

A surrogate-model methodology for thermoacoustic engine design using multilayer perceptrons

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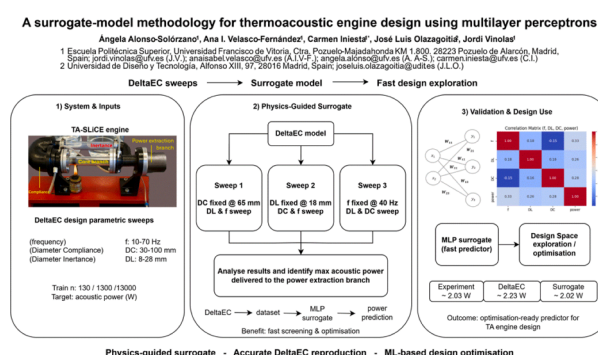
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HIGHLIGHTS

- Physics-guided neural-network surrogate for thermoacoustic power prediction.
- Accurate reproduction of DeltaEC results, enabling efficient design space exploration.
- Methodology for the design optimisation of thermoacoustic engines based on machine learning.

GRAPHICAL ABSTRACT



ARTICLE INFO

Keywords:

Thermoacoustic stirling engine
Acoustic power prediction
ANN
MLP

ABSTRACT

Thermoacoustic Stirling-like cycle engines offer a compelling solution for converting residual heat into usable energy through the generation of acoustic waves, providing a mechanically simple and environmentally friendly alternative. While recent developments have demonstrated the feasibility of using artificial neural networks for performance prediction, limited research has addressed the effect of training data volume or the experimental validation of these models. This study investigates the use of artificial neural networks to predict the acoustic power delivered to the power extraction branch in thermoacoustic engines operating at low drive ratios, using data generated through DeltaEC (Design Environment for Low-Amplitude Thermoacoustic Energy Conversion) simulations. Two training approaches were implemented: Python programming and SPSS (Statistical Package for the Social Sciences); and three datasets of increasing size were used to evaluate the impact of data volume on prediction performance. Seven design variables were analyzed, of which three emerged as the most critical. The proposed methodology enables fast and reliable prediction of thermoacoustic performance, reducing reliance on computationally expensive simulations and supporting the integration of artificial intelligence techniques into energy system design.

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<https://doi.org/10.1016/j.egyai.2026.100837>

Available online 2 July 2026

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Abbreviations

ANFIS	Adaptive Neuro-Fuzzy Inference Systems
ANN	Artificial Neural Networks
DeltaEC	Design Environment for Low-Amplitude Thermoacoustic Energy Conversion
ML	Machine Learning
MSE	Mean Squared Error
MLP	Multilayer Perceptron
RMSE	Root Mean Square Error
SGD	Stochastic Gradient Descent
SPSS	Statistical Package for the Social Sciences
SSE	Sum of Squared Errors
TA-SLiCE	Thermoacoustic Stirling-Like Cycle Engine

1. Introduction

The recovery of residual thermal energy has gained attention in response to global sustainability goals and the efforts to reduce greenhouse-gas emissions. Among various heat-recovery technologies, the conversion of heat into useful energy using thermoacoustic devices has attracted increasing attention in recent decades due to their simple construction, minimal maintenance requirements, and strong potential for energy harvesting applications [1]. Thermoacoustic engines convert heat into acoustic power by sustaining thermoacoustic oscillations in a working gas, driven by the interaction between a temperature gradient and propagating sound waves [2]. The resulting acoustic power can be harnessed to drive alternators, pumps, or thermoacoustic refrigerators, offering the added advantages of silent operation, mechanical simplicity and low-maintenance operation [3]. In a thermoacoustic Stirling-like cycle engine (TA-SLiCE), heat is applied across a regenerator (a porous medium) bounded by heat exchangers, inducing acoustic work in a manner analogous to the Stirling cycle, but without the moving pistons of conventional engines [4]. This simple architecture enhances mechanical reliability, making TA-SLiCEs especially promising for sustainable energy applications such as waste-heat recovery and thermal-to-electric power conversion [5]. The concept of a pistonless Stirling engine was first introduced by Ceperley in 1979 [6] and experimentally demonstrated two decades later by Backhaus and Swift [7], laying the foundation for a wide array of laboratory-scale TA-SLiCE

developments [8]. However, achieving high performance in a TA-SLiCE requires the careful design of the acoustic resonator network to maintain strong self-sustained oscillations and to efficiently extract usable power from the acoustic field [9]. The engine's efficiency emerges from the coupling of thermodynamic, acoustic, and heat-transfer phenomena, and is governed by numerous interdependent design parameters. As highlighted by Napolitano et al. [4], both the structural characteristics of the regenerator and the resonator geometry play a pivotal role in determining the acoustic power output and overall efficiency. Their optimization strategy has been extended to traveling-wave configurations, further broadening the design landscape for advanced thermoacoustic systems.

In a TA-SLiCE, the acoustic circuit is deliberately divided into three interdependent branches: the core branch, where heat is converted into acoustic power; the feedback branch, which includes compliance and inductance elements to establish the appropriate traveling-wave phasing; and the power extraction branch, which harnesses part of the acoustic power for external work [10]. A representative TA-SLiCE configuration is illustrated in Fig. 1. It consists of a closed-loop acoustic circuit with a regenerator at one end, supplied with thermal input from an external heat source, and a feedback loop incorporating a compliant-inertial segment on the opposite side. The feedback loop typically comprises a compliance chamber and an inductance tube, forming a passive acoustic phase-shifting network. This network ensures that pressure and volumetric velocity oscillations attain the desired phase difference, approaching that of a traveling wave, thereby promoting efficient heat-to-work conversion within the regenerator. This design enables self-sustained acoustic oscillations: the regenerator and associated heat exchangers inject acoustic power into the system, the feedback branch recirculates part of this energy to maintain oscillations, and the surplus is directed toward the power extraction branch [11]. Typically implemented as a side resonator equipped with a compliant piston or loudspeaker, the power extraction branch allows part of the acoustic power to be converted into useful external work. The acoustic power extracted through the power extraction branch serves as a key indicator of the engine's overall efficiency. It reflects how effectively the thermoacoustic cycle is sustained and how critical design parameters, such as the geometry of compliance and inductance, influence energy conversion. Even subtle modifications in compliance geometry can significantly alter the distribution of acoustic energy, thereby impacting the proportion of

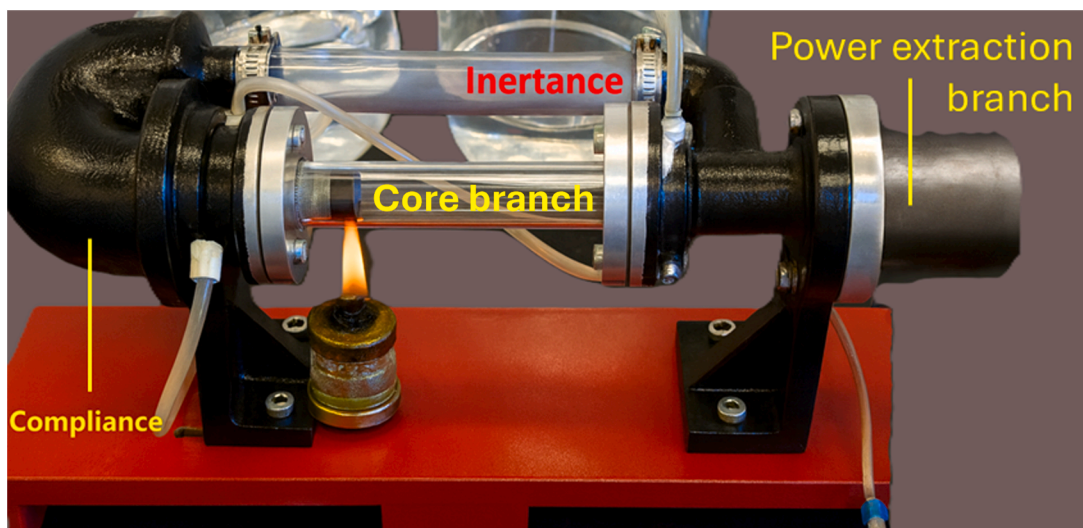


Fig. 1. Schematic of the thermoacoustic Stirling-like engine (TA-SLiCE) configuration used in this study. The system consists of the core branch which contains the regenerator sandwiched between a hot heat exchanger (heat input) and a cold heat exchanger (heat sink). The feedback branch provides the necessary acoustic compliance and inductance via an enclosed compliance chamber and a narrow inertance tube, which together tune the phase of pressure–volumetric velocity oscillations. The power extraction branch (right) is a side resonator terminating in a moving piston, through which a portion of the acoustic power is extracted as useful output.

power channeled into useful output versus that recirculated within the loop. Designing a TA-SLiCE therefore involves balancing multiple coupled parameters to ensure that the feedback loop sustains the oscillation while maximizing the delivery of acoustic power to the extraction load. Achieving this balance, however, is complicated by the intricate interdependence among numerous geometric and operating variables. Components such as the regenerator, heat exchangers, inertance tubes, and compliance chambers must be precisely configured to promote efficient coupling between thermal and acoustic oscillations within the system [12]. The multivariable and highly coupled nature of TA-SLiCEs presents a significant challenge for conventional design strategies based solely on analytical modelling or empirical tuning, which are often insufficient to capture the nonlinear interactions that govern system behavior.

Given the highly coupled and multiparametric nature of TA-SLiCE systems [13], simulation-based design has become indispensable in thermoacoustics. In this context, one powerful tool is the DeltaEC software (Design Environment for Low-Amplitude Thermoacoustic Energy Conversion), which was originally developed at Los Alamos by Swift et al. [14]. DeltaEC provides a one-dimensional network model of the engine based on Rott's linear acoustic wave theory [15], enabling the rapid calculation of pressure, flow and temperature distributions for a given geometry in steady state. The software is widely used by researchers and designers for tuning thermoacoustic engines and performing parametric sweeps; it has been applied to analyse acoustic field distributions in thermoacoustic systems [16] established for design and performance optimization [17], and demonstrated to achieve good agreement with experimental results [18]. For example, an initial DeltaEC model of the present TA-SLiCE was used to select a feasible operating frequency (≈ 23 Hz) and to estimate performance at various geometry settings. However, purely physics-based simulation and manual parameter sweeps can become unwieldy as the number of design variables grows. Iniesta et al. [19] conducted a sensitivity analysis using response surface methodology on a TA-SLiCE configuration, evaluating seven design variables including the dimensions of the compliance and inertance tubes, various branch lengths, and the resonance frequency. Their findings revealed that the operating frequency, the inner diameter of the compliance chamber, and the inner diameter of the inertance were the most influential parameters, enabling a 6% increase in efficiency through optimization. This study revealed the limitations of traditional parametric methods and highlighted the relevance of data-driven modelling for efficiently analysing the full design parameter set. Accordingly, the design of thermoacoustic engines must account for the strong interdependence between geometric and operating parameters, which critically influence acoustic performance and energy conversion efficiency. Although the architecture of a TA-SLiCE may appear simple, it encompasses a multitude of interrelated design variables (such as lengths, diameters, and resonance frequency) that govern the engine's thermoacoustic response [20]. Even in the relatively compact prototype examined in this study, seven key parameters were required to characterise its dynamic behavior. Previous sensitivity analyses on comparable TA-SLiCE configurations have demonstrated that only a subset of these variables exerts a dominant effect on performance. Among them, the diameters of the compliance chamber and inertance tube have proven to be the most critical, while the operating frequency also plays a substantial role [21]. In contrast, other geometric features, including exact branch lengths and minor loss factors, tend to have a more limited effect on performance metrics [22]. These results are consistent with theoretical expectations: the sizes of the compliance and inertance elements largely determine the acoustic impedance and phase relations within the loop, which in turn dictate how effectively heat-induced pressure oscillations are converted into net acoustic power. Tuning the drive frequency near the system's natural resonance is likewise critical to achieving optimal energy transfer. Importantly, the strong nonlinear interactions among key parameters (for example, the coupled effects of inertance and compliance geometries) further complicate manual design

and highlight the need for advanced optimization frameworks. Traditional optimization techniques and even reinforcement learning agents have been applied to identify favourable thermoacoustic designs by iteratively querying simulation models [23]. These studies demonstrate the potential for algorithmic design of TA-SLiCEs, but they also highlight the substantial computational cost of evaluating high-fidelity models and the need for surrogate models that can predict performance outcomes much faster than real-time simulations. Consequently, designing or optimizing a TA-SLiCE requires tools capable of managing high-dimensional parameter spaces and capturing the intricate, nonlinear dependencies between design inputs and engine performance.

