 2 \sum_m (1 - \delta_{nm}) \frac{mn}{m^2 - n^2} \left[\frac{\dot{f}}{L} \left((-1)^{m+n} - 1 \right) + \frac{\dot{L}}{L} (-1)^{m+n} \right] \phi_m, \quad (9)$$

$$\dot{\pi}_n = -\frac{1}{L} (n\pi)^2 \phi_n + \frac{\dot{L}}{2L} \pi_n + 2 \sum_m (1 - \delta_{nm}) \frac{nm}{m^2 - n^2} \left[\frac{\dot{f}}{L} \left((-1)^{n+m} - 1 \right) + \frac{\dot{L}}{L} (-1)^{n+m} \right] \pi_m. \quad (10)$$

In the cases where the cavity is neither moving nor expanding, namely $\dot{f}(t) = 0$ and $\dot{L} = 0$, the equations of motion reduce to a collection of decoupled harmonic oscillators. However, in general, whenever $\dot{f}(t) \neq 0$ and/or $\dot{L} \neq 0$, there will be couplings between different modes.

In the complexified space of solutions $\mathcal{S}^{\mathbb{C}}$ there is a natural Klein–Gordon product that is preserved by evolution. Concretely, let us define a solution as

$$\mathbf{U}(t) = (U_{1,\epsilon_1}(t), U_{2,\epsilon_2}(t), \dots),$$

such that $U_{n,\epsilon_n}(t) = (\phi_n(t), \pi_n(t))$ and with $U_{n,\epsilon_n=0}(t) = \phi_n(t)$ and $U_{n,\epsilon_n=1}(t) = \pi_n(t)$. Hence, the Klein–Gordon product $\mathcal{S}^{\mathbb{C}} \times \mathcal{S}^{\mathbb{C}} \rightarrow \mathbb{C}$ is defined as the map

$$\langle \mathbf{U}^{(1)}(t), \mathbf{U}^{(2)}(t) \rangle = \frac{i}{2} \sum_{n=1}^{\infty} \bar{\phi}_n^{(1)}(t) \pi_n^{(2)}(t) - \bar{\pi}_n^{(1)}(t) \phi_n^{(2)}(t),$$

with $\mathbf{U}^{(1)}(t)$ and $\mathbf{U}^{(2)}(t)$ two solutions in $\mathcal{S}^{\mathbb{C}}$. It is important to remark that it is independent of t (on solutions).

This inner product allows us to construct a basis of orthonormal solutions $(\mathbf{u}^{(I)}, \bar{\mathbf{u}}^{(I)}(t))$, with $I = 1, 2, \dots$. Concretely, we can always find solutions such that

$$\langle \mathbf{u}^{(I)}(t), \mathbf{u}^{(J)}(t) \rangle = \delta^{IJ}, \langle \mathbf{u}^{(I)}(t), \bar{\mathbf{u}}^{(J)}(t) \rangle = 0, \langle \bar{\mathbf{u}}^{(I)}(t), \bar{\mathbf{u}}^{(J)}(t) \rangle = -\delta^{IJ}. \quad (11)$$

for all t . Here, $u^{(I)}(t)$ and $\bar{u}^{(I)}(t)$ denote solutions of positive and negative Klein–Gordon norm, respectively, which in the static *in* and *out* regions reduce to the usual positive- and negative-frequency modes.

In addition, given another basis of solutions $(\mathbf{w}^{(I)}, \bar{\mathbf{w}}^{(I)}(t))$, they are related by the well-known Bogoliubov transformation

$$\mathbf{u}^{(I)}(t) = \sum_{J=1}^{\infty} \left(\alpha_{IJ} \mathbf{w}^{(J)}(t) + \beta_{IJ} \bar{\mathbf{w}}^{(J)}(t) \right). \quad (12)$$

such that

$$\begin{aligned} \alpha_{IJ} &= \langle \mathbf{w}^{(J)}(t), \mathbf{u}^{(I)}(t) \rangle = \frac{i}{2} \sum_{n=1}^{\infty} w \bar{\phi}_n^{(J)}(t) u \pi_n^{(I)}(t) - w \bar{\pi}_n^{(J)}(t) u \phi_n^{(I)}(t), \\ [1ex] \beta_{IJ} &= -\langle \bar{\mathbf{w}}^{(J)}(t), \mathbf{u}^{(I)}(t) \rangle = -\frac{i}{2} \sum_{n=1}^{\infty} w \phi_n^{(J)}(t) u \pi_n^{(I)}(t) - w \pi_n^{(J)}(t) u \phi_n^{(I)}(t). \end{aligned} \quad (13)$$

are the Bogoliubov coefficients⁶. Conversely, the equation relating the new basis of solutions with the first one reads

$$\mathbf{w}^{(I)}(t) = \sum_{J=1}^{\infty} \left(\bar{\alpha}_{IJ} \mathbf{u}^{(J)}(t) - \beta_{IJ} \bar{\mathbf{u}}^{(J)}(t) \right). \quad (16)$$

For a correct interpretation of these coefficients, it is convenient to jump into the quantum theory. The Fourier modes collected in $\mathbf{U}(t)$ must satisfy the reality condition $\bar{\mathbf{U}}(t) = \mathbf{U}(t)$. Hence, if we express it in terms of the complex basis $(\mathbf{u}^{(I)}(t), \bar{\mathbf{u}}^{(I)}(t))$, they must be written as

$$\mathbf{U}(t) = \sum_{I=1}^{\infty} a_I \mathbf{u}^{(I)}(t) + \bar{a}_I \bar{\mathbf{u}}^{(I)}(t) = \sum_{I=1}^{\infty} b_I \mathbf{w}^{(I)}(t) + \bar{b}_I \bar{\mathbf{w}}^{(I)}(t), \quad (17)$$

or, as in the last equality, in terms of the solutions in the basis $(\mathbf{w}^{(I)}(t), \bar{\mathbf{w}}^{(I)}(t))$. The complex amplitudes $(a_I, \bar{a}_I)$ and $(b_I, \bar{b}_I)$ represent the annihilation and creation variables of each basis. In the classical theory, they satisfy the Poisson algebra $\{a_I, \bar{a}_J\} = -i\delta^{IJ}$, $\{a_I, a_J\} = 0 = \{\bar{a}_I, \bar{a}_J\}$, and similarly for $(b_I, \bar{b}_I)$. One can easily see that the annihilation variables $(b_I, \bar{b}_I)$ are related to $(a_I, \bar{a}_I)$ by

$$b_I = \sum_{J=1}^{\infty} \alpha_{IJ} a_J + \bar{\beta}_{IJ} \bar{a}_J. \quad (18)$$

One can easily invert this relation to express $(a_I, \bar{a}_I)$ in the basis $(b_I, \bar{b}_I)$. The result is

$$a_I = \sum_{J=1}^{\infty} \bar{\alpha}_{IJ} b_J - \bar{\beta}_{IJ} \bar{b}_J. \quad (19)$$

In the quantum theory, the classical algebra is promoted to

$$[\hat{a}_I, \hat{a}_J^\dagger] = \delta_{IJ} \hat{\mathbf{I}}, \quad [\hat{a}_I, \hat{a}_J] = 0 = [\hat{a}_I^\dagger, \hat{a}_J^\dagger]. \quad (20)$$

⁶ These Bogoliubov coefficients satisfy the conditions

$$\sum_{K=1}^{\infty} \alpha_{IK} \bar{\alpha}_{JK} - \beta_{IK} \bar{\beta}_{JK} = \delta_{IJ}, \quad \sum_{K=1}^{\infty} \alpha_{IK} \beta_{JK} - \beta_{IK} \alpha_{JK} = 0. \quad (14)$$

$$\sum_{K=1}^{\infty} \bar{\alpha}_{KI} \alpha_{KJ} - \beta_{KI} \bar{\beta}_{KJ} = \delta_{IJ}, \quad \sum_{K=1}^{\infty} \bar{\alpha}_{KI} \beta_{KJ} - \beta_{KI} \bar{\alpha}_{KJ} = 0. \quad (15)$$

and again, we have similar expressions for $(\hat{b}_I, \hat{b}_I^\dagger)$. Now, we must note that each collection of annihilation variables defines a different vacuum state if $\beta_{IJ} \neq 0$. Concretely, the vacuum state is determined by

$$\hat{a}_I|0\rangle = 0, \quad I = 1, 2, \dots \quad (21)$$

Hence, we immediately note that

$$\hat{b}_I|0\rangle = \sum_{J=1}^{\infty} \bar{\beta}_{IJ} \hat{a}_J^\dagger |0\rangle \neq 0, \quad I = 1, 2, \dots \quad (22)$$

Therefore, the coefficients β_{IJ} codify the information about particle production in the quantum theory. They are nonvanishing when there is mixing of positive and negative frequency solutions. In quantum optics, this is equivalent to a squeezing operator. Now, if $\beta_{IJ} = 0$ for all I, J , and α_{IJ} is nonvanishing for $I \neq J$, it implies a symplectic transformation without mixing positive and negative solutions. The two bases are different but, define the same vacuum state. This is typically identified with beam splitters in quantum optics.

In order to carry out our computations, we will solve equation (10) numerically, using an explicit embedded Prince–Dormand (8,9) method of the GNU scientific library, which belongs to the family of Runge–Kutta methods. The numeric library developed for this project can be found in [60]. It carries two types of errors (the absolute and the relative errors) whose value can be specified. In our simulations, we set the absolute error to be between $[10^{-10}, 10^{-12}]$, and the relative error to zero. We also introduce a cut of N in the total number of modes we evolve, but we allow it to vary just as $N = 256$, $N = 512$, and $N = 1024$, so that we can take a Richardson extrapolation in order to probe the limit $N \rightarrow \infty$. Since we always assume that the cavity is static in the past, we always have a preferred *in* basis that we denote by $(^{(\text{in})}\mathbf{u}^{(I)}(t), ^{(\text{in})}\bar{\mathbf{u}}^{(I)}(t))$. Likewise, we will always assume that the cavity becomes static in the future. Hence, we will have a natural *out* basis $(^{(\text{out})}\mathbf{u}^{(I)}(t), ^{(\text{out})}\bar{\mathbf{u}}^{(I)}(t))$ there.

The restriction to an effectively 1 + 1-dimensional field deserves a brief justification. A planar boundary displaced along the cavity axis preserves the transverse profile of the field and therefore couples only modes within the same transverse sector; as long as this transverse symmetry is preserved, no particles are produced in the higher transverse channels. Given that transverse modes do not satisfy Dirichlet boundary conditions, they are allowed to have zero wave numbers. Hence, the longitudinal sector will behave as an effective 1 + 1 field as long as the transverse modes have a large mass gap set by the transverse cutoff $\omega_\perp \sim \pi c/d$, with d the transverse size of the cavity. In our case, the relevant dynamical scales lie well below this cutoff, $\omega, \kappa \ll \omega_\perp$, i.e. $d \ll L$, and the 1+1 description is justified. In superconducting implementations based on coplanar-waveguide resonators or transmission lines this is the single-mode regime in which the devices operate. The reduction is expected to break down in the strongly squeezed limit $L \sim d$, where the longitudinal spacing $\Delta\omega_\parallel = \pi c/L$ becomes comparable to $\omega_\perp$ and transverse channels open; the configurations considered here lie well within the regime where this does not occur. The fully three-dimensional problem, retaining the transverse channels, is left for future work.

3. Numerical analysis for expanding cavities

In this section, we will discuss two configurations of accelerating mirrors. Concretely, we will consider a cavity where only one of the mirrors accelerates, with relatively large changes in the size of the cavity with small accelerations and relatively small changes in the size of the cavity with large accelerations. These are two interesting asymptotic regimes corresponding to adiabatic and sharp dynamical configurations of the mirrors, respectively. Each regime shows advantages from a theoretical and experimental perspective. We will discuss them in the following.

We will compare our numerical results with some fitting expressions for $|\alpha_{IJ}|^2$ and $|\beta_{IJ}|^2$, which contain the most physically relevant information that we want to discuss in this manuscript. Concretely, we consider a (modified) Fulling–Davies [3, 61] spectrum of particle production of the form

$$|\beta_{IJ}^{(f)}|^2 = \frac{N_\beta \Delta\omega_I \Delta\omega_J}{\pi \kappa \omega_I} \frac{\Gamma_\beta(\epsilon, \omega_J)}{(e^{2\pi\omega_J/\kappa} - 1)}, \quad (23)$$

where $\omega_I = \pi I/L_0$ are the *in* frequencies and $\Delta\omega_I = \pi/L_0$ the corresponding gap (which in this model equals the fundamental *in* frequency); ω_J and $\Delta\omega_J$ are the *out* frequencies that depend on the trajectory of the plates and hence will be specified below for each case. Note that in the continuum limit, $\Delta\omega_I \Delta\omega_J \rightarrow d\omega d\omega'$. Moreover, N_β is a normalization factor that will also depend on the concrete

dynamical configuration of the mirrors. Its typical value is 2, in agreement with the usual Fulling–Davies spectrum, but it can also be 4 in some cases (see appendix A). Finally,

$$\Gamma_\beta(\epsilon, \omega_J) = [A_\beta + B_\beta \sin^2(\mathcal{T}_\beta \omega_J)], \quad (24)$$

is a greybody factor that shows a strong dependence on ϵ (the change in the size of the cavity) and the *out* frequencies. It accounts for the finiteness of the duration of the acceleration, producing the oscillations described in very good approximation by a sinusoidal function with a frequency characterized by $\mathcal{T}_\beta = (1 + C_\beta)\epsilon$, with C_β a very small dimensionless parameter with a weak dependence on the trajectory parameters and the *in* and *out* frequencies⁷. A_β and B_β are also dimensionless parameters, with A_β small and B_β order unit, both of which are weakly dependent on the trajectory parameters and the *in* and *out* frequencies. This parametrization is valid only for the sets of modes that reach a nearly thermal final state. These sets of thermal modes will depend on the trajectories of the mirrors. An exact thermal state will correspond to $\Gamma_\beta(\epsilon, \omega_J) = 1$.

We have also found a fitting expression for the α coefficients, although it is not as simple as the one for the β coefficients. Concretely,

$$|\alpha_{IJ}^{(f)}|^2 = \begin{cases} \frac{N_\alpha \Delta\omega_I \Delta\omega_J}{\pi \kappa \omega_I} \frac{1}{1 + D_1} \left(1 + D_1 \frac{(F\omega_I)^2}{|F\omega_I - \omega_J|^2} \right) \frac{\Gamma_\alpha(\epsilon, \omega_J) e^{2\pi\omega_J/\kappa}}{(e^{2\pi\omega_J/\kappa} - 1)}, & \omega_J < F\omega_I, \\ \frac{N_\alpha \Delta\omega_I \Delta\omega_J}{\pi \kappa \omega_I} \left(1 + D_2 \frac{(F\omega_I)^2}{|F\omega_I - \omega_J|^2} \right) \frac{\Gamma_\alpha(\epsilon, \omega_J)}{(e^{2\pi(\omega_J - F\omega_I)/\tilde{\kappa}} - 1)}, & \omega_J > F\omega_I. \end{cases} \quad (25)$$

Here, the dimensionless parameters D_1 , D_2 and F take values that are $\mathcal{O}(1)$ or smaller, which means that $|\alpha_{IJ}|^2$ reaches a maximum value when $\omega_J \simeq F\omega_I$. In addition, $\Gamma_\alpha(\epsilon, \omega_J)$ has the same functional form as $\Gamma_\beta(\epsilon, \omega_J)$ in equation (24) but with dimensionless constants A_α , B_α and C_α (the latter defining $\mathcal{T}_\alpha$). Besides, N_α is a normalization factor that depends on the trajectories of the mirrors.

Let us now compare these fitting expressions in more detail with concrete numerical examples. In all cases, we choose several parameters of the trajectories of the mirrors within the range of several experimental settings [44, 45]. The trajectories of the boundaries of the cavity are given by:

$$\begin{aligned} f(t) &= 0, \\ g(t) &= 1 + \frac{s}{2\kappa} + \frac{1}{2\kappa} [\log(\cosh(\kappa(t - t_0))) - \log(\cosh(s - \kappa(t - t_0)))], \end{aligned} \quad (26)$$

In this configuration, at $t \ll t_0$, the left boundary remains at $x^f = 0$ at all times, while the right boundary is nearly static at the initial position $x_{\text{in}}^g = 1$, while at $t \gg T \simeq t_0 + \epsilon$ (with $\epsilon = s/\kappa$) its final position will be $x_{\text{out}}^g = (1 + \epsilon)$. In the interval $[t_0, t_0 + \epsilon]$ the right boundary follows an acceleration close to the speed of light. Besides, regarding equations (23) and (25), the normalization in this case is just $N_\beta = 2 = N_\alpha$ and the out frequencies are given by $\omega_J = \pi J/(L_0 + \epsilon)$ and their gap by $\Delta\omega_J = \pi/(L_0 + \epsilon)$. We depict this configuration of the cavity in figure 1.