In parallel with simulation-based optimization, machine learning (ML) has emerged as a promising tool to model the complex input–output relationships of thermoacoustic systems. In particular, artificial neural networks (ANNs) have been used to predict key performance indicators of thermoacoustic devices [24]. These studies show that ANN-based models can capture the highly nonlinear mappings in thermoacoustic behavior with impressive accuracy (often within a few percent error), outperforming simpler regressions or analytical models especially when strong variable interactions are present. Variations and extensions of standard ANNs, including adaptive neuro-fuzzy inference systems (ANFIS), ANN ensembles optimized via genetic or imperialist competitive algorithms, and hybrid ANN-PSO models, have all been tested on thermoacoustic case studies [25]. Among the dominant intelligent modelling approaches are those that integrate data-driven learning with physical insight, for example combining an ANN predictor with known conservation-law constraints or using ML to augment low-order models [22]. In the context of engine performance prediction, which is largely a static mapping from design parameters to steady-state outputs, Zare et al. [26] proposed an ANN-based design method to predict the operating parameters of a thermoacoustic Stirling engine, demonstrating practical utility in the design stage and ensuring dynamic stability. Prior studies have explored the use of neural networks to predict the performance of thermoacoustic coolers and the electrical output/voltage in heat recovery projects using thermoacoustic systems; in several cases, ANNs compete with or outperform empirical models when representative experimental data are available [27]. These studies also explore variants such as ANFIS and ANNs optimized by evolutionary algorithms (e.g. ANN-PSO) to adjust hyperparameters and improve robustness [28].

Similar ANN-based surrogate methodologies have also been successfully applied in related thermo-fluid energy systems, including Organic Rankine Cycle applications, where neural-network models have demonstrated strong capability for predicting nonlinear thermodynamic behavior and supporting design optimization under coupled operating conditions [29]. In the context of photovoltaic thermal systems, recent studies have shown that systematic experimentation with multilayer perceptrons, together with controlled normalization procedures and data-quality management, are essential to ensure robust predictive performance under varying operating conditions [30]. In sustainable energy-system design, fuzzy-logic-assisted methodologies have further demonstrated the value of transparent ML-supported decision-making in complex engineering environments [31]. In parallel, recent engineering-oriented neural-network frameworks have reinforced the need for carefully structured datasets and physically consistent validation procedures when surrogate models are applied to strongly nonlinear coupled systems [32].

More recently, several contributions have reinforced the role of surrogate-based and ANN-based modelling as an efficient complement to classical thermoacoustic simulation tools. Rahman and Zhang [33] demonstrated that feed-forward neural networks can accurately reproduce acoustic wave parameters in thermoacoustic prime movers, achieving high prediction accuracy even with limited data. Ngcukayitobi et al. [25] extended this idea by systematically comparing ANN, ANN-PSO and ANFIS surrogates for thermoacoustic waste-heat recovery, showing that hybrid ANN-PSO architectures provide excellent

generalization capability across wide operating ranges.

In parallel, scarce works have explored hybrid simulation-and-learning strategies specifically aimed at accelerating thermoacoustic design. Zare et al. [26] used ANN surrogates to support the geometric design of thermoacoustic Stirling engines. Although this study focuses on different physical configurations, it provides methodological precedents highly relevant for the surrogate-based framework developed in the present work.

From a broader methodological perspective, the use of surrogate modelling for high-dimensional thermo-fluid systems is consistent with trends observed in energy engineering and computational acoustics. Frank [34] highlights the effectiveness of ML-based surrogate models for accelerating design tasks in complex coupled systems, while Croci [35] demonstrates the value of combining physics-based thermoacoustic models with data-driven components, particularly when experimental data are sparse or expensive to acquire. These methodological advances align with the central motivation of the present study: building reliable surrogate models that retain physical consistency while enabling systematic exploration of coupled thermoacoustic design spaces.

This study presents a methodology for predicting the acoustic power delivered to the extraction branch of TA-SLiCEs operating at low drive ratios, using artificial neural networks (ANNs), addressing several outstanding research gaps in the field. While prior work has predominantly analysed the impact of design variables on linear dynamics and system stability, less emphasis has been placed on the inverse design problem: identifying optimal parameter combinations that meet specific performance objectives, such as maximising acoustic power output, while preserving dynamic stability.

The present approach tackles this challenge by constructing an extensive dataset of DeltaEC simulations that span three key design parameters, including operating frequency, inertance diameter and compliance diameter. To investigate the influence of data volume on predictive performance, networks are trained on datasets comprising 130, 1300, and 13,000 records. In addition, two training frameworks, Python-based and SPSS-based are compared. The trained models are evaluated using a single but representative experimental data point from a previously published TA-SLiCE demonstrator, providing a practical benchmark for assessing prediction accuracy.

By integrating parametric simulation, systematic data analysis, and experimental validation, this work provides a structured and reproducible framework for the design optimization of thermoacoustic engines based on machine learning. In this study, particular attention is given to thermoacoustic engine operation at low drive ratios (typically below 10%), where the linear acoustic approximation remains valid. This regime is especially relevant because the DeltaEC modelling framework employed in this work is based on Rott's linear thermoacoustic theory, which assumes small-amplitude oscillations. Operating within this range ensures numerical stability and physical consistency of the simulation results, avoiding nonlinear effects such as flow separation or shock formation that may arise at higher drive ratios. From a practical standpoint, low drive ratio conditions are commonly encountered during the design and experimental validation stages of thermoacoustic systems, particularly in laboratory-scale prototypes. In these contexts, stable and predictable operation is essential for parameter tuning and system optimization. Therefore, focusing on this regime enables the development of a reliable surrogate model while maintaining direct relevance to real-world thermoacoustic design workflows.

This work is positioned within the emerging use of artificial intelligence in energy systems engineering. By leveraging data-driven surrogate modelling, the proposed approach enables rapid prediction of thermoacoustic performance, significantly reducing the computational cost associated with physics-based simulations.

In this paper, first, the methodology and mathematical bases are introduced. Next, the dataset and descriptive data analysis is presented in Section 3. Subsequently, the acoustic power delivered to the power extraction branch of the engines are estimated in Section 4. In Section 5,

a discussion is presented regarding several key outcomes derived from this study, with particular emphasis on the SPSS and Python configurations, outcomes comparisons. Finally, concluding remarks are proposed in Section 6.

2. Methodology

The present work follows a structured surrogate-modelling workflow designed to reproduce the converged DeltaEC response while providing a fast and physically coherent predictive tool for TA-SLiCE systems. The methodology consists of five sequential stages: data generation, feature selection, surrogate training, sensitivity-and-robustness analysis, and validation. Each stage is described below.

All models correspond to fully connected feedforward multilayer perceptrons (MLPs) with three input neurons and one output neuron. Four architectures were evaluated: 3-12-6-1, 3-9-1, 3-6-3-1, and 3-6-1, representing networks with one or two hidden layers and between 6 and 12 neurons per layer. Two activation functions were analyzed, sigmoid and hyperbolic tangent, with each individual model employing a single activation function consistently across all hidden layers. All input and output variables were normalized to the interval [0,1] using min-max scaling. The normalization parameters were computed exclusively from the training data and subsequently applied to the test data to prevent data leakage. The dataset was partitioned using a 70–30 train–test split, and an additional 15% of the training data was used as a validation subset for early stopping. Model training was performed using stochastic gradient descent (SGD) with an adaptive learning rate. The initial learning rate was set to 0.01, with a momentum coefficient of 0.9 and Nesterov acceleration enabled. A mini-batch size of 64 samples was used, and L2 regularization was applied with $\alpha = 1 \times 10^{-4}$. The maximum number of iterations was 3000, with a tolerance of 1×10^{-6} . Early stopping was implemented, terminating training if no improvement in validation performance was observed for 50 consecutive iterations.

2.1. Data generation and preprocessing

To construct the surrogate model, a comprehensive dataset was generated using DeltaEC. The numerical model of the TA-SLiCE engine was configured with fixed geometric and thermophysical properties, while three design variables were selected for parametric exploration: operating frequency (f), compliance diameter position (DC), and inertance diameter (DL). These parameters were chosen because they are the primary determinants of the acoustic power output and appear consistently in previous thermoacoustic sensitivity studies. In travelling-wave engines such as TA-SLiCE, these three variables govern the reactive impedance of the loop, the pressure-velocity phase relation, and the proximity to resonance, making them the most influential design degrees of freedom for power prediction.

The dataset used in this study consists of 13,747 samples and includes eight variables. The input variables selected for the ANN model are frequency (f), DC, and DL, which represent the main operational parameters of the thermoacoustic system. The target variable corresponds to the power output. The ranges of the variables are as follows: $f \in [10, 70]$, $DC \in [30, 100]$, $DL \in [8, 28]$, and $Power \in [0.00193, 3.3707]$. Although additional geometric parameters, including the inertance length (LL), compliance length (LC), hot heat exchanger length (LHHX), and thermal buffer tube length (LTBT), are present in the dataset, they remain constant across all samples and were therefore not included as input features.

These ranges were selected to cover the full operating region of interest for the TA-SLiCE prototype while ensuring numerical stability of DeltaEC across the explored domain. Extending the sweeps to both small and large values of DC and DL allow the dataset to include off-design conditions where performance deteriorates due to impedance imbalance. Oversized compliance chambers (large DC) can reduce acoustic

stiffness and detune the phasing, while undersized chambers concentrate reactive energy. Likewise, very narrow inertance tubes (small DL) increase viscous dissipation, whereas excessively wide tubes reduce inertive reactance and disturb the pressure–flow phase matching. Including these regions ensures that the surrogate learns the boundaries of physically meaningful behavior.

A total of 13,747 DeltaEC simulations were performed by combining the three parameters at discrete increments. Simulations that did not reach numerical convergence or produced non-physical trends (e.g., sign changes in acoustic power, discontinuities between nearby samples) were discarded. Only converged and physically consistent samples were retained for training the surrogate model. This filtering step is essential because DeltaEC may exhibit sensitivity near resonance peaks or impedance extrema, and retaining such artefacts would degrade the stability of the surrogate.

For each converged case, DeltaEC provided the acoustic power output at the resonator, which was selected as the target variable for the neural network. The use of acoustic power rather than intermediate thermoviscous fields ensures consistency across the entire dataset and facilitates direct comparison with experimental measurements.

Before training, all input variables were normalized to the [0,1] range using Min–Max scaling. This choice avoids scale imbalance among the parameters and improves the conditioning of the optimization problem. The dataset was inspected for continuity and monotonicity with respect to the three design variables, confirming that DeltaEC produces a smooth and well-behaved response surface across the retained parametric space. This ensures that the surrogate model is trained on a coherent and physically interpretable map, free from numerical artefacts introduced by the solver.

2.2. Feature selection

Although thermoacoustic performance depends on multiple geometric and thermophysical parameters (e.g., resonator length, stack porosity, heat exchanger dimensions, working fluid properties), prior analytical studies and numerical experimentation indicate that only a subset of parameters dominates the acoustic power output of standing-wave engines.

In the present work, three variables were selected as surrogate inputs: operating frequency (f), compliance chamber diameter (DC), and the inertance tube diameter (DL). These variables were identified as the critical determinants of performance based on (i) preliminary DeltaEC screening; (ii) established thermoacoustic theory regarding impedance

balance and resonance tuning; and (iii) their direct physical role in establishing resonance conditions, phasing of thermoacoustic heat exchange, and acoustic impedance. Subsequent statistical analysis of the dataset (Section 4) was used only to verify that these variables provide non-redundant predictive information, but it did not determine their selection.

To support the selection of the dominant design variables, a sensitivity and feature importance analysis was carried out using the trained MLP model (3-12-6-1, tanh activation).

Permutation feature importance was evaluated by randomly shuffling each input variable and recording the resulting drop in R^2 . Fig. 2 shows that frequency (f) and displacement length (DL) cause the largest reductions in predictive accuracy, with mean decreases of 5.05 and 5.01, respectively, while DC produces a moderate drop of 3.80. The variables LL, LC, LHTX and LTBT have practically no effect, which is expected given that they remain constant throughout the dataset and provide no discriminative information to the model.

A one-at-a-time (OAT) sensitivity analysis was also carried out, varying each input across its normalized range while the remaining variables were held at their mean values. The resulting curves (Fig. 3) reveal a pronounced nonlinear dependence of the predicted acoustic power on f , DC and DL, with clearly defined local maxima in each case. This is physically coherent with the known behavior of thermoacoustic engines, where performance is strongly tied to operating frequency and geometric configuration. The other four variables produce essentially flat responses, which further confirms their negligible role.

Finally, the sensitivity range, computed as the peak-to-peak variation in predicted output for each variable, was used to quantify the magnitude of these effects (Fig. 4). DL yields the widest range (8.09 W), followed by f (2.92 W) and DC (1.97 W). Although LL shows a non-negligible numerical range (2.56 W), this value is an artefact of forcing the sweep beyond the range present in the training data and does not reflect a real model dependency, as the permutation results confirm.

The three analyzes converge to the same conclusion: f , DL, and DC are the dominant variables governing the system response. The agreement between the different methods strengthens confidence in the choice of inputs and supports the physical interpretability of the surrogate model.

All remaining geometric and thermophysical parameters were kept fixed at their validated nominal values. This avoids unnecessary dimensionality expansion and ensures that the surrogate focuses on the parametric directions that meaningfully modify the acoustic power.

A preliminary exploratory analysis (described in Section 4)

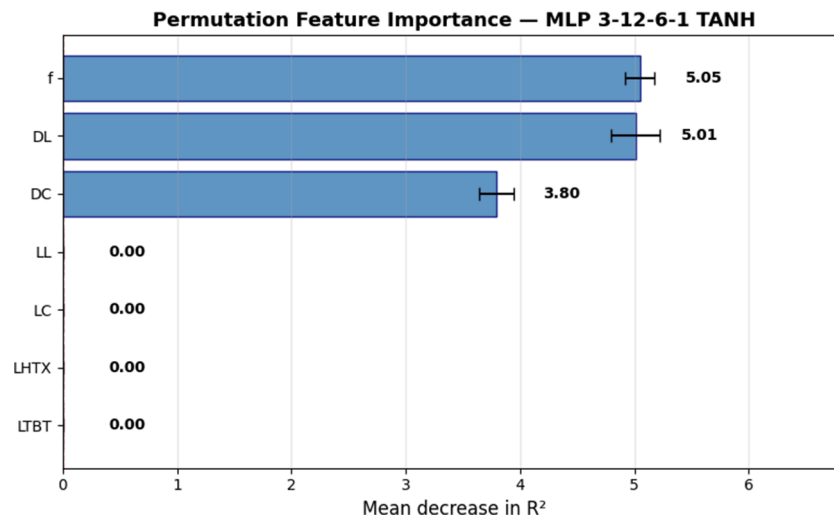


Fig. 2. Permutation feature importance analysis performed on the trained MLP (3-12-6-1, tanh activation) shows that f and DL are the dominant variables influencing the network predictions, with mean decreases in R^2 of 5.05 and 5.01, while DC presents a moderate contribution of 3.80.

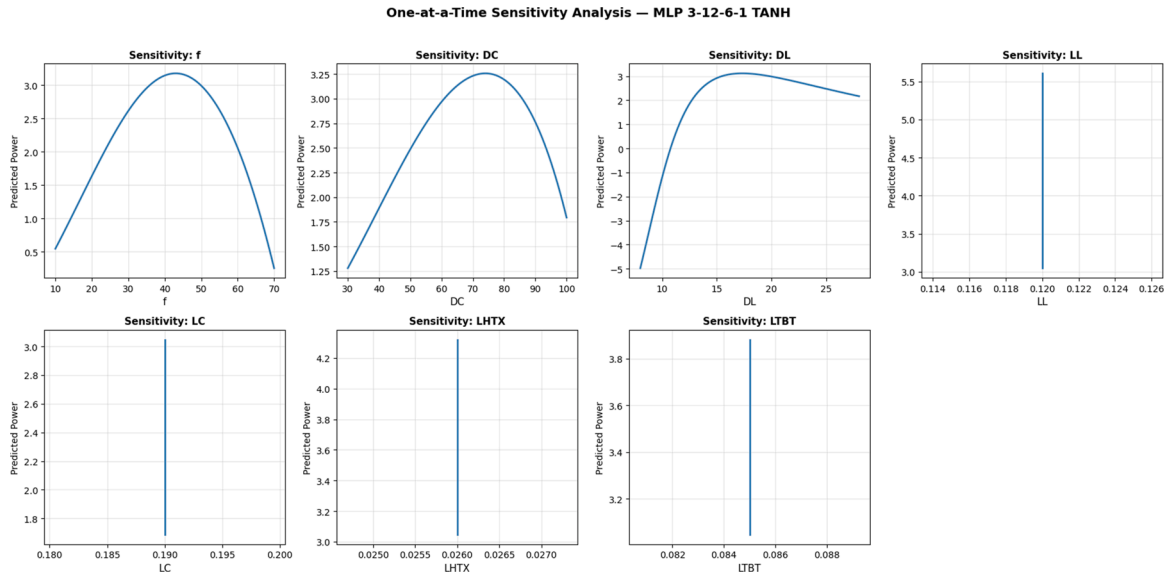


Fig. 3. One-at-a-Time sensitivity analysis of the trained MLP (3-12-6-1, tanh activation) showing that *f*, *DL* and *DC* produce the largest nonlinear variations in predicted acoustic power. The variables *LL*, *LC*, *LHTX* and *LTBT* remain nearly constant across their analysed ranges, indicating a negligible contribution to the MLP predictions within the dataset.

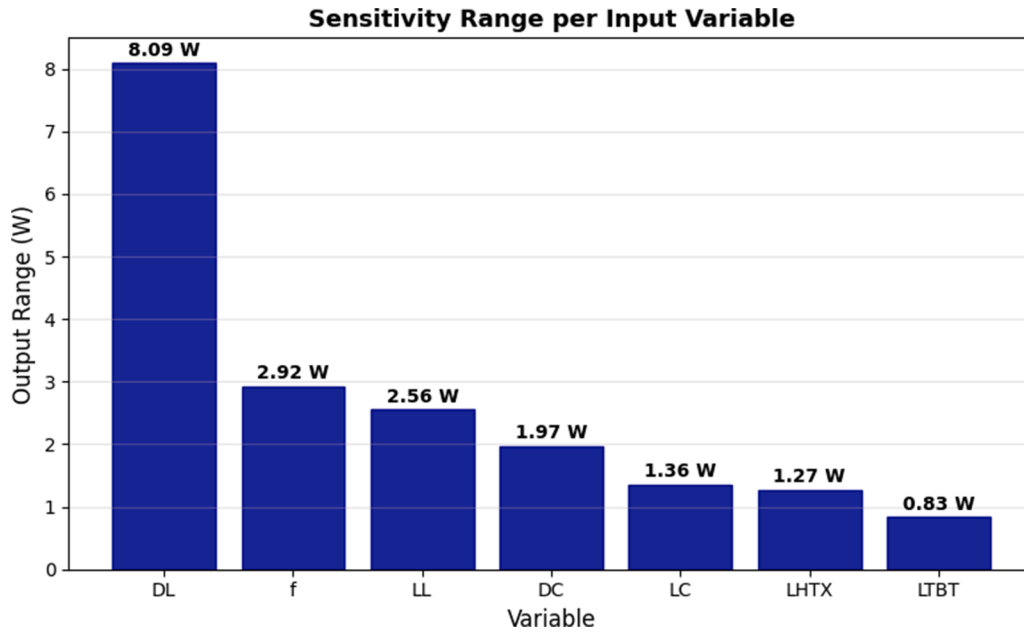


Fig. 4. Sensitivity range analysis of the trained MLP (3-12-6-1, tanh activation) showing that *DL* produces the largest variation in predicted acoustic power (8.09 W), followed by *f* (2.92 W).

confirmed that the three selected variables provide non-redundant information on the acoustic power. In this way, the surrogate model focuses on the dominant design degrees of freedom without introducing unnecessary complexity.

2.3. Surrogate model training (MLP implementation)

The surrogate model was implemented as a multilayer perceptron (MLP) that maps the three selected input variables to the corresponding acoustic power obtained from DeltaEC. The goal of this stage is to define a reproducible training protocol that can be applied consistently across all experiments rather than on the internal mathematical details of the network, which are presented in Section 3.

For all experiments, the dataset was split into two main subsets: 70% of the samples were used for training and 30% were reserved as an independent test set. This 70–30 split was applied systematically to guarantee methodological consistency.

A family of feedforward MLP architectures with three input neurons and one output neuron was defined in advance, differing in depth and width, and combined with two standard activation functions (sigmoid and hyperbolic tangent). The specific architectural layouts and the mathematical description of the learning process are presented in Section 3. All architectures shared the same input–output normalization strategy defined in Section 2.1 to ensure fair comparison across experiments.

Training in both environments followed the same logical sequence:

(i) loading and preprocessing the DeltaEC dataset; (ii) splitting the data into training and test subsets; (iii) fitting the MLP to the training data using gradient-based optimization; and (iv) evaluating the final model on the independent test set. Mean squared error (MSE) was used as the optimization objective, while root mean squared error (RMSE) and the coefficient of determination (R^2) were used as global performance metrics. The formal definitions of these metrics and the specific optimization schemes are detailed in Section 3. This separation ensures that the methodological description remains focused on the training protocol, while the mathematical foundations and numerical details are discussed where they belong.

This protocol defines the common training framework for all surrogate models considered in the study. The impact of the different architectures, dataset sizes and software environments on the resulting predictive performance is discussed in the Results section. By maintaining a unified protocol across implementations, the study isolates the effect of architectural and computational choices, enabling a fair and interpretable comparison between the surrogate models.