3.1. Small accelerations

We have studied several configurations for small accelerations. In all cases, we have found qualitatively similar results. Let us consider a concrete realization given by $\epsilon = 0.375$ and $\kappa = 33.3$. In figure 2 we show the behaviour of Bogoliubov coefficients for some frequency bands and compare them with the fitting expressions of equations (23) and (25). Here, we fix $A_\alpha = 1 = A_\beta$, and $B_\alpha = 10^{-3} = B_\beta$, $D_1 = 0.1 = D_2$ and $\tilde{\kappa} = 1.6\kappa$. The upper panel shows several (relatively) infrared *in* modes ω_I as functions of the *out* frequencies ω_J . We see that for low ω_J , the computed Bogoliubov coefficients agree very well with equations (23) and (25), with $C_\alpha = C_\beta$ and F suitable functions of ω_I . For infrared *in* modes, we see that $C_\alpha \rightarrow 0$ and $C_\beta \rightarrow 0$. We show in appendix B the behaviour of C_α , C_β and F , specifically, in figure 27. At very high ω_J we have checked numerically that the Bogoliubov coefficients decay as ω_J^{-n} with $n > 3$. On the other hand, the lower panel shows several (relatively) ultraviolet *in* modes ω_I as functions of the *out* frequencies ω_J . We see that for low ω_J the computed beta coefficients still agree well with equation (23). Nevertheless, the beta coefficients reach quicker the behaviour ω_J^{-n} , departing from equation (23) at lower *out* frequencies. This indicates that these *in* modes did not have enough time to thermalize.

⁷ These oscillations are common in scenarios similar to the one considered here. See, for instance, the β -coefficients in equation (7) of [62].

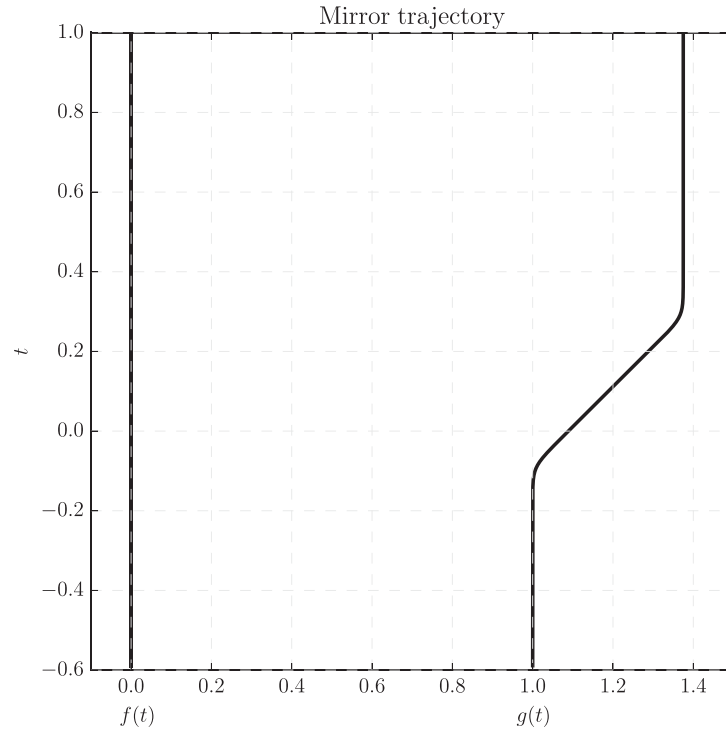


Figure 1. Trajectories: Typical trajectory given by equation (26), in this case with $\epsilon = 0.375$ and $\kappa = 33.3$.

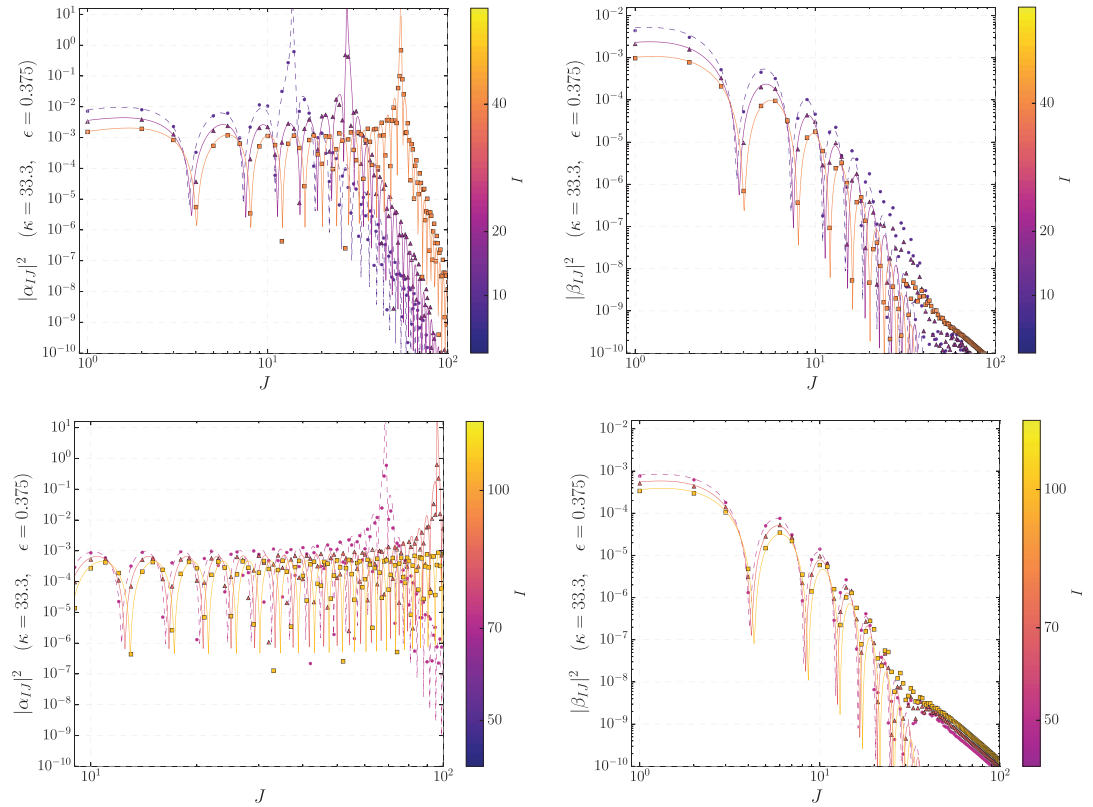


Figure 2. Bogoliubov coefficients: These plots correspond to the trajectories given by equation (26), with $\epsilon = 0.375$ and $\kappa = 33.3$. The upper panel for the infrared *in* modes. The lower panel for the ultraviolet *in* modes. The lines interpolating points correspond to the fitting expressions in equations (23) and (25) for the right and left panels, respectively.

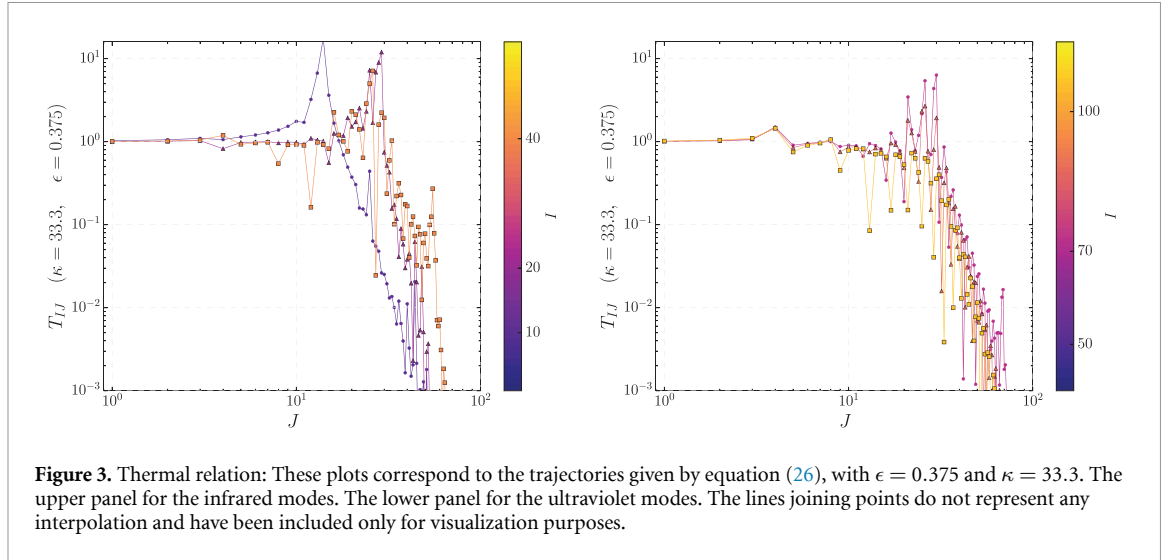


Figure 3. Thermal relation: These plots correspond to the trajectories given by equation (26), with $\epsilon = 0.375$ and $\kappa = 33.3$. The upper panel for the infrared modes. The lower panel for the ultraviolet modes. The lines joining points do not represent any interpolation and have been included only for visualization purposes.

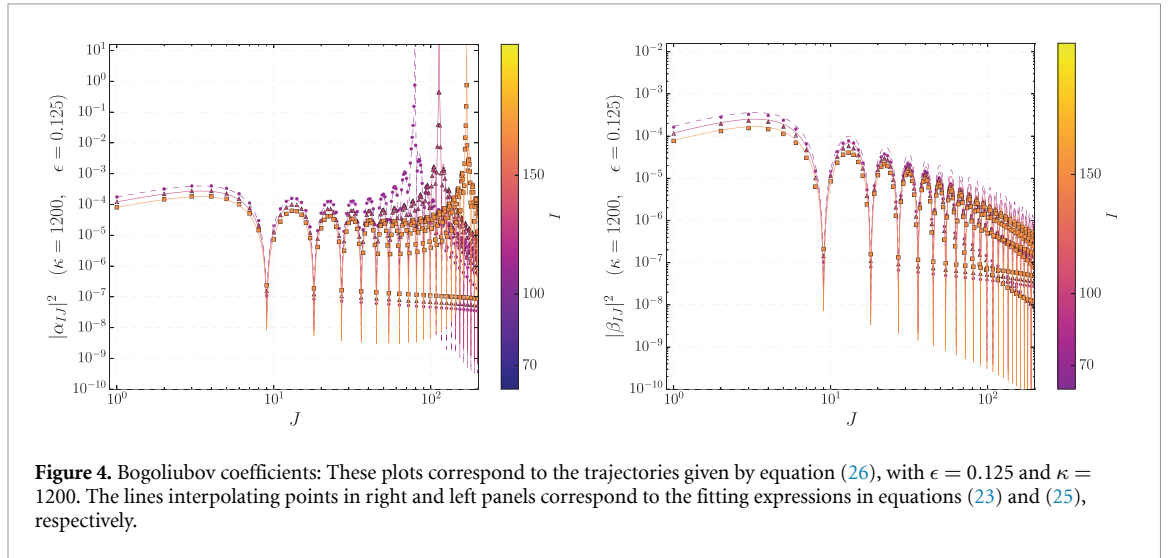


Figure 4. Bogoliubov coefficients: These plots correspond to the trajectories given by equation (26), with $\epsilon = 0.125$ and $\kappa = 1200$. The lines interpolating points in right and left panels correspond to the fitting expressions in equations (23) and (25), respectively.

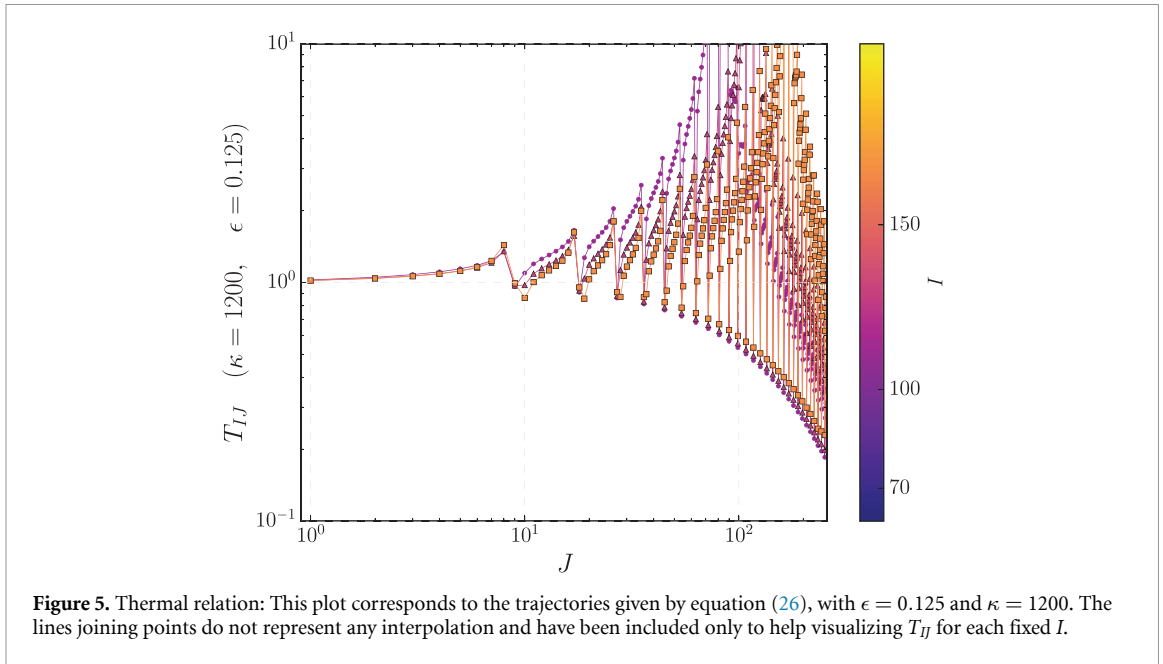
To gain insight into the thermal properties of these modes, in figure 3 we show the thermality function

$$T_{IJ} = \frac{|\alpha_{IJ}|}{|\beta_{IJ}|e^{\pi\omega_J/\kappa}}, \quad (27)$$

for several values of the *in* modes I . As we can see, for this trajectory of the mirrors, in the limit of infrared *out* modes and sufficiently large *in* modes, the thermality condition is near the unit and, hence, it is approximately fulfilled. Therefore, the trajectories in equation (26) are found to yield more resilient thermal signatures regarding the detailed balance condition in equation (27). On the other hand, for very ultraviolet *in* modes (large I), this detailed balance condition is met only for very infrared *out* modes (small J).

3.2. Large accelerations

We have also studied several sharp configurations, namely, for large accelerations κ and small changes ϵ in the size of the cavity. Here, the right mirror transitions very quickly from a steady configuration to a new one, sharply approaching the speed of light. The concrete configuration we will show here is given by $\epsilon = 0.125$ and $\kappa = 1200$. Other configurations with large accelerations show similar results. In figure 4 we show the behaviour of the Bogoliubov coefficients for modes in the frequency band $I \in (70, 150)$ and $J \in (1, 150)$, and compare them with the fitting expression of equation (23). In these simulations, we set $A = 1.0$ and $B = 10^{-4}$. We show the Bogoliubov coefficients for several *in* modes ω_I as functions of the *out* frequencies ω_J . We see that for very low ω_I , the computed beta coefficients agree very well



with equations (23) and (25). The parameters D_1 , D_2 , F and $\tilde{\kappa}$ are shown in appendix B, concretely, in figure 28 as functions of ω_J for the two trajectories shown here.

For lower or higher *in* modes compared to those shown here, the fitting expression for $|\alpha_{IJ}|^2$ still agrees very well, but the one for $|\beta_{IJ}|^2$ shows an amplitude that disagrees with the fitting expression. Hence, those modes depart strongly from a thermal distribution and, therefore, we do not consider them here. On the other hand, in figure 5 we show the thermal relation T_{IJ} in figure (3) for several values of the *in* modes I . As we can see, only the most infrared *out* modes yield $T_{IJ} \simeq 1$ and, hence, follow a thermal distribution. However, we also see departures from thermality in those modes where either $|\alpha_{IJ}|$ or $|\beta_{IJ}|$ reach their local minima, even in the infrared sector. In consequence, they do not follow a thermal distribution.

In addition to these configurations of the mirrors, we have also studied other interesting situations. We summarize here the main results, but for details, see appendix A. Concretely, one of the most interesting configurations corresponds to a cavity expanding symmetrically, namely, the two boundaries follow the same trajectory but in opposite directions. In this case, we recover a thermal spectra qualitatively similar to the ones discussed in this section. However, there are some differences. The configuration of the boundaries yields a spectrum where even and odd modes are decoupled. Consequently, the amplitude of the spectrum is twice the one discussed above. The other natural configuration corresponds to a cavity that moves rigidly (i.e. its length from the viewpoint of the laboratory remains constant). We found no thermal spectrum for infrared modes, except for the fundamental mode. This mode has occupation number that agrees very well with a thermal distribution. Hence, we can safely conclude that this configuration can be used to simulate an Unruh-de Witt detector. Finally, we have considered cavities that collapse, instead of expand. They are the time reversal of previous expanding configurations. In these situations, we observe no thermal spectra. However, they will be useful for the study of next section. There, we will consider trajectories where the mirrors return to its original position in a finite period of time, in order to compare with those studied in [55]. In this way, we can truly discuss if the above thermal spectra are robust against transients like the ones introduced by this kind of configurations.

4. Numerical analysis for expanding-collapsing cavities

In this section, we explore the last family of configurations we will study in this paper. Concretely, there is a natural question one can ask: *What is the robustness of the spectrum if an expanding cavity follows a subsequent collapse in such a way that the cavity reaches its original configuration in the end?* This kind of

configurations has been recently discussed in [55] for Minkowski spacetimes with one moving boundary. However, the thermal properties of the spectra show some limitations, as we will see. In this work, we have simulated them but for a field confined in a cavity, where one of the boundaries follows the trajectories in [55], obtaining very good agreement in all cases. In section 4.1 we show the details for two of these families of configurations.

Our aim in this section is to discuss their ability to produce a thermal spectrum, and compare with similar expanding-collapsing configurations but for the trajectories studied in our manuscript. In all cases, the latter trajectories will be of the form (26) for the expansion and its time reverse for the collapse, for simplicity, but with different values of the acceleration parameter.

In order to proceed, we can either carry out the whole simulation numerically, or take advantage of the simulations we have performed for the expanding cavity, as we have done for the case of collapsing cavities in appendix A, along with a composition of two Bogoliubov transformations. We will proceed this way here. Specifically, we need to compose two Bogoliubov transformations, one consisting of the *in* vacuum evolved to a time where the cavity has expanded (or collapsed), and the one of the *out* vacuum state evolved backwards in time from the final collapsed (expanded) cavity to the time where we match with the evolved *in* state. We discuss this in more detail in the following.

Let us express the *in* basis in some auxiliary basis at some intermediate instant where the cavity is neither expanding nor collapsing:

$${}^{(\text{in})}\mathbf{u}^{(I)}(t) = \sum_{J=1}^{\infty} \alpha_{IJ} {}^{(\text{aux})}\mathbf{u}^{(J)}(t) + \beta_{IJ} {}^{(\text{aux})}\bar{\mathbf{u}}^{(J)}(t). \quad (28)$$

Now, in order to deal with the collapsing evolution, the auxiliary basis is evolved to times where the *out* basis is the standard one and has the form

$${}^{(\text{aux})}\mathbf{u}^{(I)}(t) = \sum_{J=1}^{\infty} \gamma_{IJ} {}^{(\text{out})}\mathbf{u}^{(J)}(t) + \rho_{IJ} {}^{(\text{out})}\bar{\mathbf{u}}^{(J)}(t). \quad (29)$$

We can compose both transformations so that we get

$$\begin{aligned} {}^{(\text{tot})}\alpha_{IJ} &= \sum_{K=1}^{\infty} \alpha_{IK} \gamma_{KJ} + \beta_{IK} \bar{\rho}_{KJ}, \\ {}^{(\text{tot})}\beta_{IJ} &= \sum_{K=1}^{\infty} \alpha_{IK} \rho_{KJ} + \beta_{IK} \bar{\gamma}_{KJ}. \end{aligned} \quad (30)$$

However, we must note that γ_{IJ} and ρ_{IJ} are not independent of α_{IJ} and β_{IJ} . Actually, we can consider the *out* basis in the asymptotic future evolved backwards to the intermediate time where the cavity is neither expanding nor collapsing, and express it in the auxiliary basis. The corresponding transformation will be given by

$${}^{(\text{out})}\bar{\mathbf{u}}^{(I)}(t) = \sum_{J=1}^{\infty} \alpha_{IJ} {}^{(\text{aux})}\bar{\mathbf{u}}^{(J)}(t) + \beta_{IJ} {}^{(\text{aux})}\mathbf{u}^{(J)}(t). \quad (31)$$

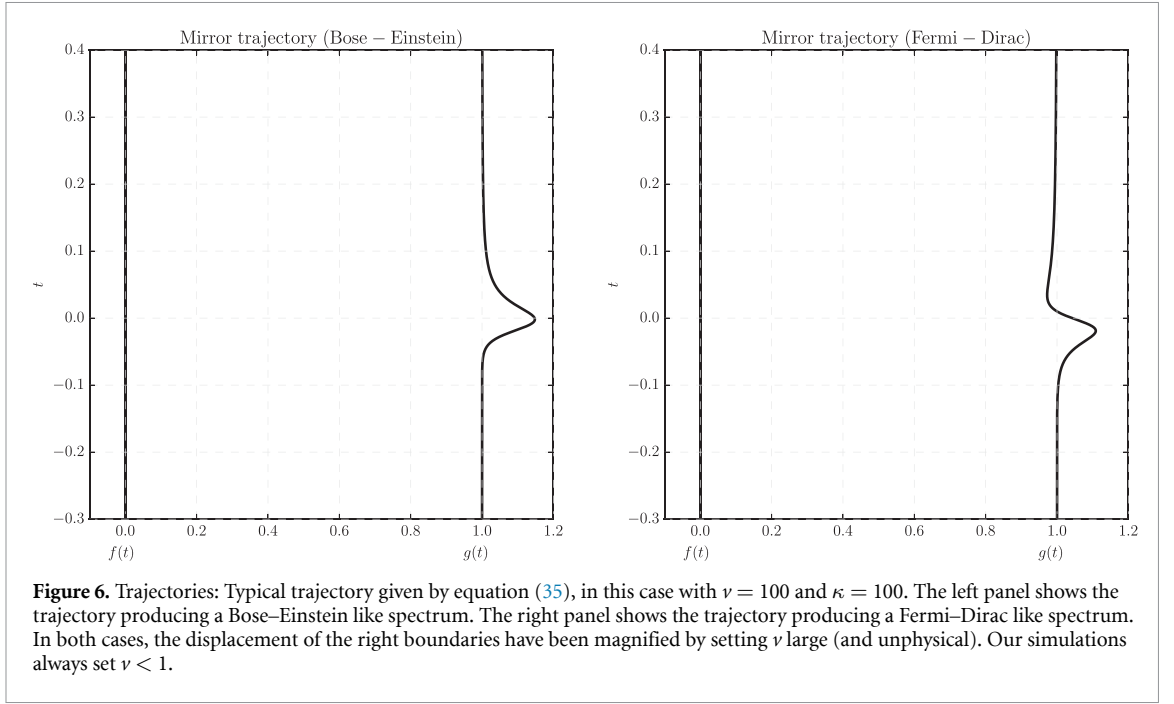
Note that the elements of the *out* basis, ${}^{(\text{out})}\bar{\mathbf{u}}^{(I)}$, (and also those of the *aux* basis) have positive norm when evolved backward in time, while ${}^{(\text{out})}\mathbf{u}^{(I)}$ have negative norm in this backwards evolution, since the time direction is reversed.

Now, we can invert equation (31) getting

$${}^{(\text{aux})}\mathbf{u}^{(I)}(t) = \sum_{J=1}^{\infty} \alpha_{IJ} {}^{(\text{out})}\bar{\mathbf{u}}^{(J)}(t) - \bar{\beta}_{IJ} {}^{(\text{out})}\mathbf{u}^{(J)}(t). \quad (32)$$

Hence, identifying equations (29) and (32) we conclude that

$$\gamma_{IJ} = \alpha_{IJ}, \quad \rho_{IJ} = -\bar{\beta}_{IJ}. \quad (33)$$



In total, after one cycle, the *in* basis can be written in terms of the *out* basis as

$${}^{(\text{in})}\mathbf{u}^{(I)}(t) = \sum_{J=1}^{\infty} {}^{(\text{tot})}\alpha_{IJ} {}^{(\text{out})}\mathbf{u}^{(J)}(t) + {}^{(\text{tot})}\beta_{IJ} {}^{(\text{out})}\bar{\mathbf{u}}^{(J)}(t), \quad (34)$$

with the Bogoliubov coefficients given in equation (30) with γ and ρ expressed as in equation (33).

For a chain of N expansion-collapse cycles, the same reasoning allows to obtain the end state in terms of some Bogoliubov coefficients that result from the successive composition (equation (30)) of intermediate coefficients. We analyse these chains in section 4.3.

4.1. Bose and Fermi Dirac

Before we start studying the configurations for the trajectories given in (26), we will make a small detour so that we discuss several properties of some of the trajectories in [55], that also correspond to mirrors that fly away and return to its original position. The main difference with our work is that, there, there is only one boundary. Concretely, the trajectories we are interested in are, respectively,

$$z_{-}(t) = \frac{\nu}{\kappa} \frac{W(e^{\kappa t})}{(1 + W(e^{\kappa t}))^3}, \quad \text{and} \quad z_{+}(t) = \frac{\nu}{\kappa} \frac{(1 - W(e^{\kappa t})) \sqrt{W(e^{\kappa t})}}{2(1 + W(e^{\kappa t}))^3}, \quad (35)$$

with ν the maximum speed of the boundary and $W(x)$ the Lambert function. In figure 6 we show the typical trajectories for both configurations.

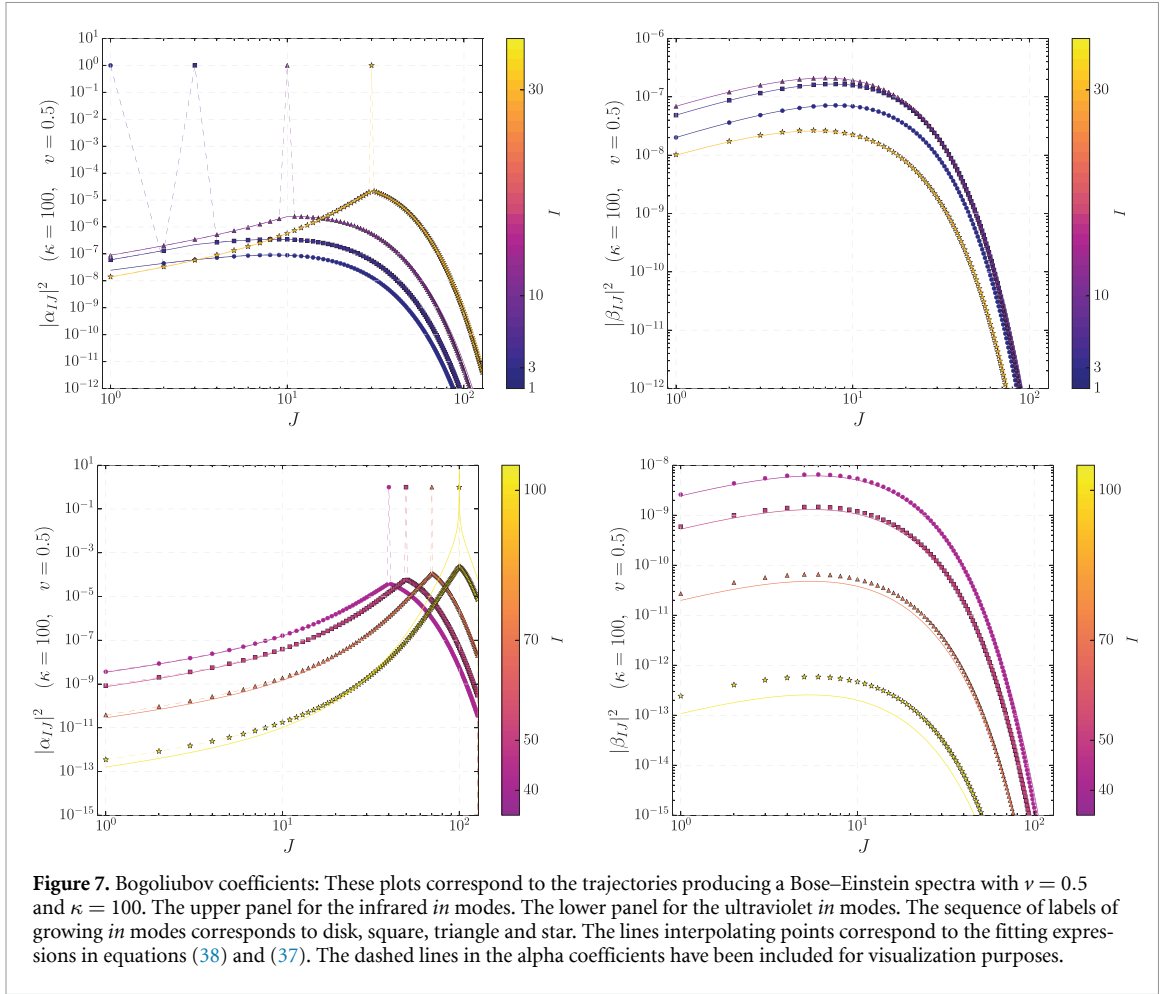
In the limit of small velocities $|\dot{z}_{\pm}(t)|$, i.e. small ν , the spectrum can be computed in closed form. The respective beta Bogoliubov coefficients can be computed approximately at leading order in ν ,

$$|\beta_{\omega\omega'}^{\mp}|^2 = \frac{4\nu^2 |\omega\omega'|}{\pi \kappa^5} \frac{|\omega + \omega'|}{e^{2\pi|\omega + \omega'|/\kappa} \mp 1}. \quad (36)$$

The alpha Bogoliubov coefficients can be easily derived by replacing $\omega' \rightarrow -\omega'$ in equation (36).