2.4. Sensitivity-and-robustness analysis

An optional methodological stage considers the evaluation of how the surrogate model responds to controlled perturbations of the input parameters. In this stage, predefined percentage variations are applied to the normalized inputs, and the corresponding changes in the predicted acoustic power are computed. These perturbations are applied independently to each input and, when relevant, to combinations of parameters to assess whether the surrogate behaves smoothly across the full operating domain defined in Section 2.1.

The objective of this step is to define a framework for assessing the stability, smoothness and physical coherence of the surrogate model under input uncertainty. This analysis also provides a qualitative indication of whether the surrogate extrapolates sensibly near the boundaries of the DeltaEC design space, without requiring additional simulations. The specific perturbation levels, analysis procedures and quantitative outcomes are presented, if included, in the Results and Discussion sections.

2.5. Validation protocol

The validation of the surrogate model was organized into two complementary stages to ensure both numerical reliability and physical plausibility.

First, a numerical validation was conducted using an independent test subset of the DeltaEC simulations. This step assesses the generalization capability of the surrogate within the same parametric domain used for data generation. The procedure involves comparing predicted and reference outputs on the test set, checking error metrics and visually inspecting the agreement between the modelled and simulated response surfaces. This validation stage ensures that the surrogate has learned the mapping imposed by DeltaEC without overfitting to the training data.

Second, an experimental validation was performed using a representative operating point of the TA-SLiCE prototype. The corresponding input conditions were imposed in the DeltaEC model and in the trained surrogate, and the resulting power predictions were compared with the measured value. This comparison provides a consistency check between the surrogate, the underlying numerical model and the physical system. Although only a single operating point is available experimentally, this step offers an external reference that complements the purely numerical validation.

Together, these two validation levels establish a coherent protocol for assessing whether the surrogate model is not only accurate with respect to its training source (DeltaEC) but also compatible with the observed behavior of the real thermoacoustic prototype. This hierarchical validation structure also clarifies the respective roles of numerical fidelity (test-set evaluation) and physical plausibility (experimental

comparison).

3. Neural network formulation

3.1. General ANN structure

Artificial Neural Networks (ANNs) are nonlinear function approximators composed of interconnected layers of neurons with trainable weights. In this work, multilayer perceptrons (MLPs) were employed to construct surrogate models capable of predicting thermoacoustic power from the three selected physical variables (f , DC , and DL). Each network receives the normalised input vector and propagates it through one or more hidden layers, where linear transformations of the form $xW + b$ are applied prior to nonlinear activation. The output layer contains a single neuron representing the predicted acoustic power.

A typical ANN consists of an input layer, one or more hidden layers, and an output layer. In the present study, the input layer includes the three normalized covariates (f , DC , and DL), while the output layer produces a single value corresponding to the acoustic power. Fig. 5 illustrates the general structure of the architecture used, in which each neuron receives weighted signals from all neurons in the preceding layer.

The internal operation of each neuron is based on a weighted linear combination of the inputs followed by a bias term. This computation can be expressed in general form by Eq. (1), which represents the pre-activation calculation:

$$(x_1, x_2) \cdot \begin{pmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \end{pmatrix} + (b_1, b_2, b_3) = (y_1, y_2, y_3) \quad (1)$$

where the weights W_{ij} determine the relative influence of each input, and the bias terms b_j shift the activation, improving the model's ability to fit the data. After this linear transformation, each neuron applies its corresponding activation function, enabling the network to capture nonlinear relationships between the input variables and the generated power.

In this work, sigmoid and hyperbolic tangent activation functions were employed depending on the selected architecture. Both functions introduce nonlinearity into the network and allow the surrogate model to accurately approximate the thermoacoustic behavior simulated with DeltaEC. Together, normalization, the MLP architecture, and nonlinear activations enable the network to represent the underlying nonlinear mapping between the design parameters and the resulting thermoacoustic power.

3.2. Network architectures and experimental design

The following subsections describe the specific MLP architectures and experimental design used in this study, building on the methodological framework introduced in Section 2.

The experimental design incorporated four different neural architectures for all the methodologies, evaluating performance across varying levels of computational complexity. This comparative configuration was intentionally designed to provide a controlled assessment of the sensitivity of predictive performance to architectural depth, neuron distribution, activation function selection, and dataset size.

- Architecture 1: 3–12–6–1 (input-hidden-hidden-output)
- Architecture 2: 3–9–1 (input-hidden-output)
- Architecture 3: 3–6–3–1 (input-hidden-hidden-output)
- Architecture 4: 3–6–1 (input-hidden-output)

These architectures were chosen to explore how depth and width affect the surrogate's ability to approximate the nonlinear DeltaEC mapping, ranging from compact single-hidden-layer networks to deeper models with increased representational capacity.

Data scalability was analysed through identical dataset partitioning

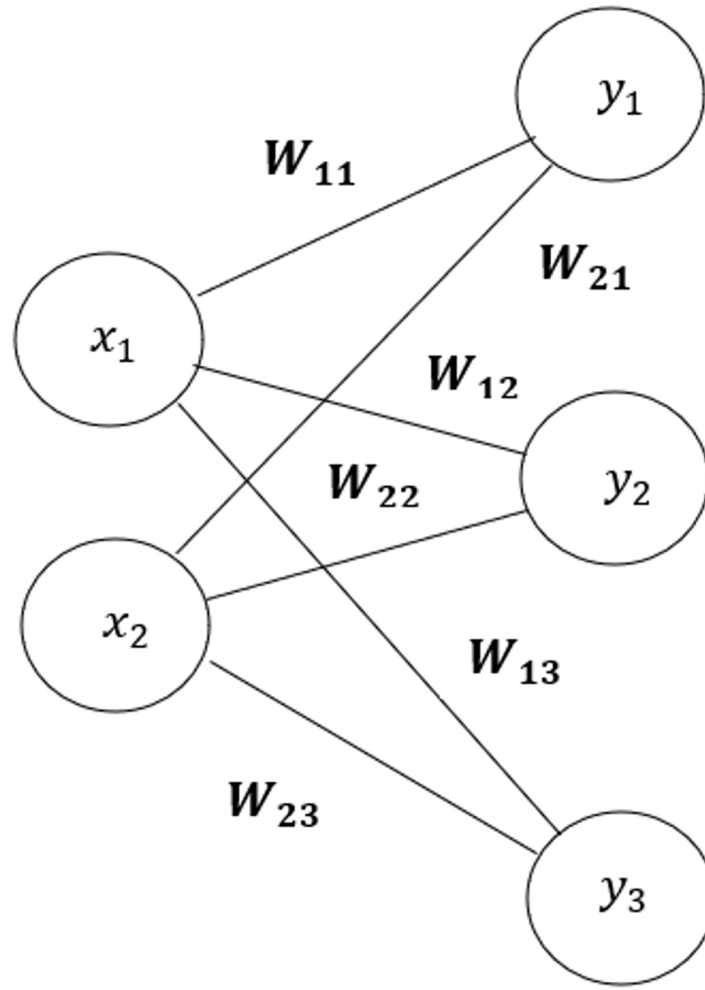


Fig. 5. Architecture of the neural network.

protocols across three different volume conditions: limited sample size ($n = 130$), standard sample size ($n = 1300$), and expanded sample size ($n = 13,000$). In all experimental configurations, a 70–30 train-test partition ratio was implemented, consistent with the methodological description in Section 2. This design enables systematic evaluation of how the surrogate’s performance degrades or stabilises as the amount of available training data is reduced.

All input and output variables received uniform normalization procedures, mapping values to the standardized interval $[0,1]$. This normalization protocol was applied across all methodological implementations to eliminate scale-based influences on model performance.

Both sigmoid and hyperbolic tangent activation functions were evaluated across all architectural configurations, allowing direct comparative assessment of their impact on predictive performance. The gradient descent optimization algorithm served as the consistent foundation for parameter optimization across implementations, though with methodology specific adaptations as required by each computational framework.

The investigation employed two different computational methodologies for thermoacoustic power prediction, sharing the fundamental design elements described above while differing in implementation specifics. This dual-methodology approach allows the robustness of the surrogate formulation to be evaluated independently of the software environment, providing an additional layer of methodological validation.

3.3. Python methodology

The dataset division represented a fundamental step, implemented through a 70–30 split, where 70% of the data was used for model training and the remaining 30% was reserved as an independent test set for final evaluation. This partition was carried out using the `train_test_split` function with randomization enabled to guarantee statistical representativeness in both subsets.

In addition, using Python, an internal validation mechanism was incorporated within the training phase by allocating 15% of the training data to monitor learning performance and apply the early stopping technique when no significant improvement was observed.

The overall study follows a methodical controlled protocol using MLP architectures applied to predict the power estimation in thermoacoustic systems. Experimental reproducibility was ensured through a fixed random seed, systematically applied to all stochastic processes, from synaptic weight initialization to data partitioning.

3.3.1. Workflow

The methodological implementation followed a structured workflow using specialized machine learning frameworks, primarily TensorFlow and scikit-learn.

The training of neural networks proceeded through a series of well-defined stages designed to guarantee efficient learning. Data pre-processing constituted the initial phase, involving data cleaning, normalization, and partitioning into training and testing subsets. The architectural design of the network was defined according to the

predefined architectures: 3-12-6-1, 3-9-1, 3-6-3-1, and 3-6-1.

Training configuration included the specification of performance metrics, such as error measures and determination coefficients, as well as the selection of the optimization algorithm, stochastic gradient descent (SGD) with momentum. Model training was then performed by exposing the network to the input data, enabling parameter adjustment through the backpropagation algorithm.

Upon completion of training, each model was evaluated on the test set to verify its operational consistency and predictive capability using statistical indicators such as the Coefficient of Determination (R^2), Mean Squared Error (MSE) and Root Mean Square Error (RMSE).

3.3.2. Learning mechanism

The learning rate (α) determines the magnitude of each weight update and is therefore a critical hyperparameter. If α is too large, the optimization trajectory may overshoot the minimum and diverge. If it is set too low, convergence becomes excessively slow. In practice, suitable values typically range between 10^{-1} and 10^{-4} , and the optimal selection often requires empirical adjustment. Training proceeds cyclically:

1. Data are propagated forward through the network to produce predictions.
2. The loss is evaluated by comparing predictions with target values.
3. The weights are updated to minimise this loss through gradient descent.

The training phase employed an enhanced SGD optimizer configured as follows: an initial learning rate of 0.01 with adaptive reduction when performance plateaus; a momentum factor of 0.9 with Nesterov acceleration to anticipate the gradient direction, and a mini-batch size of 64 samples, achieving a balance between computational efficiency and gradient stability. The corresponding weight update rule is expressed in Eq. (2) as:

$$W_{t+1} = W_t - \alpha(\beta v_t + \nabla J(\theta_t)) - \lambda W_t \quad (2)$$

where W_t denotes the weight matrix at iteration t , $\beta = 0.9$ is the momentum coefficient, v_t is the accumulated momentum vector, $\nabla J(\theta_t)$ represents the gradient of the loss function, and $\lambda = 1 \times 10^{-4}$ corresponds to the L2 regularization term used to improve generalization by penalizing excessively large weights.

To prevent overfitting, early stopping was applied, stopping training after 50 consecutive epochs without improvement or once the loss reached a tolerance threshold of 1×10^{-6} . The number of epochs was defined in advance, with each epoch representing a complete pass through the training set. This iterative process allows gradual refinement of the network, promoting a progressive reduction in prediction error over successive training iterations. The training was configured with a maximum of 3000 epochs, but an early stopping mechanism was applied. If the validation performance doesn't improve for 50 consecutive epochs, the process stops earlier, preventing overfitting and reducing computation time.