In our setting, we consider a field enclosed in a cavity with moving Dirichlet boundary conditions. To compare with the above results, we will assume that the left boundary remains static while the right boundary undergoes either trajectory $z_{\pm}(t)$ in equation (35). The expressions that fit our numerical results agree with those in equation (36), but with different normalization factors, as expected, since we are dealing with a field confined in a cavity. Concretely,

$$|\beta_{IJ}^{\mp}|^2 \simeq \frac{2\nu^2 \Delta\omega_I \Delta\omega_J |\omega_I \omega_J| |\omega_I + \omega_J|}{\kappa^5 \pi (e^{2\pi|\omega_I + \omega_J|/\kappa} \mp 1)}, \quad (37)$$

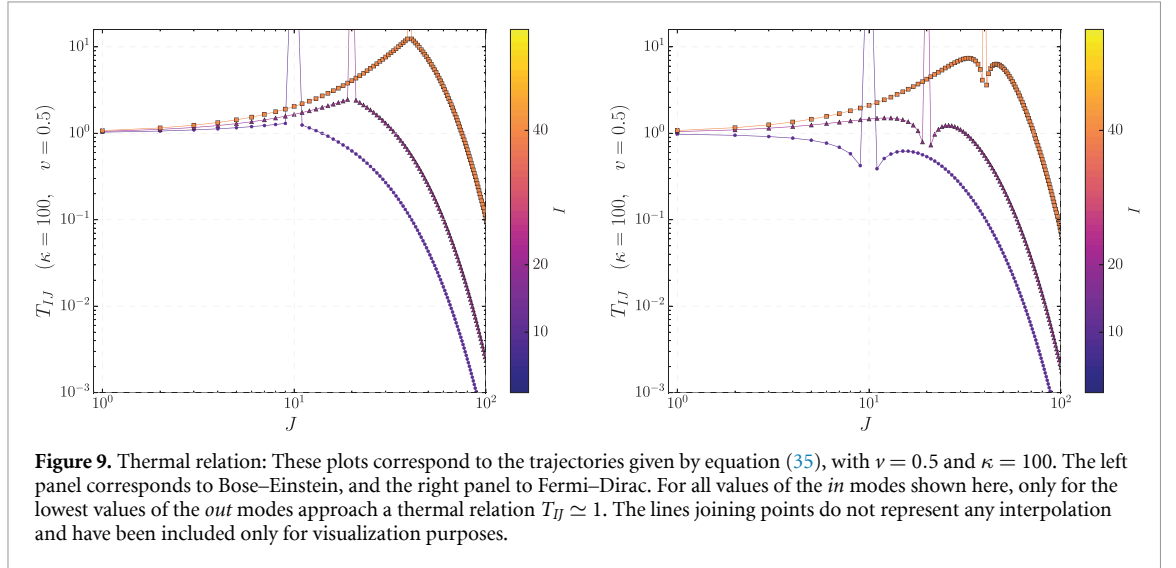
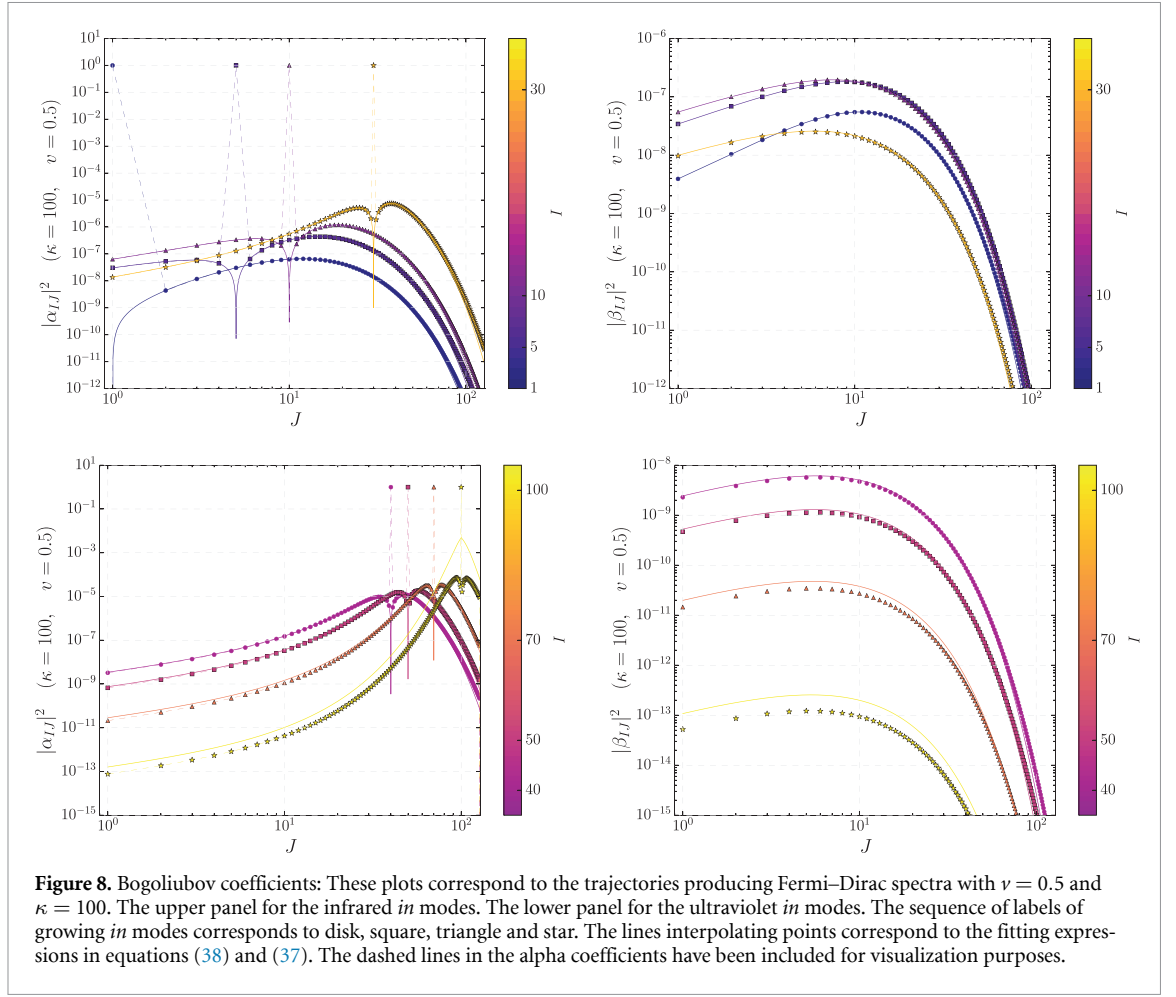


and

$$|\alpha_{IJ}^\mp|^2 \simeq \delta_{IJ} + \frac{2v^2 \Delta\omega_I \Delta\omega_J |\omega_I \omega_J| |\omega_I - \omega_J|}{\kappa^5 \pi (e^{2\pi |\omega_I - \omega_J|/\kappa} \mp 1)}. \quad (38)$$

In figures 7 and 8 we compare them with numerical results obtained from the library [60]. In both cases, we observe very good agreement for infrared *in* and *out* modes, and departures for ultraviolet modes, as expected. However, there are several comments in order. The first one is that the spectra are not thermal in the sense of Fulling and Davies, or the one obtained in the main body of this manuscript. As we can see in equations (36) and (37), the exponents depend on both the *in* and the *out* frequencies, besides the extra dependence in the numerators in the amplitude. Therefore, the spectrum is not thermal strictly speaking. Second, the normalization factors in the Bogoliubov coefficients are different because the normalization of the field modes inside a cavity is different with respect to those of the unconfined field with only one boundary. In addition, the alpha coefficients are nearly equal unit when the *in* and *out* modes coincide. This is because the alpha coefficients very well approximate a Kronecker delta. In the continuous spectrum case with only one boundary, they must nearly approximate a Dirac delta function. Note that the analytical expressions are valid only for small values of the maximum speed of the boundaries. For other values with $v \simeq 1$ there are no analytical results. Here, our numerical tools [60] could probe these extreme regimes. Finally, since the spectra in equation (37) very well reproduces our simulations, one can compute the thermality function in equation (27) and check that it is only satisfied for trajectories with $\omega_I \gg \kappa \gg \omega_J$, as we already discussed in section 2.

One can see in figure 9 (and also by direct inspection) that, in general, the detailed balance condition in equation (3) is only fulfilled in the limit $\omega' \gg \kappa \gg \omega$. Unlike in the Davies and Fulling setting, there is an extra condition $\kappa \gg \omega$ for the spectrum to be thermal. It means that, for sufficiently high *in* frequencies, only *out* infrared frequencies had enough time to thermalize. Therefore, there is some notion of thermality in these spectra, provide the above restrictions are fulfilled.



4.2. One (symmetrically) expanding and collapsing mirror

Let us now turn to the cavity boundary trajectories given by equation (26). Concretely, from some time $t \simeq t_0$ up to some time $t \simeq t_f$, the mirror accelerates, expanding the cavity, and then breaks to a (nearly) static position. Then, from $t \simeq t_f$ up to some time $t \simeq 2t_f$ follows the time-reversed trajectory (26). Hence, the mirror reaches its original position following a time-symmetric trajectory. In addition, the left boundary remains at $x^f = 0$ at all times. In summary, the right mirror expands from $x_{\text{in}}^g = 1$ to $x_{\text{int}}^g = (1 + \epsilon)$ and then collapses to $x_{\text{out}}^g = 1$. Here, we fix equations (23) and (25) with normalization

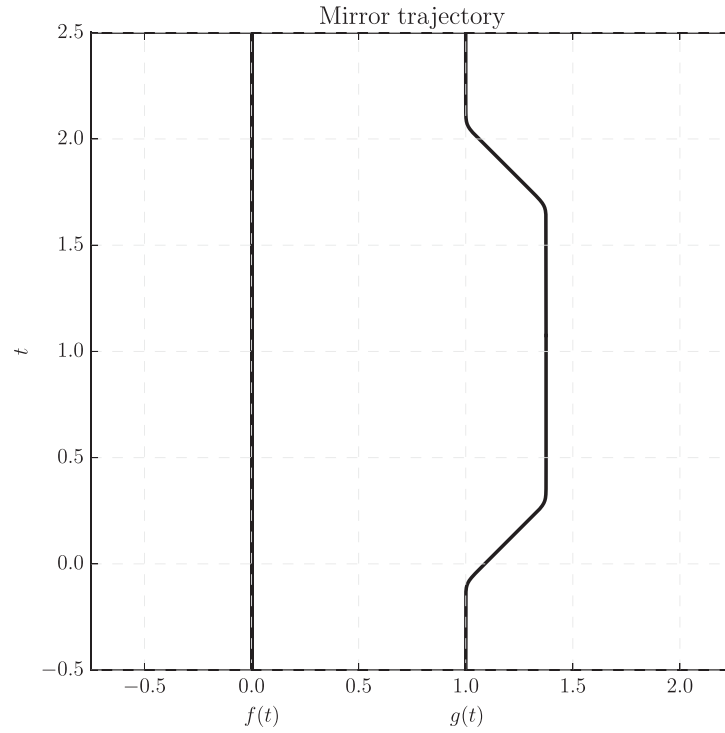


Figure 10. Trajectories: Typical trajectory given by equation (26), followed by the same trajectory reversed in time, in this case with $\epsilon = 0.375$ and $\kappa = 33.3$.

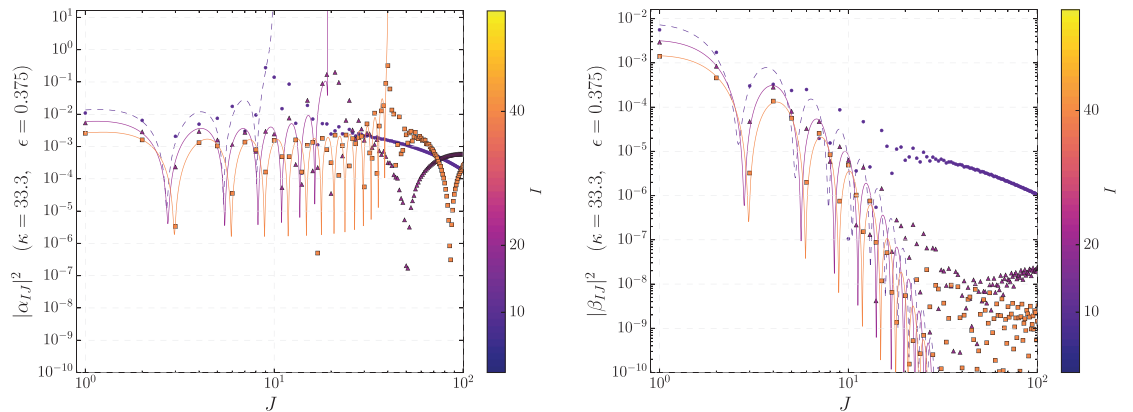
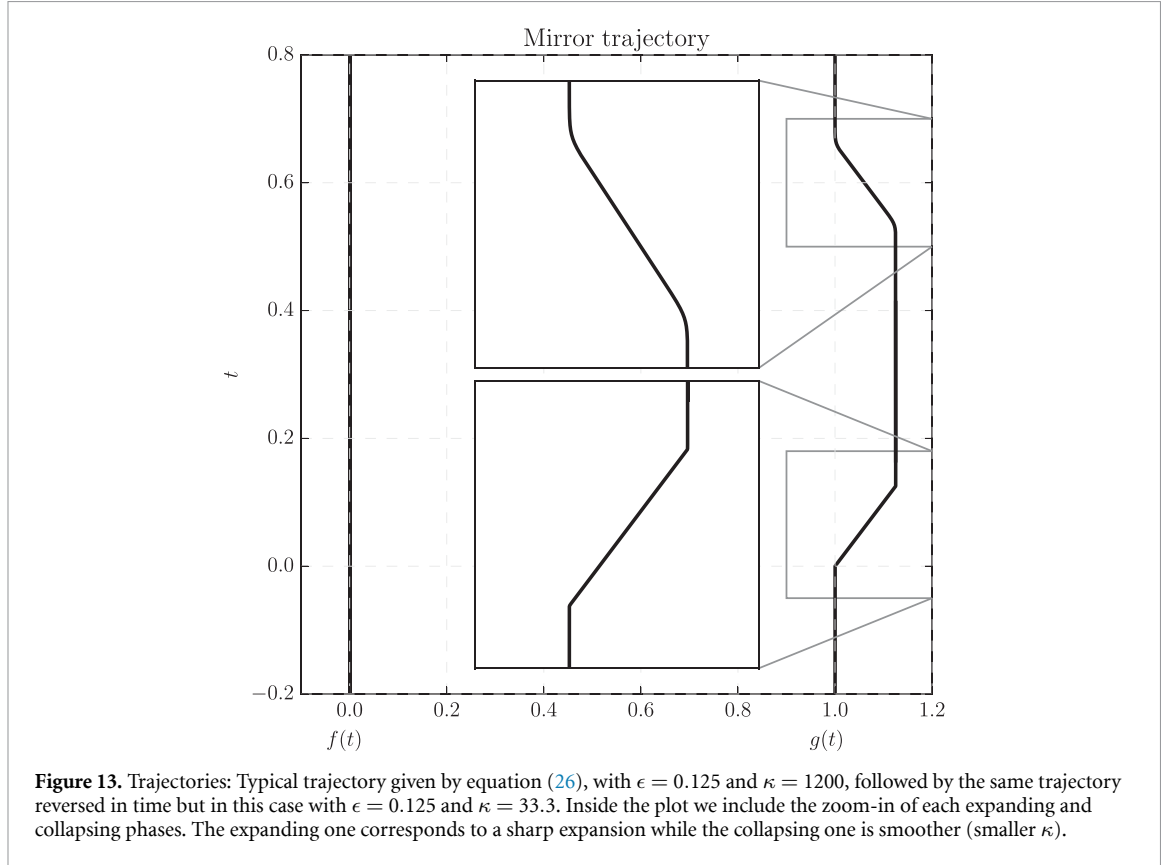
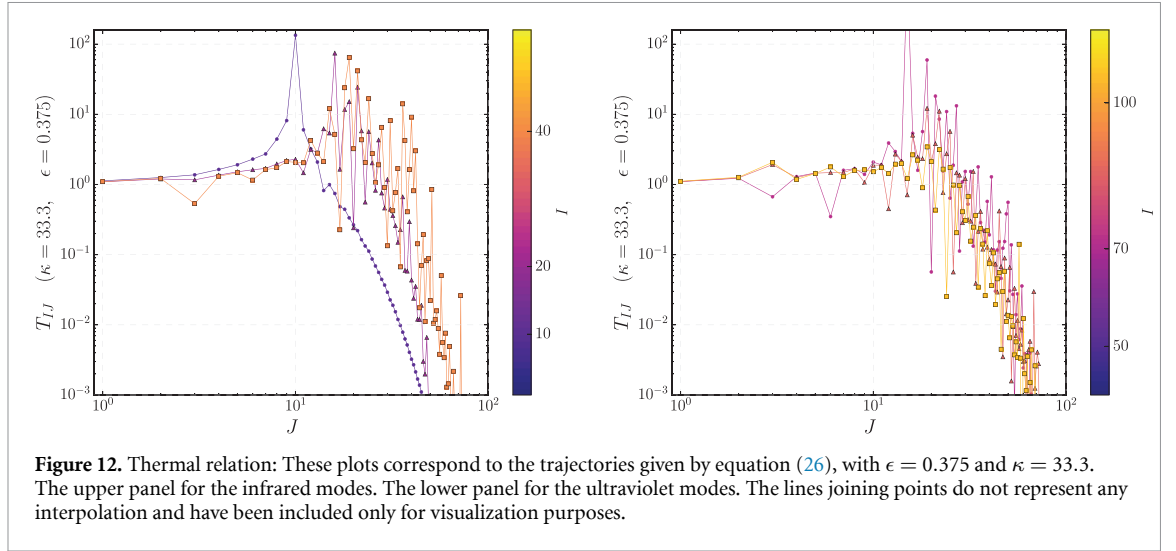


Figure 11. Bogoliubov coefficients: These plots correspond to a boundary undergoing a trajectory given by equation (26) and then its time reversed trajectory returning to its initial position, with $\epsilon = 0.375$ and $\kappa = 33.3$. The lines interpolating points correspond to the fitting expressions in equations (23) and (25) for the right and left panels, respectively.

$N_\beta = 2 = N_\alpha$ and we have the same *in* and *out* frequencies given by $\Delta\omega_I = \pi/L_0$. We depict this configuration of the cavity in figure 10.

For the sake of brevity, we will show here a concrete realization given by $\epsilon = 0.375$ and $\kappa = 33.3$. In figure 11 we show the behaviour of the Bogoliubov coefficients for the same frequency bands as in figure 2. We also compare with the fitting expressions of equations (23) and (25), only for infrared modes (UV modes show a different behaviour, and we did not find a simple fitting expression)⁸. We surprisingly see that there is qualitative agreement between the two cases. This demonstrates that the thermal spectrum shows robustness against particle production because of the return of the right mirror to its original position following the time-reversed trajectory. On the other hand, the UV sector is sensitive to this new configuration of the cavity, with a stronger particle production in the UV modes.

⁸ We fix $A_\alpha = 1 = A_\beta$, and $B_\alpha = 10^{-3} = B_\beta$. The other parameters D_1 , $C_\alpha = C_\beta$ and F are suitable functions of ω_J similar to the ones shown in figure 27. Hence, we do not show them here.



We also show in figure 12 the corresponding thermality function. As we see in both panels, the thermality function T_{IJ} is still order unit for the lowest *out* frequencies. On the other hand, for UV *out* frequencies, departures from unit are stronger, compared with figure 3.

4.3. One (asymmetrically) expanding and collapsing mirror

Now that we have seen that the infrared sector is robust against a time-symmetric collapse of the cavity returning to its original position, we will analyse what happens if the cavity returns to its position but following a different collapsing trajectory. Concretely, we are interested in a cavity that undergoes a sharp expansion and then an adiabatic collapse such that it returns to its original position, namely, the returning trajectory follows a small acceleration trajectory compared with the one during expansion. We show in figure 13 this configuration of the cavity.

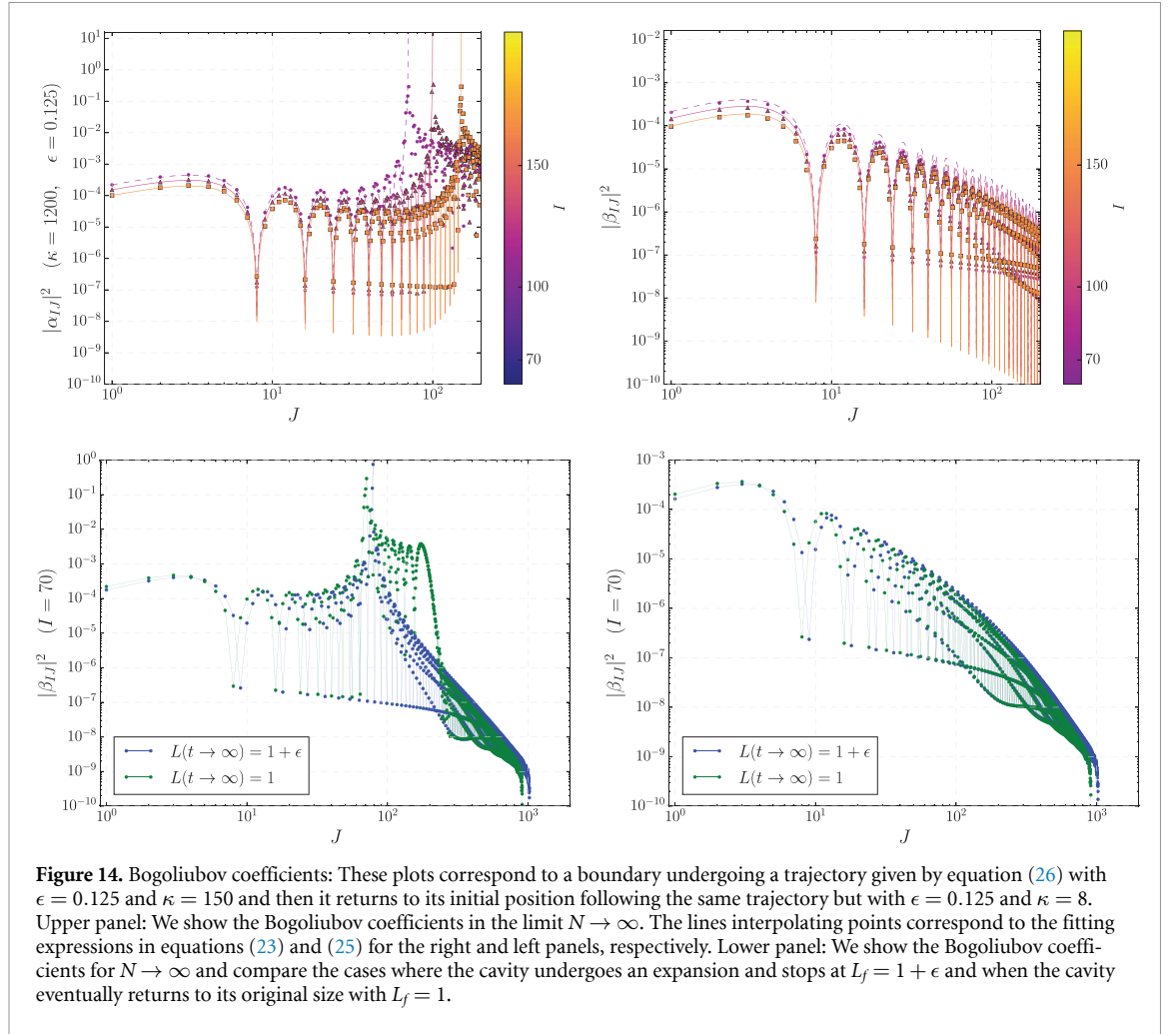


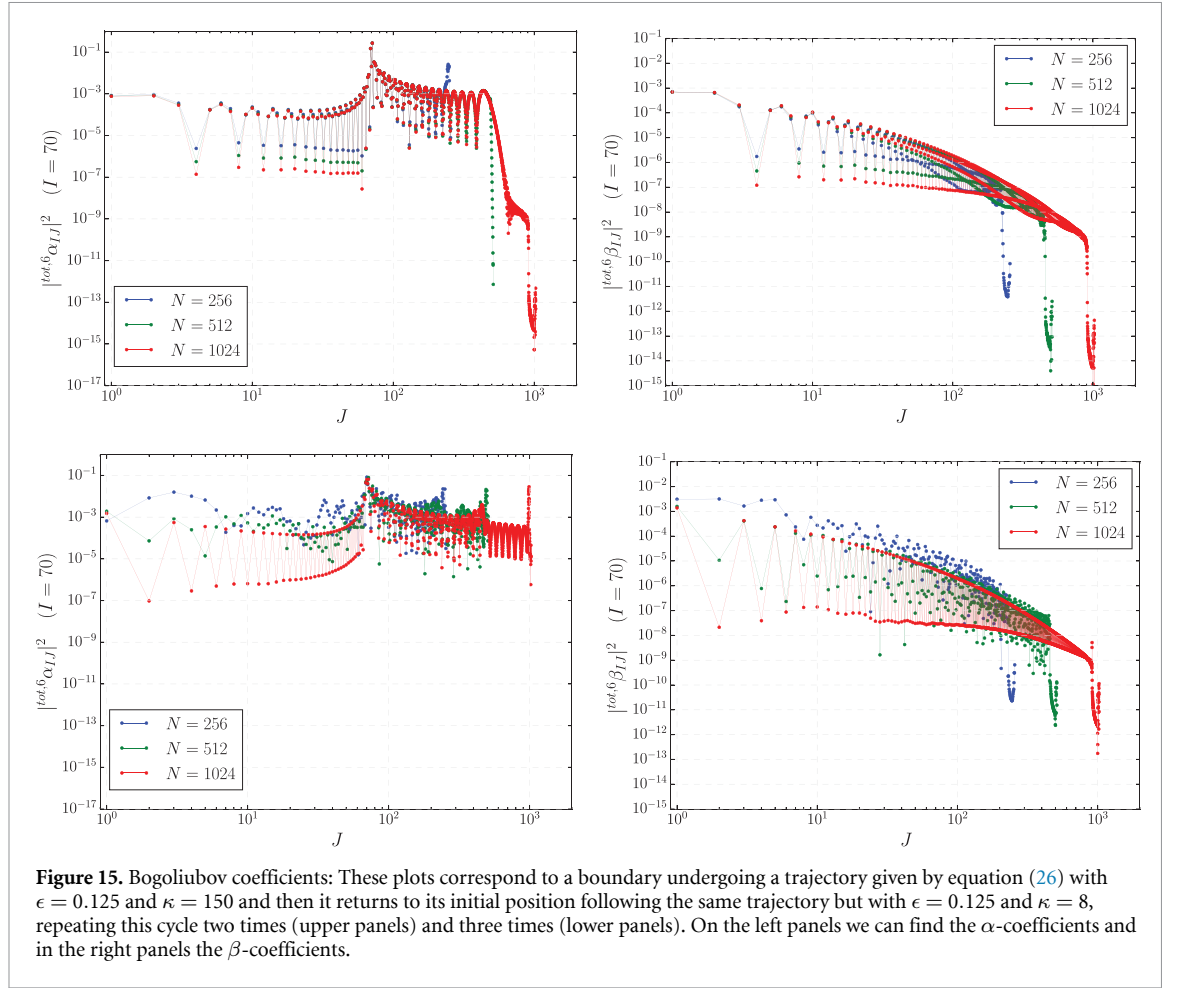
Figure 14. Bogoliubov coefficients: These plots correspond to a boundary undergoing a trajectory given by equation (26) with $\epsilon = 0.125$ and $\kappa = 150$ and then it returns to its initial position following the same trajectory but with $\epsilon = 0.125$ and $\kappa = 8$. Upper panel: We show the Bogoliubov coefficients in the limit $N \rightarrow \infty$. The lines interpolating points correspond to the fitting expressions in equations (23) and (25) for the right and left panels, respectively. Lower panel: We show the Bogoliubov coefficients for $N \rightarrow \infty$ and compare the cases where the cavity undergoes an expansion and stops at $L_f = 1 + \epsilon$ and when the cavity eventually returns to its original size with $L_f = 1$.

As we see in the upper panels of figure 14, the Bogoliubov coefficients in the limit $N \rightarrow \infty$ present a few (expected) qualitative differences with respect to the case of a cavity that expands and stops, without returning to its original position—see figure 4. Concretely, the parameters of the fitting expression remain the same. We must just take into account that the *out* frequencies are different. This is translated into a small change in the amplitude of the coefficients and the corresponding modification in the oscillations of the grey-body factor. In addition, the strongest modifications appear in the ultraviolet part of the spectrum. Unfortunately, we cannot take the limit $N \rightarrow \infty$ via Richardson extrapolation, due to the appearance of spurious effects in the UV regime. Therefore, in the lower panels of figure 14, we show the Bogoliubov coefficients for a simulation with $N = 1024$, for the two cases: a cavity that expands and does not return to its original position and a cavity that expands and returns adiabatically to its initial position. Here, we clearly see how the α -coefficients get strong modifications in an intermediate band frequency, whereas we do not see considerable particle production since the β -coefficients remain nearly the same. Hence, the following question arises naturally: *How robust is the thermal spectrum against several cycles of expansion and collapse of the cavity?* We will address this question in the next subsection.

A final comment on the thermal properties of the final state is in order. Since the Bogoliubov coefficients remain the same in the infrared sector, except for the mentioned changes in the *out* frequencies and shift of the oscillations, the thermality function T_{IJ} remains qualitatively the same as in figure 19. Therefore, we can safely conclude that, for those configurations in which the cavity returns adiabatically to its original size, thermal properties will be preserved.

Finally, we compose several cycles of expansion and collapse to analyse whether thermality is preserved in this scenario. For the sake of simplicity, we compute the evolution by composing the Bogoliubov transformation in equation (34) a number of times.

In figure 15 we show the Bogoliubov coefficients for the mirror undergoing two cycles of expansion and contraction in the upper panels and three cycles in the lower panels. As we can see, the thermal structure is qualitatively present if the number of cycles is less than three. However, we can see that we



lost convergence in the infrared sector for different values of N already at the third cycle. As the number of cycles increases, the amplitudes of both types of Bogoliubov coefficients grow very rapidly, until numerical precision is eventually lost. This growth appears to be physical rather than numerical in origin: a periodically driven boundary meets the parametric-resonance condition of the dynamical Casimir effect, so that the mode amplitudes grow exponentially once the driving crosses a parametric resonance of the cavity (a Floquet-type instability). The accompanying loss of numerical convergence is a consequence of this growth rather than its cause. A dedicated stability analysis, e.g. of the Floquet exponents or the Mathieu-type instability bands, would render this quantitative and is left for future work.

5. Conclusions

In this work, we have presented a rigorous and exhaustive numerical analysis of particle production in cavities with moving reflective boundaries, utilizing the dynamical Casimir effect as an analogue for Hawking radiation. These robustness statements hold within a closed-system, unitary description; the role of environmental decoherence is discussed at the end of this section. Our analysis encompasses multiple physically relevant configurations, including expanding, collapsing, concatenations of expanding/collapsing boundaries, and rigidly accelerating cavities.

Our results demonstrate that the emergence of a genuine thermal spectrum is highly sensitive to the frequency range and to the parameters governing the acceleration, with infrared modes reaching thermal equilibrium more easily. For all considered scenarios, the most energetic ultraviolet modes present substantial deviations from the ideal thermal distribution.

Among the explored trajectories, scenarios featuring a single moving mirror or two symmetrically accelerating mirrors prove to be the most prone to describing a final thermal state of the field. Furthermore, we have shown that if a cavity expands and subsequently returns to its original size adiabatically, the thermal aspects of the spectrum are not considerably modified. Beyond standard thermality, we successfully verified that specific mirror trajectories can generate particle spectra following

Bose–Einstein or even Fermi–Dirac statistics within our confined cavity model, despite the underlying degrees of freedom being purely bosonic. However, we found that these non-standard thermal signatures require the stringent condition $\omega' \gg \kappa \gg \omega$ to satisfy the detailed balance condition.

Conversely, we found that rigidly accelerating cavities fail to produce a robust thermal state. The underlying physical reason is that, while the right boundary induces a red-shift compatible with thermality, the particle production on the left boundary induces a blue-shift, which heavily contaminates the spectrum with ultraviolet modes and destroys the overall thermal balance. However, the most infrared mode (fundamental mode of the cavity) seems to thermalize, what opens the possibility of using this setting as an Unruh–de Witt detector. Additionally, when the cavity follows a concatenation of expansion and collapse cycles, the qualitative thermal structure is preserved for up to three cycles. Nevertheless, increasing the number of cycles leads to an extremely rapid growth in the amplitude of the Bogoliubov coefficients. This signals the onset of a genuine physical instability—a parametric resonance induced by the periodicity of the boundary motion—rather than a numerical artefact: the exponential growth is the physical response of the driven cavity, and the attendant loss of numerical convergence is a symptom of it, arising once the finite mode cutoff can no longer follow the ultraviolet cascade. This instability ultimately spoils the thermal character of the radiation and calls for a dedicated Floquet analysis.

To quantitatively characterize the deviations from exact thermality and provide a predictive framework, we introduced a set of fitting expressions for the Bogoliubov coefficients that incorporate greybody factors. These factors exhibit oscillatory behaviour as functions of frequency due to the finite duration of the acceleration. This parametrization provides a robust methodology for future analogue gravity experiments to help distinguishing genuine physical thermal signals from spurious experimental artefacts. By identifying the distinct roles played by acceleration parameters, mode selection, and boundary configurations, this study might contribute to the design of more precise future analogue gravity experiments and deepens our theoretical understanding of how quantum field theoretic effects manifest in non-stationary backgrounds.

The analysis has been carried out for a closed system undergoing unitary evolution. In a realistic implementation the dominant limitation is the coupling to the environment: dissipation and dephasing degrade the coherence of the produced state and tend to wash out the thermal signature, which can then be resolved only if the radiation is generated within the coherence time τ_c of the device. The robustness of quantum entanglement between Hawking modes and their partners in this particular setting has already been discussed in [53] by some of the authors. We found that there is a critical temperature (per mode) where quantum entanglement vanishes. We also found that this critical temperature has no strong dependence on the acceleration κ . To what extent the thermal features identified here survive a more realistic environment, and how the open-system dynamics can be mitigated, requires a treatment beyond the present scope; decoherence and entanglement degradation in analogue systems have been studied in [63, 64] and, for the ion-ring analogue, in [65, 66]. We regard this open-system analysis as the natural continuation of the present study.