In addition, the use of relatively compact network architectures, together with L2 regularization, early stopping, and fixed random initialization, contributed to stable training behavior across the evaluated dataset sizes. Although a fixed 70–30 train–test split was adopted to ensure methodological consistency between all experiments and software environments, additional exploratory tests using alternative random partitions showed only minor variations in the global performance metrics. This suggests that the reported trends are primarily governed by dataset size and network architecture rather than by a particular data partition. Future extensions of the methodology could incorporate repeated random sub-sampling or k-fold cross-validation strategies to provide additional statistical characterization in low-data regimes.

Under this optimization framework, the Python implementation

achieved stable and reproducible convergence across all tested architectures and dataset sizes, ensuring that performance differences observed in the Results section arise from the experimental design—rather than uncontrolled variability in the training process.

3.3.3. Evaluation metrics

A comprehensive set of metrics was employed to quantitatively assess model performance on both training and testing datasets:

The primary optimization criterion utilized in this investigation is the quadratic loss function, commonly referred to as MSE. This metric quantifies prediction accuracy by computing the arithmetic mean of squared residuals between estimated and observed values. Its mathematical formulation is expressed in Eq. (3) as:

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2 \quad (3)$$

where N represents the total observation count in the evaluated dataset, y_i denotes the actual value of the target variable for observation i , and $\hat{y}_i$ indicates the corresponding model prediction. The quadratic nature of this function disproportionately penalises substantial deviations, making it particularly valuable for applications where large prediction errors carry significantly greater consequences than minor inaccuracies. Furthermore, the RMSE is the square root of the MSE, expressing the average prediction error in the same units as the target variable.

Beyond direct error measurements, our evaluation framework incorporates the Coefficient of Determination (R^2), a normalized statistical measure expressing the proportion of variance in the dependent variable successfully captured by the predictive model in Eq. (4):

$$R^2 = 1 - \frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{\sum_{i=1}^N (y_i - \bar{y})^2} \quad (4)$$

Where $\bar{y}$ represents the arithmetic mean of all actual values.

This metric generates values typically ranging from 0 to 1, with higher values indicating superior predictive performance. An R^2 value of 1 represents perfect prediction, while a value of 0 suggests the model performs no better than simply predicting the mean value for all observations. The scale-invariant nature of R^2 facilitates direct performance comparisons across different target variables and studies.

In addition, the Mean Absolute Error (MAE), in Eq.(5), was employed to provide a complementary measure of model performance:

$$MAE = \frac{1}{N} \sum_{i=1}^N |y_i - \hat{y}_i| \quad (5)$$

Where N is the total number of observations in the dataset, y_i represents the actual value of the target variable for observation i , and $\hat{y}_i$ corresponds to the predicted value generated by the model.

These definitions are common to both Python and SPSS implementations, ensuring a consistent basis for the comparative analysis presented in the Results section.

3.4. SPSS methodology

The modelling and evaluation procedure was independently reproduced using IBM SPSS Statistics (version 27), employing the Multilayer Perceptron (MLP) module to ensure methodological consistency with the Python-based approach. Unlike Python, SPSS provides an integrated environment in which data preprocessing, normalization, training, validation, and performance evaluation are internally managed, while still allowing the user to define network architecture, activation functions, and stopping criteria.

Following the same protocol adopted in the Python implementation, the dataset was randomly partitioned into two independent subsets: 70% of the data were assigned to training and 30% to testing, ensuring representative sampling in both subsets. Input variables were

automatically normalized by SPSS using z-score standardization, transforming each predictor to zero mean and unit variance. This normalization step guarantees numerical stability during training and prevents variables with larger magnitudes from dominating the learning process.

For a consistent methodological comparison, the same MLP architectures explored in Python were implemented in SPSS: 3-6-1, 3-9-1, 3-6-3-1, and 3-12-6-1. In all cases, the input layer consisted of the three selected physical parameters: frequency (f), compliance diameter (DC), and inductance diameter (DL), while the output layer contained a single neuron corresponding to the predicted thermoacoustic power. Both sigmoid and hyperbolic tangent (tanh) activation functions were tested in the hidden and output layers to assess their influence on predictive performance.

Network training was performed using the batch backpropagation algorithm, with synaptic weights initialized randomly under a fixed random seed to ensure reproducibility. The stopping criterion was defined as one consecutive iteration without a decrease in the Sum of Squared Errors (SSE), which acts as an early stopping mechanism and reduces the risk of overfitting. This rule halts training once further weight updates no longer yield measurable improvements in prediction accuracy.

3.4.1. Evaluation and metrics

SPSS directly reports the Sum of Squared Errors (SSE) and the Relative Error for both training and testing datasets. From these quantities, additional performance metrics were computed to enable direct comparison with the Python-based models. The Mean Squared Error (MSE) is defined by the Eq. (6):

$$MSE = \frac{SSE}{N} \quad (6)$$

where N denotes the number of observations in the evaluated subset. The Root Mean Squared Error (RMSE) is then obtained by the Eq. (7):

$$RMSE = \sqrt{MSE} \quad (7)$$

providing an error measure expressed in the same physical units as the target variable (power).

Model goodness-of-fit was further quantified using the coefficient of determination (R^2), computed from the SSE as shown in the Eq. (8):

$$R^2 = 1 - \frac{SSE}{\sum_{i=1}^N (y_i - \bar{y})^2} \quad (8)$$

where y_i represents the observed power values and $\bar{y}$ their mean. This formulation ensures consistency with the Python implementation and provides a normalized measure of how well the model explains the variance of the dependent variable.

During training, SPSS iteratively updates the synaptic weights through forward and backward propagation cycles, minimizing the SSE by adjusting the weights in the direction opposite to the error gradient. Convergence behavior and residual distributions were monitored to verify numerical stability and the absence of systematic prediction bias.

4. Data

The quality of the predictive model depends on the relevance and representativeness of the data used for its training and validation. This section describes how the dataset used in this study was generated and processed. DeltaEC, a tool widely used in the design of thermoacoustic engines [11], was used to obtain the simulations. The modelling of the TA-SLiCE engine and the definition of the input parameters are explained. The ranges of variation for these parameters were established based on the literature and a preliminary analysis that identified these factors as the most influential. A descriptive and exploratory analysis of the data obtained is then described, as well as the pre-processing

techniques applied. In addition, the procedures used for detecting and handling potential outliers in the simulation results are summarized, focusing on their impact on data quality prior to model training. This provides a consistent bridge between the numerical generation stage and the surrogate modelling framework described in Sections 2 and 3.

4.1. DeltaEC variables

The present study focuses specifically on operating conditions characterized by low drive ratios (typically <10%), where the linear thermoacoustic approximation remains valid and DeltaEC simulations can reliably capture the system behavior. The ability to explore 'what if' scenarios makes simulation-based design an invaluable approach for mapping the thermoacoustic design space before physical prototyping. In this context, DeltaEC was used to generate a synthetic dataset for training the surrogate neural network models described in Sections 2 and 3.

The simulations replicate a toroidal TASLiCE demonstrator (overall length 0.4 m, air at 100 kPa mean pressure) whose core branch and heat exchangers remain invariant. To analyze the effect of the feedback-branch geometry on acoustic power delivered to the power extraction branch, three primary design variables within the DeltaEC model were varied: the operating frequency (f), the compliance chamber diameter (DC), and the inductance tube diameter (DL), while keeping all other geometrical and operating values constant. Variations in the compliance and inductance dimensions strongly affect the acoustic impedance and phase relationships in the feedback loop, while the drive frequency dictates how closely the system operates to resonance, thereby controlling energy transfer efficiency [36].

Accordingly, a structured campaign was conducted. This involved three independent DeltaEC simulation sweeps. These were used to systematically explore the three-dimensional parametric space. This space is defined by operating frequency, compliance chamber diameter and inductance tube diameter. Fig. 6 illustrates the process of generating this simulation data in the form of a flow diagram, where each branch represents two parametric sweeps with one variable fixed, following the same parameter bounds and sampling strategy detailed in Section 2.1.

In the first sweep, DL was varied from 8 to 28 mm and f from 10 to 70 Hz, each in 76 increments, while DC was held constant at 65 mm, representing an intermediate compliance volume. The second sweep fixed DL at 18 mm and varied both f from 10 to 70 Hz and DC from 30 to 100 mm, again in 76 steps each. In the third sweep, f was fixed at 40 Hz, and the parameter space was explored across DC (30–100) mm and DL (8–28) mm.

The bounds shown in Fig. 6 were chosen based on a combination of literature guidance and practical design limits. Some thermoacoustic traveling-wave engines typically operate in the low-frequency range of ≈ 10 –70 Hz, with designs converging on compliance (acoustic spring) chambers ≈ 30 –100 mm in diameter and inductance (acoustic mass) tubes ≈ 8 –28 mm in diameter [37]. These ranges are well supported in both simulations and experiments. Notably, one high-efficiency looped engine achieved optimal phasing at ≈ 56 Hz by coupling a 26.5 mm inner-diameter inductance tube with an enlarged compliance chamber measuring approximately 102 mm in diameter [38]. Similarly, a multi-stage 5 kW thermoacoustic generator used ≈ 80 mm diameter heat-exchanger "compliance" sections together with long slim resonance tubes of ≈ 20 mm ID to tune a 70 Hz acoustic loop [39], while a more compact engine demonstrated a 40 mm main loop coupled via a 15 mm feedback tube for proper traveling-wave action [40].

Acoustically, the compliance and inductance elements form an LC network that does not add or remove energy but instead stores it per cycle, shifting the phase of pressure and flow oscillations [41]. The inductance tube induces a velocity lag (like an inductive reactance in an electric circuit) whereas the compliance volume causes a velocity lead (capacitive reactance), each approaching a $\approx 90^\circ$ phase difference between pressure and oscillatory volume flow [42]. These reactive effects

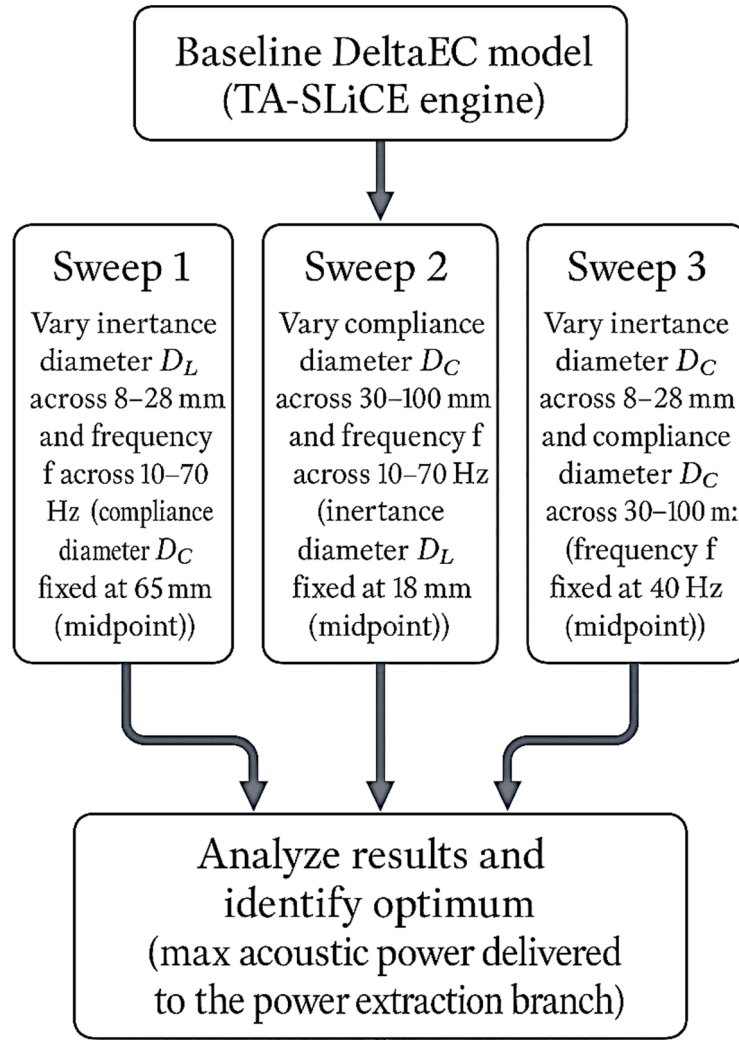


Fig. 6. Flowchart of the DeltaEC parametric study used to generate the training dataset. Starting from a baseline TA-SLiCE model, three sweeps are executed. The collected simulation data are then analyzed to identify the optimal design to obtain the maximum acoustic power delivered to the power extraction branch.

raise the acoustic impedance magnitude in the loop and create a circulating reactive power (associated with standing-wave energy exchange) in the feedback path. By adjusting the inertance and compliance dimensions, designers minimise the reactive power delivered to the engine's thermoacoustic core, so that the pressure and velocity at the regenerator are nearly in phase (a traveling-wave condition) and the active acoustic power output is maximized [43]. The nominal midpoint values shown in Fig. 6, lie near this regime.