The radiation can be detected by amplifying the output field close to the quantum limit with a Josephson or traveling-wave parametric amplifier (JPA/TWPA) and reading it out through heterodyne measurement of the power spectrum. Because the dynamical-Casimir process produces photons in correlated pairs, whereas thermal noise is uncorrelated, the signal is most cleanly isolated through cross-correlations between the output ports [35, 44], which suppress the incoherent thermal background and expose the pair structure of the effect. The greybody oscillations quantified by the fitting expressions appear in the frequency-resolved spectrum; resolving them requires adequate spectral resolution and averaging, again assisted by the correlation measurement. Reading out the field requires opening the cavity to external lines, which is itself a loss channel, so that measurement access and coherence must be traded off in any such experiment.

Acknowledgments

The authors would like to thank Gerardo García-Moreno for helpful discussions and his insightful comments. Financial support is provided by the Spanish Government through the projects PID2020-119632GB-I00, and PID2019-105943GB-I00 (with FEDER contribution).

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Appendix A. Numerical results for particle production in other trajectories

A.1. Two symmetrically expanding mirrors

We consider now the trajectories of the boundaries of the cavity given by:

$$\begin{aligned} f(t) &= -\frac{s}{2\kappa} - \frac{1}{2\kappa} [\log(\cosh(\kappa(t-t_0))) - \log(\cosh(s-\kappa(t-t_0)))], \\ g(t) &= 1 + \frac{s}{2\kappa} + \frac{1}{2\kappa} [\log(\cosh(\kappa(t-t_0))) - \log(\cosh(s-\kappa(t-t_0)))]. \end{aligned} \quad (39)$$

In this configuration, at $t \ll t_0$, both mirrors are nearly static at the initial positions $x_{\text{in}}^f = 0$ and $x_{\text{in}}^g = 1$, respectively, while at $t \gg T \simeq t_0 + \epsilon$ (with $\epsilon = s/\kappa$) their final positions are $x_{\text{out}}^f = (-\epsilon)$ and $x_{\text{out}}^g = (1 + \epsilon)$, respectively. In the interval $[t_0, t_0 + \epsilon]$ both mirrors accelerate reaching a maximum speed (very close to the speed of light), and then they decelerate until they reach their respective final (stationary) positions. Moreover, regarding the spectrum in equations (23) and (25), we will have *out* frequencies given by $\omega_I = \pi J/(L_0 + 2\epsilon)$ and their gap by $\Delta\omega_I = \pi/(L_0 + 2\epsilon)$. The normalization of the coefficients in the fitting expressions is given by $N_\alpha = 4 = N_\beta$. This value has a simple explanation. The trajectories are symmetric with respect to the center of the cavity. This results in a decoupling between the odd and even modes of the field. Therefore, particle production will double on each mode since the number of modes that can be occupied with particles now reduces to half the number one has when all modes are coupled, e.g. compared with the previous case of one moving mirror. In figure 16 we show the concrete configuration of this expanding cavity.

Small accelerations: As in the previous configuration, we have small accelerations, and we found qualitatively similar results in all cases. Let us consider the same concrete realization given by $\epsilon = 0.375$ and $\kappa = 33.3$. In figure 17 we show the numerically computed Bogoliubov coefficients in the band frequency $(I, J) \in (20, 100)$ and the fitting expressions of equations (23) and (25). In all cases, we fix $A_\alpha = 1 = A_\beta$, and $B_\alpha = 10^{-3} = B_\beta$. The upper panel includes several (relatively) infrared *in* modes ω_I with respect to the *out* frequencies ω_J . For low ω_J we see very good agreement between the computed Bogoliubov coefficients and the fitting expressions in equations (23) and (25), with C_α , C_β , $\tilde{\kappa}$, D_1 , D_2 and F suitable functions of ω_I . For the *in* modes considered here, we see that $|C_\alpha| \ll 1$ and $|C_\beta| \ll 1$. We show the behaviour of C_α , C_β , $\tilde{\kappa}$, D_1 , D_2 and F in figure 29. At very high ω_J we have checked numerically that the Bogoliubov coefficients decay as ω_J^{-n} with $n > 3$ (although we do not show here the results for the α coefficients). On the other hand, the lower panel shows several (relatively) ultraviolet *in* modes ω_I as functions of the *out* frequencies ω_J . We see that for low ω_J the computed β coefficients still agree well with equation (23). Nevertheless, the β coefficients reach quicker the behaviour ω_J^{-n} , departing from equation (23) at lower frequencies. This indicates that these *in* modes did not have enough time to thermalize. All of this qualitatively agrees with the results obtained for one moving mirror, namely, the trajectory (26).

As before, we can probe the thermal properties of these modes. We have again computed the thermality function given in equation (27). In figure 18 we show T_{IJ} for several values of the *in* modes I . As we can see, for this trajectory of the mirrors, in the limit of infrared *in* and *out* modes, the thermality condition is near unit and, hence, it is approximately fulfilled. However, for UV *in* modes (large I), this condition is met only for very infrared *out* modes.

Large accelerations: For the case of large accelerations, we have analysed several configurations. The mirrors transition very quickly from a steady configuration to a new one sharply approaching the speed of

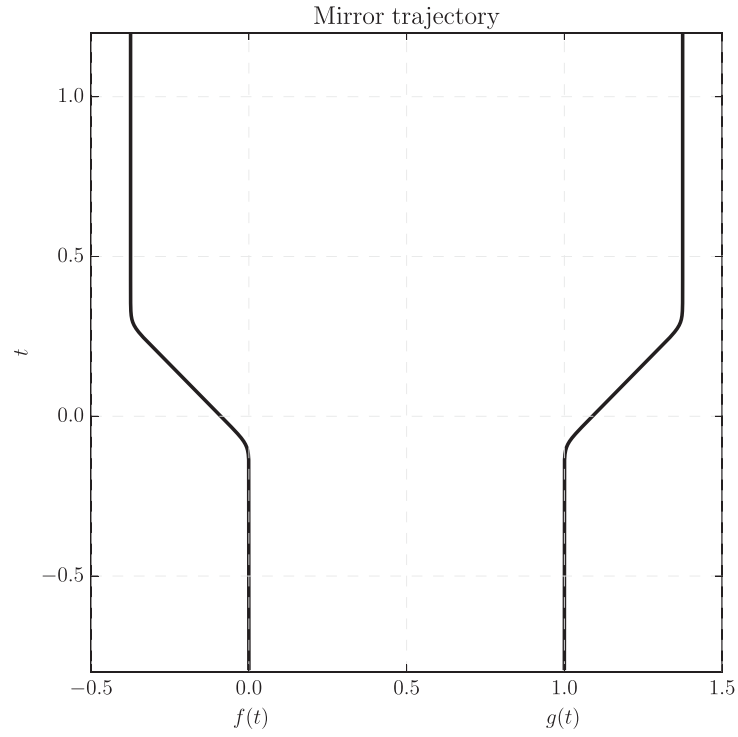


Figure 16. Trajectories: Typical trajectory given by equation (26), with $\epsilon = 0.375$ and $\kappa = 33.3$. In this case, both boundaries undergo the same trajectory resulting in a symmetric expansion of the cavity.

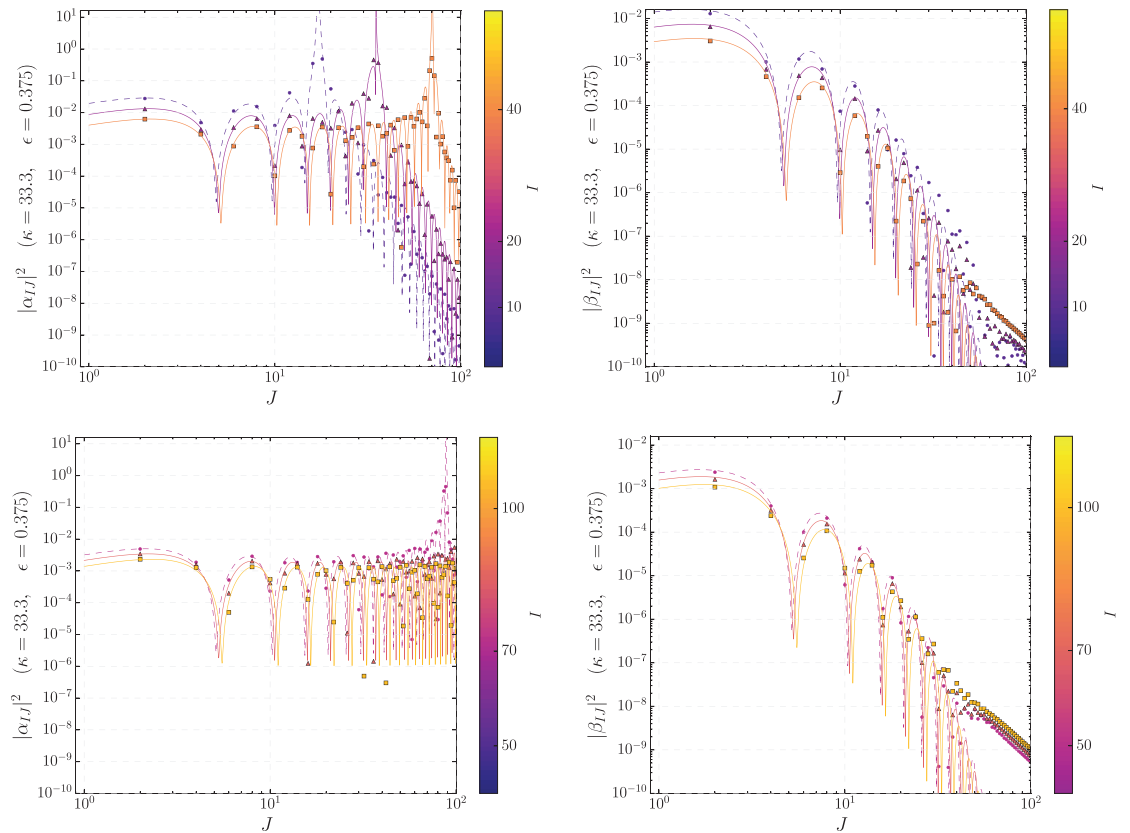


Figure 17. Bogoliubov coefficients: These plots correspond to the trajectories given by equation (39), with $\epsilon = 0.375$ and $\kappa = 33.3$. The upper panel for the infrared modes. The lower panel for the ultraviolet modes. The lines interpolating points correspond to the fitting expressions in equations (23) and (25).

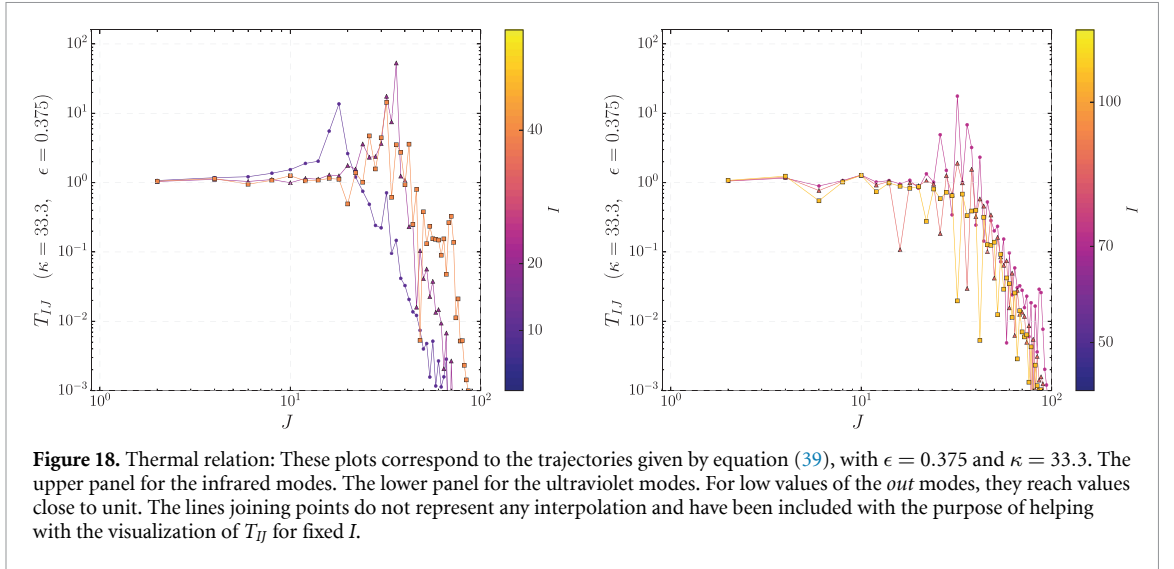


Figure 18. Thermal relation: These plots correspond to the trajectories given by equation (39), with $\epsilon = 0.375$ and $\kappa = 33.3$. The upper panel for the infrared modes. The lower panel for the ultraviolet modes. For low values of the *out* modes, they reach values close to unit. The lines joining points do not represent any interpolation and have been included with the purpose of helping with the visualization of T_{IJ} for fixed I .

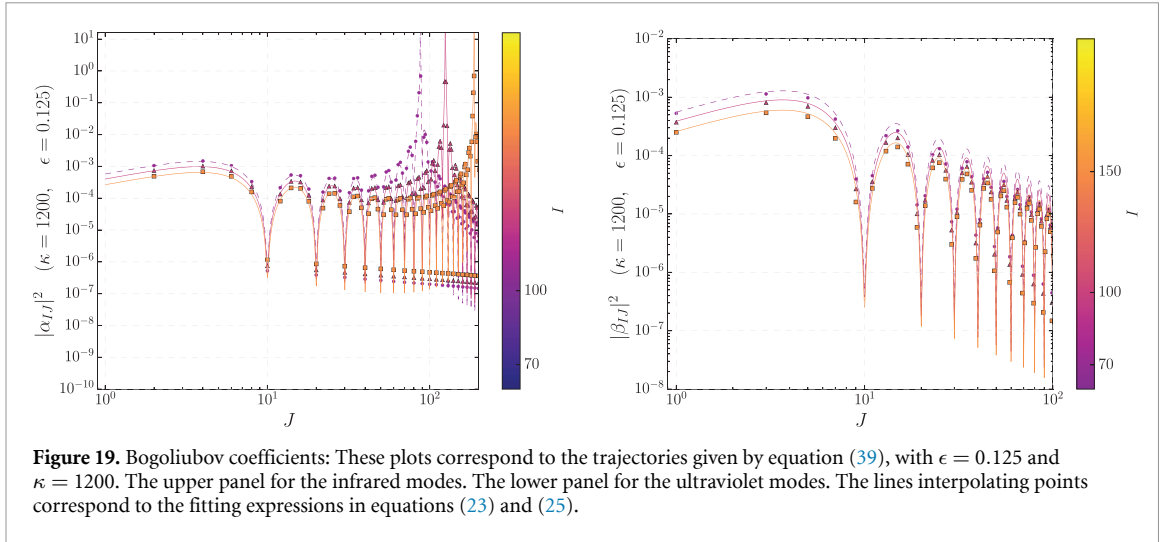


Figure 19. Bogoliubov coefficients: These plots correspond to the trajectories given by equation (39), with $\epsilon = 0.125$ and $\kappa = 1200$. The upper panel for the infrared modes. The lower panel for the ultraviolet modes. The lines interpolating points correspond to the fitting expressions in equations (23) and (25).

light for some period of time to suddenly break and stop in new stationary positions. The concrete configuration we will show here is given by $\epsilon = 0.125$ and $\kappa = 1200$. Other configurations with large accelerations show similar results. In figure 19 we show the behaviour of the modes in the frequency band $((I, J) \in (70, 150))$ and compare them with the fitting expression of equations (23) and (25). In our simulations, we set $A_\alpha = 1.0 = A_\beta$ and $B_\alpha = 10^{-3} = B_\beta$. Besides, $C_\alpha = 1.0 = C_\beta$. We show the Bogoliubov coefficients for several *in* modes ω_I as functions of the *out* frequencies ω_J . We see that for relatively low ω_I , the computed beta coefficients agree very well with equations (25) and (23). The parameters D_1 , D_2 , F and $\tilde{\kappa}$ are plotted in figure 30 as functions of ω_I .

For lower or higher *in* modes compared to those shown here, the fitting expression for $|\alpha_{IJ}|^2$ still shows good agreement with the numerical results, but for $|\beta_{IJ}|^2$ we see important departures from the fitting expression. In conclusion, those modes depart strongly from a thermal distribution, and hence we do not consider them here. On the other hand, in figure 20 we show the thermal relation T_{IJ} in equation (27) for several values of the *in* modes I . As we can see, the most infrared *out* modes yield $T_{IJ} \simeq 1$, following a thermal distribution. However, we also see departures from thermality in those modes where either $|\alpha_{IJ}|$ or $|\beta_{IJ}|$ reach their minima, departing from a thermal distribution. The lower or higher *in* modes show a stronger departure of T_{IJ} from the unit.

A.2. An accelerating rigid cavity

The trajectories of the boundaries of a rigid cavity (in the reference frame of the laboratory) are given by:

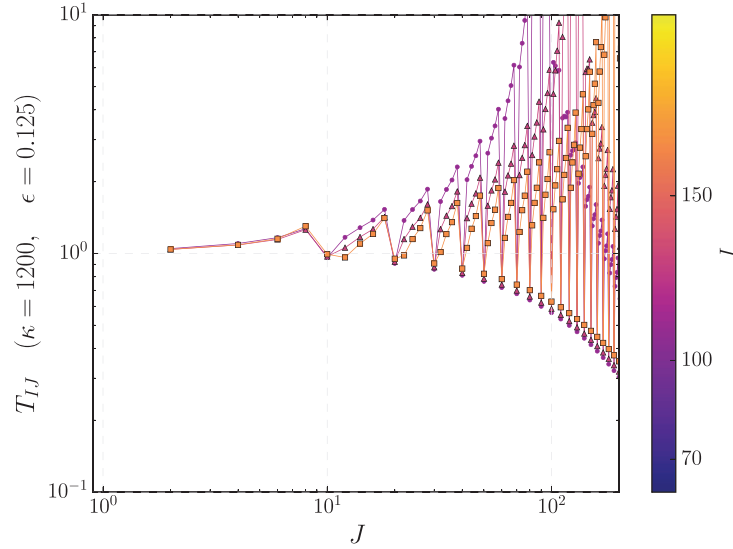


Figure 20. Thermal relation: These plots correspond to the trajectories given by equation (39), with $\epsilon = 0.125$ and $\kappa = 1200$. For all values of the *in* modes shown here, only for low values of the *out* modes approach a thermal relation $T_{IJ} \simeq 1$. The lines joining points do not represent any interpolation and have been included only for visualization purposes.

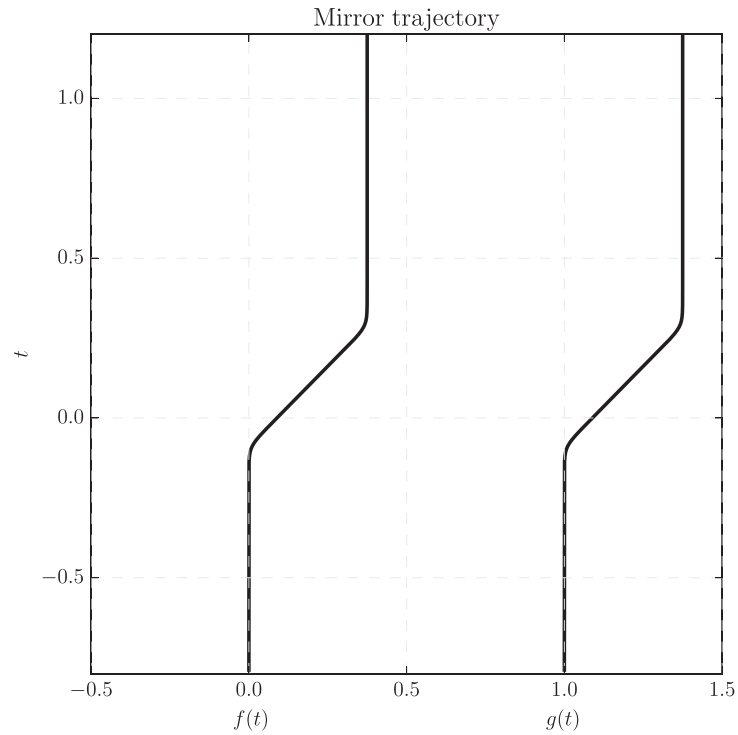
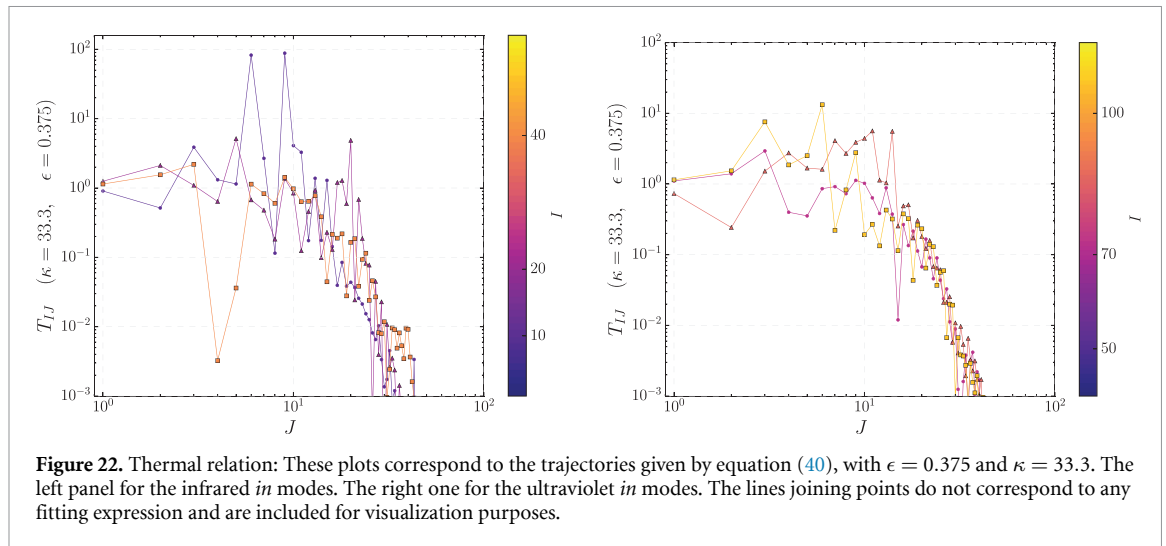


Figure 21. Trajectories: Typical trajectory given by equation (26), with $\epsilon = 0.375$ and $\kappa = 33.3$. In this case, both boundaries undergo the same trajectory resulting in a rigid displacement of the cavity with respect to the laboratory reference frame.

$$\begin{aligned} f(t) &= \frac{s}{2\kappa} + \frac{1}{2\kappa} [\log(\cosh(\kappa(t-t_0))) - \log(\cosh(s-\kappa(t-t_0)))], \\ g(t) &= 1 + \frac{s}{2\kappa} + \frac{1}{2\kappa} [\log(\cosh(\kappa(t-t_0))) - \log(\cosh(s-\kappa(t-t_0)))], \end{aligned} \quad (40)$$

As in previous configurations, at $t \ll t_0$, both mirrors are nearly static at initial positions $x_{\text{in}}^f = 0$ and $x_{\text{in}}^g = 1$. Then, at $t \gg T \simeq t_0 + \epsilon$ (with $\epsilon = s/\kappa$) their final position will be $x_{\text{out}}^f = (1 + \epsilon) = x_{\text{out}}^g$. In the interval $[t_0, t_0 + \epsilon]$ the mirrors accelerate, reaching a speed close to the speed of light. Besides, regarding equations (23) and (25), and the *out* frequencies are given by $\omega_J = \pi J/L_0$ and their gap by $\Delta\omega_J = \pi/L_0$. We show the concrete configuration of this rigidly moving cavity in figure 21.



In this configuration, we were not able to find accurate fitting expressions for $|\alpha_{IJ}|$ or $|\beta_{IJ}|$. Hence, we observed no thermal behaviour of the Bogoliubov coefficients for neither small or large accelerations. Furthermore, the thermality function defined in (27) considerably departs from the unit. Actually, for some *out* modes it can be of the order of 10^1 to 10^2 . For higher modes, we see a similar behaviour as can be seen in figure 22. Hence, we conclude that the particle production for these trajectories is not associated with a thermal state of the field. The reason behind this non-thermality must be the particle production on the left boundary, that contaminates the spectrum with ultraviolet modes, those that are reflected by a surface that is producing a blue-shift, rather than a red-shift, like the right boundary.

Despite the strong deviation from the detailed balance condition, we can see that the fundamental mode of the cavity does not depart much from thermality. In all case, the thermality function of this mode, regardless of the *in* mode under consideration, is very close to the unit. We therefore conclude that this setting can be used to simulate an Unruh–de Witt detector. However, one should be careful because, to simulate such a device precisely, the cavity must be rigid with respect to its comoving reference frame, rather than the reference frame of the laboratory. Unfortunately, such a configuration is more involved and will be studied elsewhere.

A.3. Collapsing cavities

In the main text we have discussed particle production for several configurations that involve expanding cavities, following trajectories that approach the Fulling–Davies accelerating mirror. Let us now comment on the effects of the corresponding time-reversed configurations. Namely, the case of collapsing cavities.

From a computational perspective, it is not necessary to carry out additional simulations. Concretely, if we have that the evolved *in* states are written on the *out* basis ($t \rightarrow +\infty$), as

$${}^{(\text{in})}\mathbf{u}^{(I)}(t) = \sum_{J=1}^{\infty} \alpha_{IJ} {}^{(\text{out})}\mathbf{u}^{(J)}(t) + \beta_{IJ} {}^{(\text{out})}\bar{\mathbf{u}}^{(J)}(t), \quad (41)$$

one can easily invert this relation and express the *out* basis evolved backwards in time ($t \rightarrow -\infty$) as

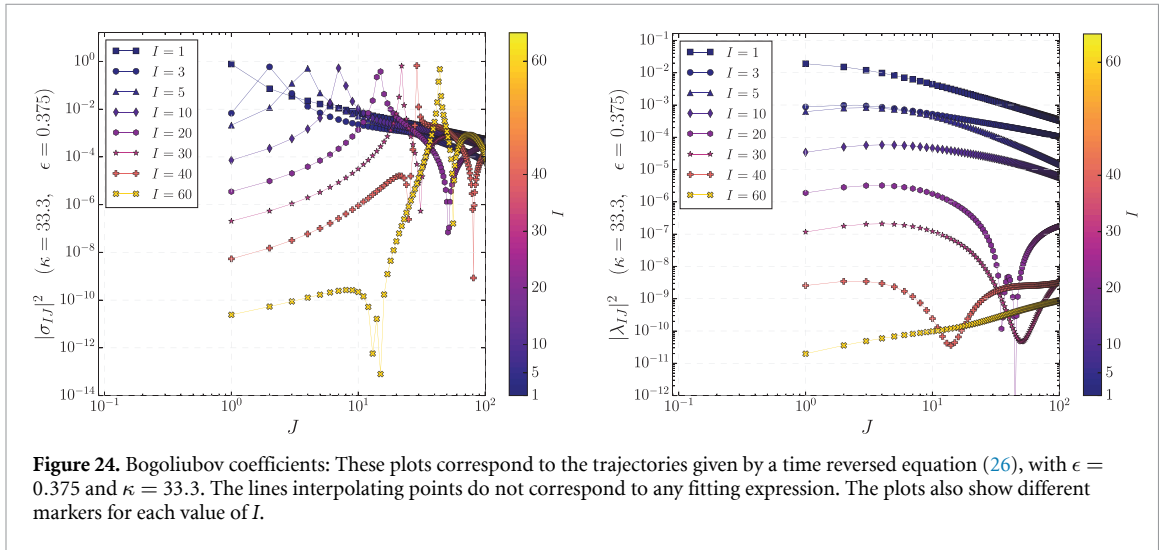
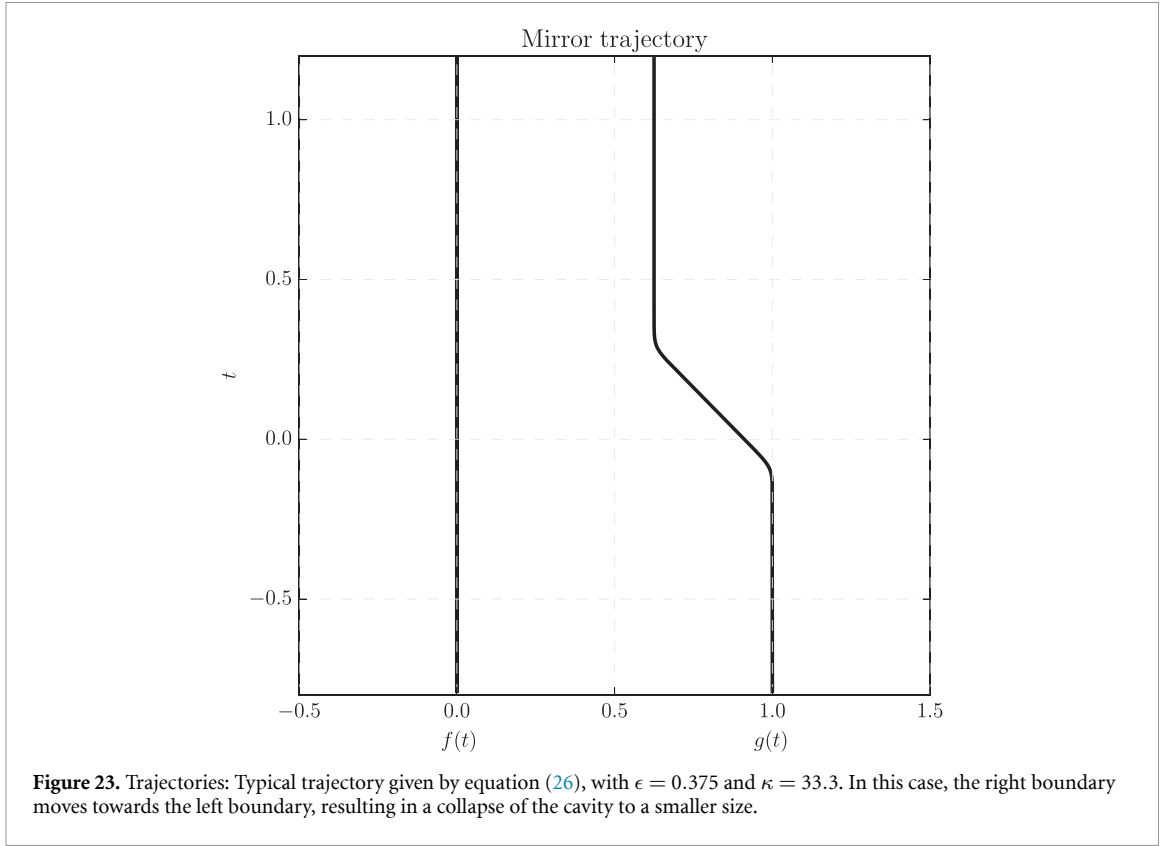
$${}^{(\text{out})}\mathbf{u}^{(I)}(t) = \sum_{J=1}^{\infty} \sigma_{IJ} {}^{(\text{in})}\mathbf{u}^{(J)}(t) + \lambda_{IJ} {}^{(\text{in})}\bar{\mathbf{u}}^{(J)}(t), \quad (42)$$

with $\sigma_{IJ} = \bar{\alpha}_{IJ}$ and $\lambda_{IJ} = -\beta_{IJ}$. Hence, particle production and mode-mixing can actually be studied through the very same numerical Bogoliubov coefficients of the trajectories in the main text. In the following, we comment the numerical results for these kind of trajectories.

A.3.1. One collapsing mirror

For the time-reversed trajectory considered in equation (26), the left cavity remains stationary at all times at $x_{\text{in}}^f = 1$, while the right starts at $x_{\text{in}}^g = (1 + \epsilon)$ and accelerates collapsing the cavity until it reaches $x_{\text{out}}^g = 1$ ⁹. We depict in figure 23 the concrete configuration of this asymmetric collapsing cavity.