By exploring the entire range from these limiting cases to the known optimum, the dataset captures how each variable influences the acoustic power output to the power extraction branch under both optimal and off-design conditions. This factorial design, comprising three dense two-dimensional grids, ensured comprehensive coverage of the parametric domain. After merging the datasets and eliminating duplicates at intersecting points, a total of 13,747 unique operating conditions were obtained. The resulting dataset captures the key dependencies that govern thermoacoustic performance at junction. The resulting dataset is fully consistent with the definition of inputs adopted in Section 2.1 and was subsequently passed to the preprocessing and modelling stages described in Sections 2 and 3.

4.2. Descriptive data analysis

4.2.1. Preparation and exploration of the data

The initial phase of the study consisted of acquiring and characterizing the dataset on thermoacoustic engines. The data was loaded from a CSV file, renaming the columns to facilitate their identification and subsequent manipulation.

An initial exploratory analysis was conducted to understand the distribution and characteristics of each variable. This process revealed that the target variable 'power' presents a pronounced concentration of samples around 3.09 W, indicating frequent operating conditions with similar acoustic output. This behavior results from the dense sampling of the parameter sweeps around mid-range values of, DC , f and DL , where DeltaEC predicts higher thermoacoustic conversion efficiency. Fig. 7 provides a histogram of the power distribution with a fitted density curve, helping visualise the overall behavior of the thermoacoustic engine within the sampled design space.

4.2.2. Correlation analysis and variable selection

To identify which variables had the greatest impact on generated power, the Pearson correlation matrix was calculated for all numerical variables in the dataset (Fig. 5). The results showed that all three main parameters, frequency (f), inertance diameter (DL), and compliance diameter (DC), showed moderate positive correlations with the

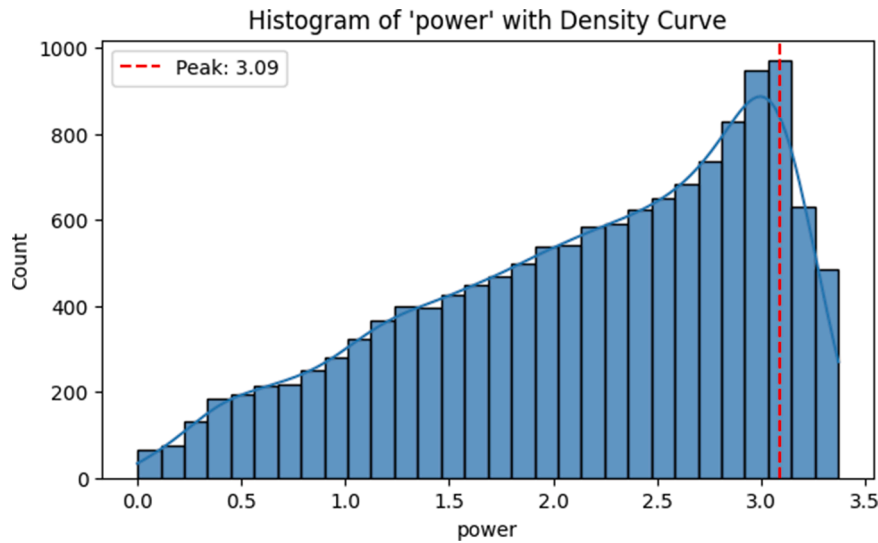


Fig. 7. Distribution of acoustic power values obtained from DeltaEC simulations. Histogram and fitted density curve showing concentration of samples around mid-range operating conditions, corresponding to higher thermoacoustic efficiency regions.

generated power. Specifically, frequency (f) presented the highest correlation with power ($r = 0.334460$), followed by compliance diameter (DC) ($r = 0.280200$) and inertia diameter (DL) ($r = 0.256867$). The complete correlation matrix between these variables is presented in Table 1.

The analysis in Fig. 8, guided the decision to select the three mentioned variables (f , DL and DC) as the main predictive features for developing neural network models, given their relationship level with the target variable. This is consistent with the feature-selection rationale presented earlier in Section 2.2, confirming that the selected parameters provide non-redundant and physically meaningful contributions.

5. Results

5.1. Python ANN

The initial exploratory analysis of the dataset, presented in the previous section, enabled a detailed characterization of the available information and its suitability for thermoacoustic power prediction. As a starting point for the modelling stage, a MLP framework was implemented to examine the influence of architectural complexity, activation function selection, and data availability on predictive performance.

All models were trained using SGD with backpropagation, a fixed random seed to ensure reproducibility, and a uniform 70–30 train–test split to maintain methodological consistency.

For each experimental configuration, the network was trained and evaluated on the independent test set. Model performance was quantified using metrics, including MSE, RMSE, MAE and R^2 , allowing a comparison of predictive capability across activation functions, architectures, and data scenarios.

In the limited data regime ($n = 130$), the sigmoid activation function demonstrated superior generalization capacity, achieving a coefficient of determination $R^2 = 0.9928$ with the minimal network architecture (3-6-1). This outperformed the best tanh configuration, which reached

$R^2 = 0.9845$ using the intermediate 3-9-1 architecture. The error metrics further confirmed this advantage, with sigmoid yielding a mean squared error of 0.000782 compared to 0.001693 for tanh, representing a 32% reduction in root mean squared error (0.028 vs. 0.041). This suggests that sigmoid's properties may provide implicit regularization benefits in low-data regimes, preventing overfitting despite limited training examples.

However, this performance relationship reversed when scaling to the medium dataset ($n = 1300$). In this scenario, the tanh activation function showed superior convergence, achieving $R^2 = 0.9995$ with the balanced 3-6-3-1 architecture, outperforming sigmoid's best configuration ($R^2 = 0.9775$). The difference in error metrics was even more pronounced, with tanh's mean squared error (0.000223) being approximately 48 times lower than sigmoid's error (0.010714). Both activation functions achieved optimal weight distributions using the 3-6-3-1 architecture in this data regime, indicating this architecture provides an optimal balance between network capacity for this problem domain.

The large dataset scenario ($n = 13,000$) reinforced tanh's advantages, maintaining $R^2 = 0.9992$ with the most complex architecture (3-12-6-1), compared to sigmoid's best performance of $R^2 = 0.9927$ using the 3-6-3-1 configuration. The error differential remained important, with tanh producing an MSE of 0.000546 versus 0.005108 for sigmoid. Consequently, tanh used the deeper hidden layer representation capacity of the 3-12-6-1 architecture, while sigmoid remained optimal with the balanced 3-6-3-1 structure, highlighting the reduced effectiveness of the sigmoid function when applied to larger network architectures.

Overall, the Python results show a clear interaction between dataset size, activation function and architectural depth: sigmoid excels when data are scarce, whereas tanh becomes progressively more advantageous as the amount of training data increases.

To complement the quantitative metrics reported above and provide insight into the optimization dynamics, Fig. 9 shows a representative learning curve corresponding to the 3-12-6-1 MLP trained on the large dataset ($n = 13,000$). In contrast to the production runs, where an early-stopping criterion was applied to terminate training upon stagnation of the test error, the curve in Fig. 9 corresponds to a full, unstopped training run extended to 889 epochs. This deliberate choice allows the long-term behavior of both errors to be examined: training and test RMSE decrease monotonically and remain closely overlapped throughout the entire optimization, with final values of 9.16×10^{-3} and 9.20×10^{-3} respectively. The negligible gap between the two curves, sustained well beyond the typical early-stopping point, confirms that the

Table 1
Correlation matrix between variables.

	f	DL	DC	Power
f	1.000000	0.176350	−0.149279	0.334460
DL	0.176350	1.000000	0.158446	0.256867
DC	−0.149279	0.158446	1.000000	0.280200
Power	0.334460	0.256867	0.280200	1.000000

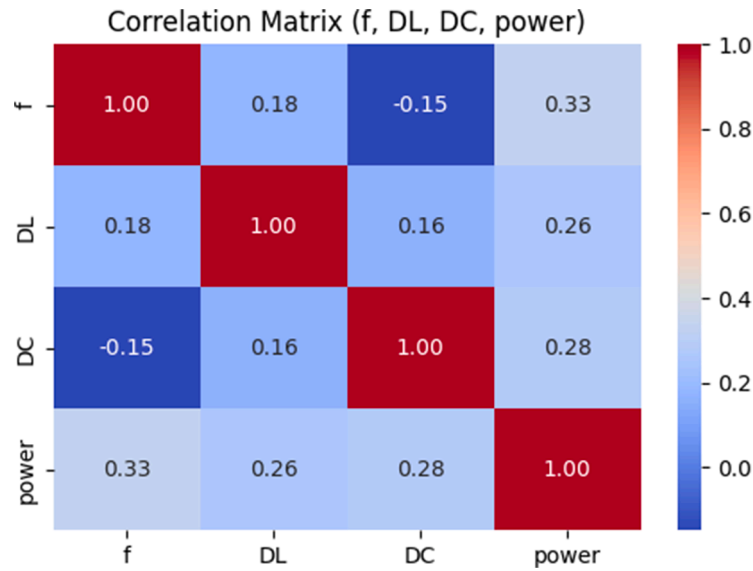


Fig. 8. Correlation heatmap between input variables (frequency f , compliance diameter DC , inertance diameter DL) and acoustic power. The plot highlights moderate positive correlations between the selected design variables and the predicted power output.

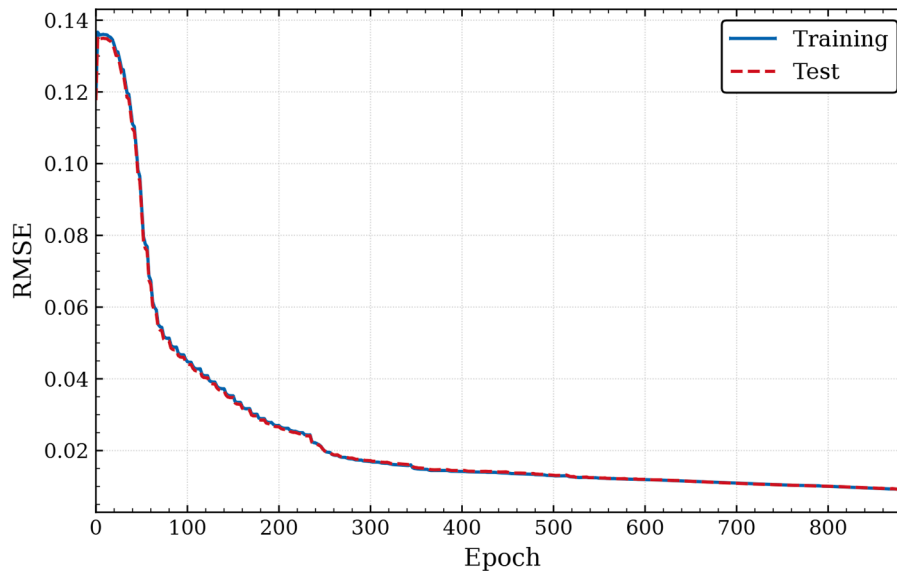


Fig. 9. Learning curve for the 3-12-6-1 MLP on the large dataset (n = 13,000): root-mean-square error (RMSE) on the training (solid blue) and test (dashed red) sets versus epoch. The figure shows a full, unstopped training run (early stopping deactivated) to expose the long-term behavior of both errors. The monotonic decrease and the negligible gap between curves confirm stable convergence and the absence of overfitting beyond the early-stopping point used in the main experiments.

selected architecture and training scheme do not induce overfitting even under prolonged optimization. Analogous convergence behavior was observed across the remaining configurations and is therefore omitted for brevity.

5.2. SPSS

In parallel with the Python-based analysis, multilayer perceptron (MLP) neural networks were developed using IBM SPSS Statistics to evaluate the influence of dataset size, network architecture, and activation function on predictive performance. To ensure comparability, three subsets containing 130, 1300, and 13,000 randomly selected observations were extracted from the full dataset. Each subset was divided into training (70%) and testing (30%) samples using random case assignment.

All SPSS models employed the same network architectures explored

in the Python study (3-6-1, 3-9-1, 3-12-6-1, and 3-6-3-1), with the three physical parameters frequency (f), compliance diameter (DC), and inertance diameter (DL) as input variables, and the generated acoustic power as the output. SPSS allows independent specification of activation functions for the hidden and output layers; therefore, both sigmoid and hyperbolic tangent ($\tanh$) functions were systematically evaluated.

Prior to training, the input variables were normalized internally by SPSS using standardization to zero mean and unit variance, ensuring numerical stability and balanced contribution of all predictors. Network training was performed using batch backpropagation with random weight initialization and a fixed random seed to guarantee reproducibility. The stopping criterion was defined as one consecutive iteration without a reduction in the sum of squared errors, allowing the learning process to terminate automatically once convergence was achieved.

The model's performance was first evaluated internally using SPSS, calculating the sum of squared errors (SSE) and relative error for both

the training and test samples. To maintain consistency with the Python-based evaluation, these results were then used to obtain the RMSE and R^2 values by applying the equations indicated in Section 3.4, allowing for a direct quantitative comparison between the two modelling environments.

The results obtained from the SPSS-trained neural networks reveal a clear improvement in predictive accuracy as the dataset size increases. For the intermediate (1300 samples) and large (13,000 samples) datasets, most architectures achieved coefficients of determination above 0.99 on the test set, indicating excellent generalization capability and stable convergence.

Regarding activation functions, SPSS models employing the sigmoid function in both the hidden and output layers generally outperformed those using tanh, particularly for medium and large datasets. These configurations consistently produced lower RMSE values and higher R^2 scores, reflecting a close agreement between predicted and experimental power values. In contrast, networks trained with the smallest dataset (130 samples) exhibited greater variability and sensitivity to activation function selection. In this low-data regime, tanh-based configurations showed signs of overfitting, characterized by strong training performance but reduced accuracy on the test set.

Overall, the SPSS results mirror the trends observed in the Python implementation, confirming the strong dependence of neural network performance on dataset size and activation function choice, while demonstrating that SPSS provides a robust and reliable framework for surrogate modelling of thermoacoustic power.

6. Discussion

In Table 2, the prediction accuracy of the models developed in Python and those built in SPSS is compared across three different data-size scenarios. With only 130 samples available, the neural network trained in Python (3-6-1, sigmoid activation) produced better results ($R^2 = 0.9928$, RMSE = 0.0280, MAE = 0.0196) than the best SPSS configuration ($R^2 = 0.9586$, RMSE = 0.1697, MAE = 0.435). The Python model achieved a lower error under limited data conditions.

With a medium-sized dataset (1300 samples), both tools improved. A tanh network (3-6-3-1) reached very high accuracy ($R^2 = 0.9995$, RMSE = 0.0149, MAE = 0.0106), while the best SPSS model (a sigmoid network 3-12-6-1), even using a deeper structure, still produced slightly larger errors ($R^2 = 0.9976$, RMSE = 0.0404, MAE = 0.118). These results indicate that the Python implementation achieves lower error magnitudes, both in terms of squared deviations (RMSE) and absolute deviations (MAE), reflecting a more accurate and stable approximation of the target variable.

Using the largest dataset (13,000 samples), both environments converged toward the same architecture (3-12-6-1). However, Python continued to perform slightly better ($R^2 = 0.9992$, RMSE = 0.0234, MAE = 0.0174 versus $R^2 = 0.9957$, RMSE = 0.0424, MAE = 0.211 in SPSS). This means the difference is not just in the network architecture, but also in how each platform performs its mathematical optimization.

Python handled small datasets more efficiently and preferred tanh activations as model complexity increased. SPSS, on the other hand,

consistently relied on sigmoid functions and required larger datasets to match Python's performance. Although both platforms identified the same deep network as optimal for large datasets, Python delivered more accurate predictions when using tanh activation, especially beyond the 1000 samples.

The fact that both platforms converge toward the same architecture when using the largest dataset suggests that, once the data volume is high enough, the influence of sample selection decreases. With broader and more balanced coverage of the parameter space, both approaches receive sufficient information to identify a similar structural solution. It becomes important to ensure representative coverage of the full operating domain in these models.

The results confirm that both Python and SPSS can produce highly accurate models for predicting power in thermoacoustic engines when sufficient data are available. However, Python demonstrates a slight advantage for small sample sizes, likely due to its finer control over regularization and learning-rate adjustment. Conversely, SPSS offers a robust and accessible framework, achieving comparable predictive performance with minimal configuration effort. These findings suggest that SPSS can be effectively employed for neural modelling in experimental energy systems, particularly when computational simplicity and reproducibility are prioritized over marginal precision gains.

6.1. Experimental validation

The DeltaEC model was configured to replicate the geometry of the experimental TA-SLiCE demonstrator reported by Iniesta et al. [10], and was employed to determine a suitable operating frequency (≈ 23 Hz) while estimating the expected acoustic power output across a range of design configurations.

The experimental configuration, including instrumentation and measurement methodology, has been previously described in detail in Iniesta et al. [44], where a dedicated low-cost sensor system for acoustic measurements was characterized. In that work, acoustic power is obtained from calibrated pressure measurements using piezoresistive sensors, and the associated uncertainty, mainly governed by sensor accuracy, phase estimation, and signal processing, was systematically evaluated.

DeltaEC predicted an acoustic power output of approximately 2.23 W at a frequency of approximately 22.7 Hz for a baseline configuration (compliance chamber diameter DC ≈ 63 mm and inertance tube diameter DL ≈ 15 mm). This closely matched the experimental performance of 2.03 W and guided the selection of 23 Hz as the operating frequency.

Building on these insights, the first Python validation focuses on a particular condition defined by DC = 63 mm, $f = 22.7$ Hz, and DL = 15 mm. For this setup, the experimentally measured acoustic power was 2.03 W, whereas the theoretical DeltaEC had a reference value of 2.23 W. A comparative analysis is presented in Table 3 and in Fig. 6. The reported experimental value corresponds to a well-characterized operating point of the demonstrator, obtained using the validated measurement framework described in [44]. It should be noted that the purpose of this experimental point is not to validate the system across multiple

Table 2
Comparative results of Python and SPSS.

Samples	Method	Architecture	Activation	MSE Test	RMSE Test	R2 Test	MAE Test
130	Python	3-6-1	Sigm	0.000782	0.027960	0.992831	0.019600
130	SPSS	3-6-1	Tanh	0.028821	0.169766	0.958629	0.435000
130	SPSS	3-6-3-1	Tanh	0.030872	0.175704	0.955685	0.468000
1300	Python	3-6-3-1	Tanh	0.000223	0.014929	0.999531	0.010616
1300	SPSS	3-12-6-1	Sigm	0.001636	0.040446	0.997658	0.118000
1300	SPSS	3-6-3-1	Sigm	0.001851	0.043027	0.997350	0.134000
13000	Python	3-12-6-1	Tanh	0.000546	0.023361	0.999216	0.017368
13000	SPSS	3-12-6-1	Sigm	0.002917	0.042463	0.995773	0.211000
13000	SPSS	3-6-3-1	Sigm	0.002348	0.044374	0.996598	0.170000

Table 3

Results of the different models for the validation point with Python.

Samples	Architecture	Activation function	Predicted power (W)	% DeltaEC error	% Experimental error
130	3-9-1	Tanh	2.2924	2.80	12.93 %
130	3-6-1	Sigm	2.2951	2.92	13.06 %
1300	3-6-1	Sigm	2.0340	-8.79	0.20 %
1300	3-12-6-1	Sigm	2.0886	-6.34	2.89 %
13000	3-6-1	Sigm	2.0163	-9.58	-0.67 %
13000	3-12-6-1	Sigm	2.1035	-5.67	3.62 %

operating regimes, but to provide a point-wise confirmation of the computational framework. In this work, DeltaEC is adopted as a physics-based digital twin, used to generate the dataset for training the surrogate model. The experimental demonstrator operates at a single stable condition, and the agreement between experimental and simulated results confirms the physical consistency of the DeltaEC model. This additional reference is incorporated to reinforce the reliability of the computational approach, while the generalization capability of the surrogate model is governed by the parametric coverage of the simulation dataset. The comparison across different dataset sizes in Fig. 10 (left), between experimental data, predicted results, and DeltaEC reference values, provides the foundation for validating the models and assessing their consistency with real physical behavior.

For the smallest dataset with 130 samples, the configurations using tanh and sigmoid activation functions show higher experimental errors, ranging from 12.93% to 13.06%, compared with those trained on larger dataset.

When the dataset increases to 1300 samples, the accuracy of the models improves notably. The 3-6-1 sigmoid configuration reaches an experimental error of 0.20%, while the 3-12-6-1 sigmoid network achieves 2.89%. These results indicate that using a larger dataset improves the model's capability to reproduce the experimental behavior of the thermoacoustic system with greater precision.

For the largest dataset with 13,000 samples, the models continue to perform with high precision. The 3-6-1 sigmoid model presents an experimental deviation of 0.67%, and the 3-12-6-1 sigmoid configuration attains 3.62%, both showing close agreement with the experimental and theoretical DeltaEC reference values.

The models developed in Python reveal low prediction errors, remaining below 4% for the largest dataset.

The Fig. 10 shows that the best performing model was the 3-6-1 architecture with sigmoid activation, which had an error of 0.20% compared to the experimental value of 2.03 W. All models fell within the range between of 0.2%- 13.06-% experimental error, as appears in

Table 3.