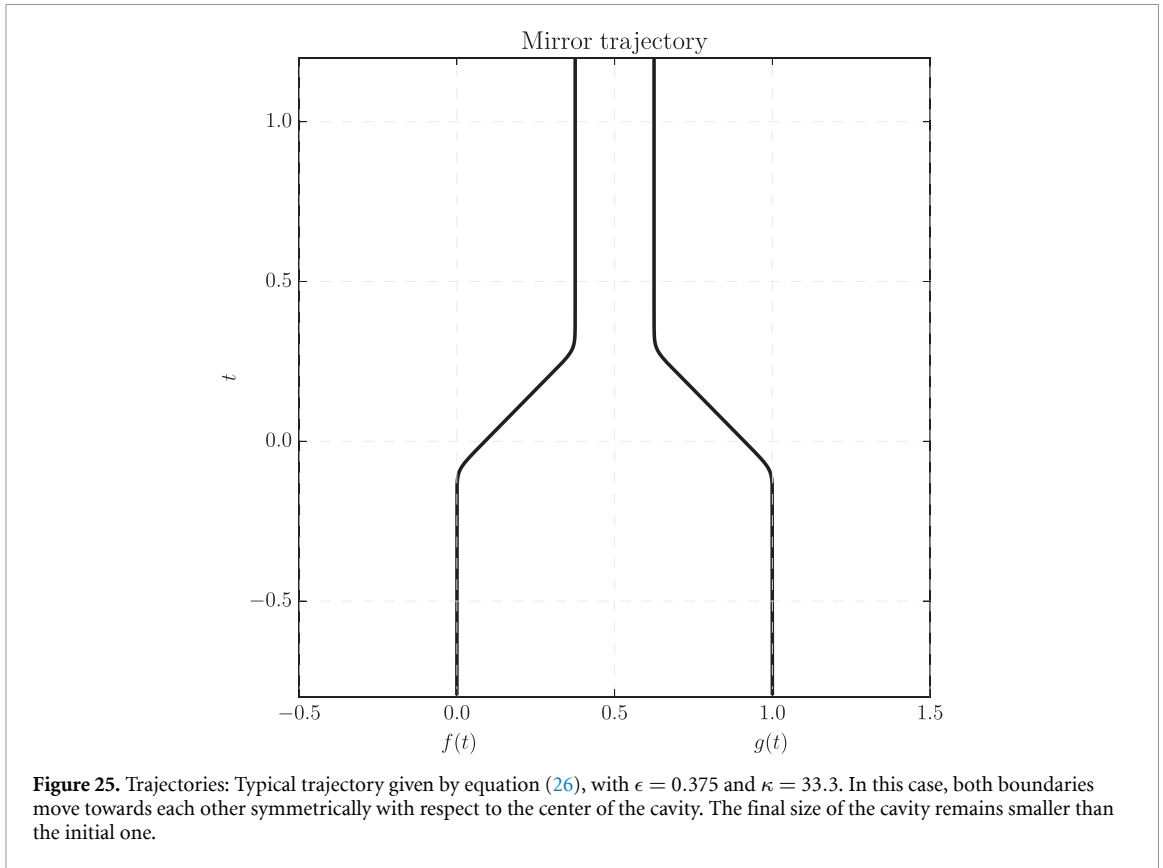
⁹ It is important to note the difference with previous trajectories, where the initial length of the cavity is $L = 1$, while here is $L = 1 + \epsilon$.



As we have analysed several simulations for relatively small accelerations, we summarize our findings with a concrete example given by $\epsilon = 0.375$ and $\kappa = 33.3$. We have not found analytical fitting expressions for them, since their behaviour with respect to *in* modes for different given values of *out* modes is nontrivial.

For instance, let us consider the Bogoliubov coefficients for several values of the *in* modes I for $|\sigma_{IJ}|^2$ and $|\lambda_{IJ}|^2$ (i.e. *in* modes J for $|\alpha_{IJ}|^2$ and $|\beta_{IJ}|^2$). As we see in figure 24, $|\sigma_{IJ}|^2$ grows as a function of J until it reaches a peak at $J \simeq I$, and then it oscillates, resembling a Bessel function. This behaviour of $|\sigma_{IJ}|^2$ involves a strong mode mixing. On the other hand, $|\lambda_{IJ}|^2$ shows a nontrivial behaviour as a function of J (*out* modes) for fixed values of I . We also see that the higher the J , the smaller is the amplitude of $|\lambda_{IJ}|^2$, indicating that there is less particle creation in the ultraviolet sector.

We have also seen that the Richardson extrapolation for $|\sigma_{IJ}|^2$ shows relatively good results, in the sense that there is convergence at low values of I and J , but the convergence is lost for high I and very small and high values of J (for intermediate values of J the convergence is still preserved). On the other



hand, the convergence of $|\lambda_{IJ}|^2$ is less robust, specially for high values of J . It does not seem to be a problem of our numerical methods, but a consequence of the cutoff we introduce on each simulation.

The above discussed picture does not change qualitatively when relatively large accelerations are considered.

A.3.2. Two symmetrically collapsing mirrors

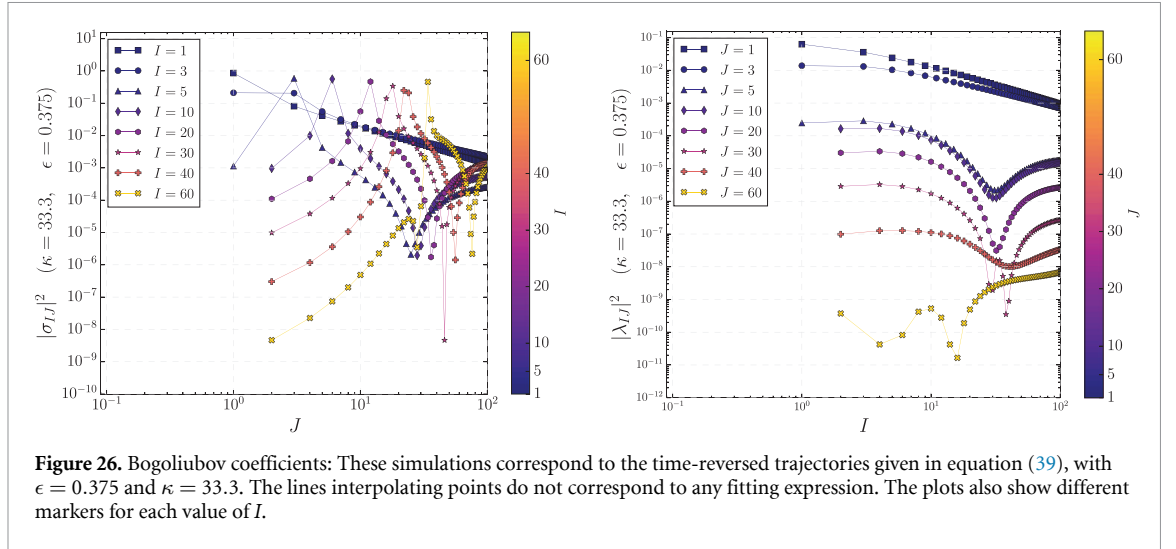
Another (collapsing) trajectory we have analysed corresponds to the time-reversed configuration of in equation (39). Here, the left and right cavities start at $x_{\text{in}}^f = -(\epsilon)$ and $x_{\text{in}}^g = (1 + \epsilon)$, respectively. Then, they accelerate towards each other, until they stop at positions $x_{\text{out}}^f = 1 = x_{\text{out}}^g$. In figure 25 we show this configuration of a symmetrically collapsing cavity.

Small accelerations: In this case of smooth trajectories, all the simulations performed, for different values of the acceleration, show similar results. We summarize them with a concrete representative example given by $\epsilon = 0.375$ and $\kappa = 33.3$. As in the previous case, we report only the numerical results. No simple fitting expressions have been found.

In figure 26 we show $|\sigma_{IJ}|^2$ and $|\lambda_{IJ}|^2$ for several values of the *in* modes I . Concretely, $|\sigma_{IJ}|^2$ again starts growing as a function of J until it reaches a peak at $J \simeq I$, and then it oscillates, with a behaviour similar to a Bessel function. This behaviour of $|\sigma_{IJ}|^2$ involves a strong mode mixing. On the other hand, $|\lambda_{IJ}|^2$ has a nontrivial behaviour as a function of J (*out* modes) for fixed values of I . In addition, its amplitude decreases with J , which indicates less particle creation in the UV sector. Moreover, for this trajectory, even and odd modes are decoupled.

As in the previous configuration (one moving mirror and small accelerations), we have seen that the Richardson extrapolation for $|\sigma_{IJ}|^2$ is relatively good. We see convergence at low values of I and J , but convergence worsens for high I , and very small and high values of J (intermediate values of J near the peak preserve convergence). Regarding the convergence of $|\lambda_{IJ}|^2$, for high values of I and J , we see that it is lost (higher values in the cutoff in the number of modes yield smaller $|\lambda_{IJ}|^2$). Again, it does seem to be a consequence of the cutoff introduced on each simulation rather than a problem of our numerical methods.

As a final comment, we must note that in [67] it is claimed that a thermal spectrum arises in some asymptotic regime. We have not been able to find such thermal regime, at least for the cases studied here.



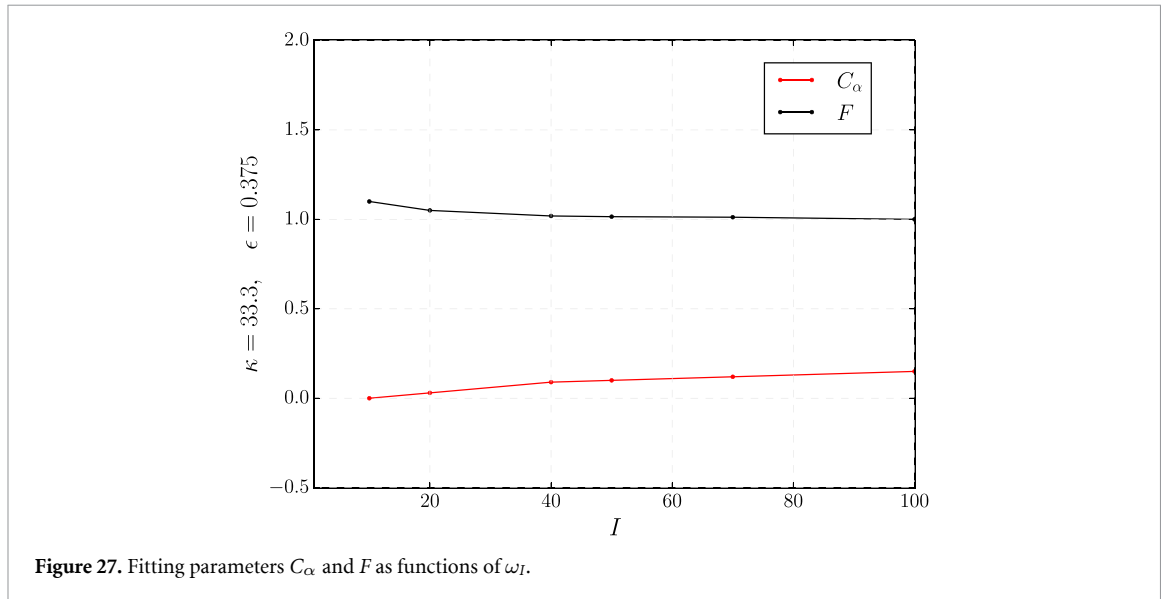
Appendix B. Fitting coefficients

In this section, we show the behaviour of several coefficients in the fitting expressions of $|\alpha_{IJ}^{(f)}|^2$ and $|\beta_{IJ}^{(f)}|^2$. We start with the trajectory in equation (26) with $\kappa = 33.3$ and $\epsilon = 0.375$. We show the parameters that change as functions of ω_I , that is, $C_\alpha = C_\beta$ and F in figure 27. The other parameters are nearly constant, and their value is given in the main text of the corresponding section.

For the trajectory in equation (26) with $\kappa = 1200$ and $\epsilon = 0.125$, again we show in figure 28 the parameters that change as functions of ω_I . In this case, they correspond to D_1 , D_2 , F and $\tilde{\kappa}$. The other parameters are nearly constant, and their value is given in the main text of the corresponding section.

We now consider the trajectory in equation (39) with $\kappa = 33.3$ and $\epsilon = 0.375$. We show the parameters that change as functions of ω_I , namely, $C_\alpha = C_\beta$, $\tilde{\kappa}$, D_1 , D_2 , and F in figure 29. The other parameters are nearly constant, and their value is given in the main text of the corresponding section.

For the same trajectory in equation (39) but with $\kappa = 1200$ and $\epsilon = 0.125$ we show the parameters that change as functions of ω_I , in this case $\tilde{\kappa}$, D_1 , D_2 , and F in figure 30. The other parameters are nearly constant, and their value is given in the main text of the corresponding section.



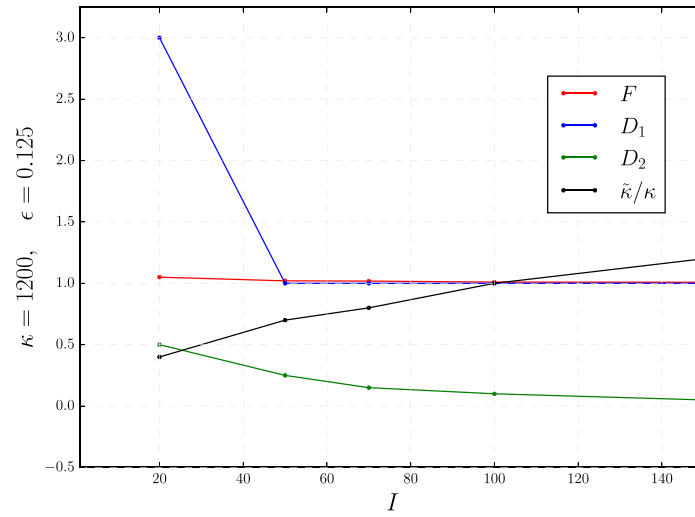


Figure 28. Fitting parameters D_1 , D_2 , F and $\tilde{\kappa}$ as functions of the in modes ω_I .

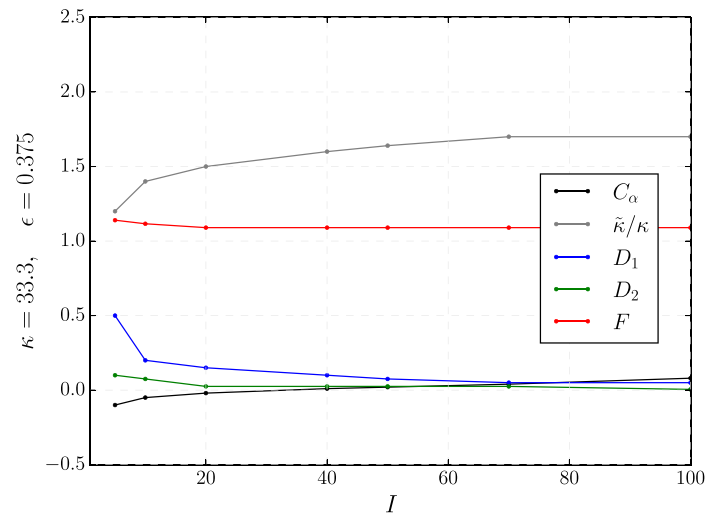


Figure 29. Fitting parameters $C_\alpha = C_\beta$, $\tilde{\kappa}$, D_1 , D_2 , and F as functions of ω_I .

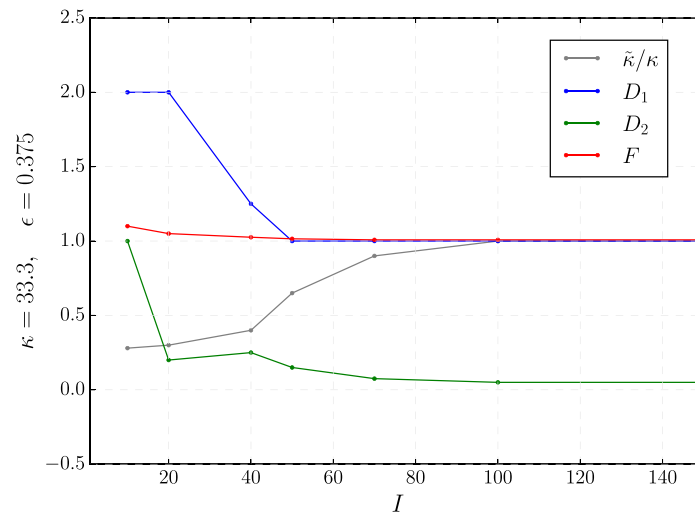


Figure 30. Fitting parameters $\tilde{\kappa}$, D_1 , D_2 , and F as functions of ω_I .

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