On the other hand, the validation of neural networks trained with SPSS is based on a specific operating condition defined by $f = 22.7$ Hz, $DC = 63$ mm, and $DL = 15$ mm. Under these parameters, the experimentally measured acoustic power reached a value of 2.03 W, while the theoretical value predicted by the DeltaEC model was 2.23 W. The comparative analysis presented in Table 4 and in the Fig. 11, which contrast the experimental measurements, the results predicted by SPSS and the reference values of the DeltaEC model, forms the basis for validating the neural models obtained with SPSS and confirming their consistency with the actual thermoacoustic behavior of the system.

The comparison of predictions shown in Table 4 shows the general agreement between the power values predicted by the neural networks generated with SPSS and the value obtained through simulation (DeltaEC) and in the experimental test of the thermoacoustic engine. In all cases, the models present very low prediction errors, confirming the ability of MLP architectures to capture the non-linear relationships between the physical input parameters (f , D C and DL) and the acoustic power generated.

For the smallest sample (130 data points), the 3-6-3-1 network with the tanh activation function shows anomalous behavior compared to the other networks, although we had already intuited the overtraining of this network, which would explain an experimental error percentage of 14.36% when the other networks remain below 6.24%. Similarly, only this network overestimates the predicted power by 4.10%, while the rest of the networks estimate below the simulated value with DeltaEC. Except for the 3-6-3-1 network with a tanh activation function in both the hidden layer and the output layer for a sample of 130 data points, the rest of the networks, regardless of the number of data points in the sample, have prediction error percentages of <10% compared to the value simulated with DeltaEC and the experimental value. These results reflect the greater stability and generalization of the model as the number of observations increases, demonstrating that with large data volumes, SPSS converges towards an accurate and reproducible representation of the thermoacoustic power generation process.

In summary, the differences between the predicted, simulated and experimental values remain within very narrow margins, <10%, confirming that the neural networks developed in SPSS can faithfully reproduce both the theoretical and empirical behavior of the system.

6.1.1. Comparison between SPSS and Python configurations

The analysis of the best-performing SPSS and Python configurations is presented in Table 5 and in Fig. 12. It shows different performance trends across the various dataset sizes.

In Fig. 12, it is shown that while SPSS performs more reliably with

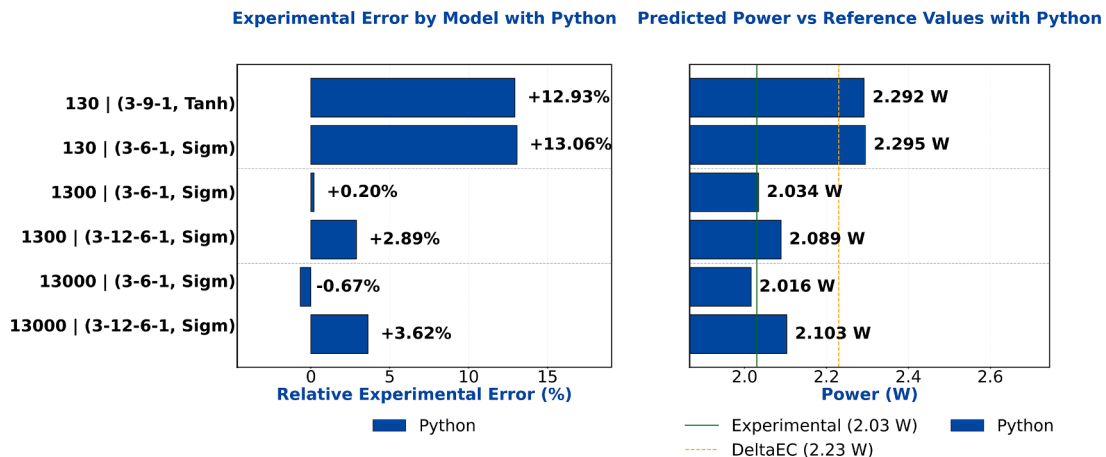


Fig. 10. Predicted power vs reference values with Python ($f = 22.7$ Hz, $DC = 63$ mm, $DL = 15$ mm) (left), and relative Experimental Error by model with Python (right).

Table 4

Results of the different models for the validation point with SPSS.

Samples	Architecture	Activation HL	Activation OL	Predicted Power	DeltaEC error %	Experimental error %
130	3-6-1	Tanh	Tanh	2.156160	−3.31	6.21
130	3-6-3-1	Tanh	Tanh	2.321470	4.10	14.36
1300	3-12-6-1	Sigm	Sigm	2.131634	−4.41	5.01
1300	3-6-3-1	Sigm	Sigm	2.152493	−3.48	6.03
13000	3-12-6-1	Sigm	Sigm	2.156595	−3.29	6.24
13000	3-6-3-1	Sigm	Sigm	2.149180	−3.62	5.87

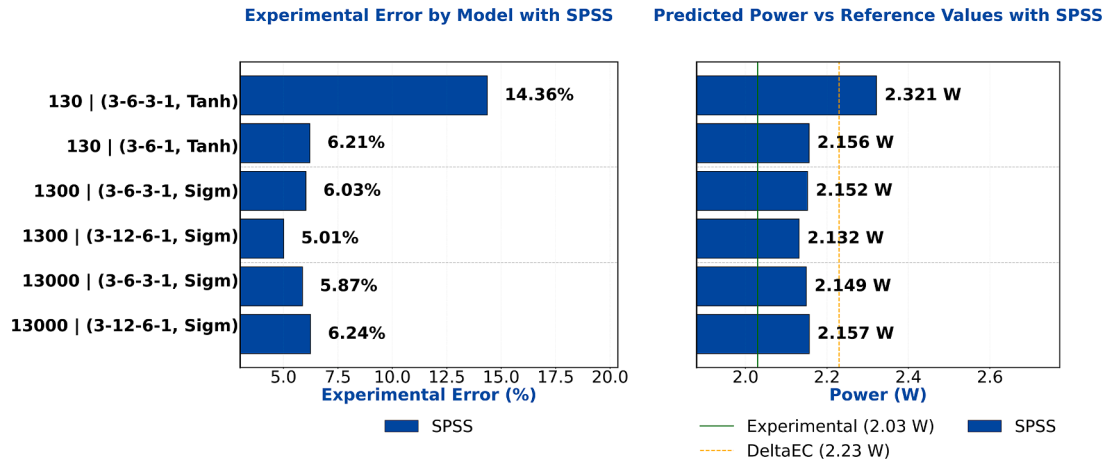


Fig. 11. Predicted power vs reference values with SPSS (left), and experimental error by model with SPSS (right).

Table 5

Comparison between SPSS and Python results for different neural network architectures under the point of validation (DC = 63 mm, $f = 22.7$ Hz, DL = 15 mm).

Sample	Architecture	Activation	SPSS Error (%)	Python Error (%)	SPSS Power (W)	Python Power (W)
130	3-6-1	Tanh	6.21	—	2.156	—
130	3-6-3-1	Tanh	14.36	—	2.321	—
130	3-6-1	Sigm	—	13.06	—	2.295
130	3-9-1	Tanh	—	12.93	—	2.292
1300	3-6-1	Sigm	—	0.20	—	2.034
1300	3-12-6-1	Sigm	5.01	2.89	2.132	2.089
1300	3-6-3-1	Sigm	6.03	—	2.152	—
13000	3-6-1	Sigm	—	−0.67	—	2.016
13000	3-12-6-1	Sigm	6.24	3.62	2.157	2.104
13000	3-6-3-1	Sigm	5.87	—	2.149	—

small datasets, Python gains accuracy as the amount of data increases, showing stronger similarity with experimental observations.

For the smallest dataset, comprising 130 samples, the SPSS models show experimental errors between 6.21% and 14.36%, while the Python models under the same conditions showed higher errors, ranging from 12.93% to 13.06%. These results indicate that SPSS provides more precise estimations when only a limited number of data points are available.

When the dataset size increases to 1300 samples, the performance gap between both methods narrows. The SPSS model with a 3-12-6-1 sigmoid architecture achieved an experimental error of 5.01%, whereas the equivalent Python model reached 2.89%. This suggests that as the amount of data grows, Python's predictive accuracy improves notably.

For the largest dataset of 13,000 samples, Python slightly outperforms SPSS. The Python model with a 3-6-1 sigmoid configuration achieved an experimental error of −0.67%, compared to 5.87% obtained by the SPSS 3-6-3-1 sigmoid model.

The results in Table 5 confirm how Python models become more stable with increasing data volume, taking advantage of a larger and more representative dataset.

6.2. Learning curve analysis for different dataset volume

To determine the optimal dataset size for thermoacoustic power prediction, a learning curve analysis, presented in Fig. 13, was conducted using the highest-performing model configuration. Based on previous experimental results, the 3-12-6-1 architecture with hyperbolic tangent (tanh) activation function was selected as the reference model for this analysis due to its superior generalization capabilities.

The training set size was varied from small subsets to the complete available training data, with performance metrics computed at each increment to establish the relationship between training sample quantity and predictive accuracy. For each evaluated training fraction (10%, 20%, etc.), only that portion of the training set was used to fit the model and compute the corresponding performance metrics. For the 100% case, the full training dataset was employed to determine the maximum achievable performance.

The learning curve analysis showed three different performance regions that characterise the model's learning dynamics:

Initial zone (0–2000 samples): The initial learning phase showed accelerated performance gains, with the coefficient of determination (R^2) showing fast improvement as the training set. This phase demonstrates the model's efficient pattern extraction capabilities even with limited training examples.

Medium improvement zone (2000–6000 samples): In this intermediate region, the model achieved R^2 values exceeding 0.98, with the mean squared error decreasing considerably.

Final zone (>6000 samples): Beyond a 6000–8000 training examples, performance metrics stabilized, with R^2 values converging asymptotically toward 0.99. After this point, adding more training data offered minimal benefit, with almost no improvement in accuracy and small variations.

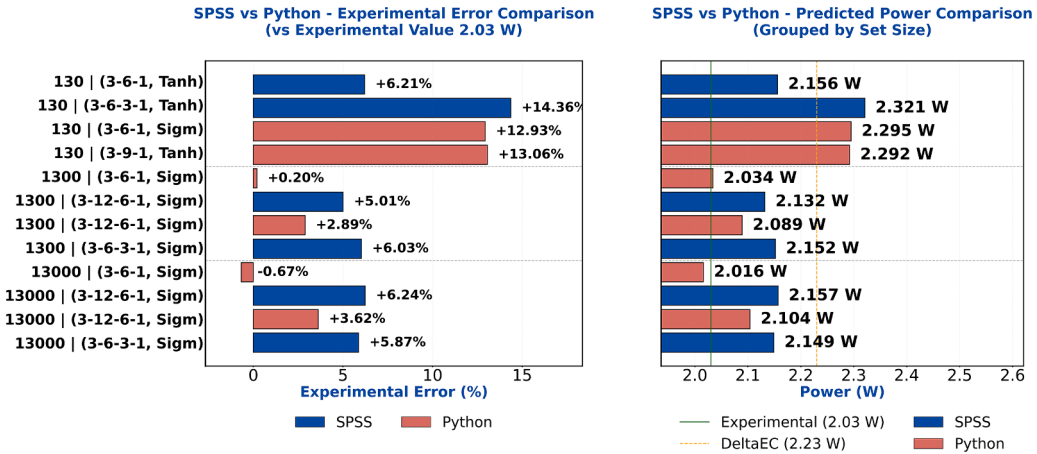


Fig. 12. Experimental error comparison (SPSS vs Python) left), and predictive power comparison (SPSS vs Python) (right).

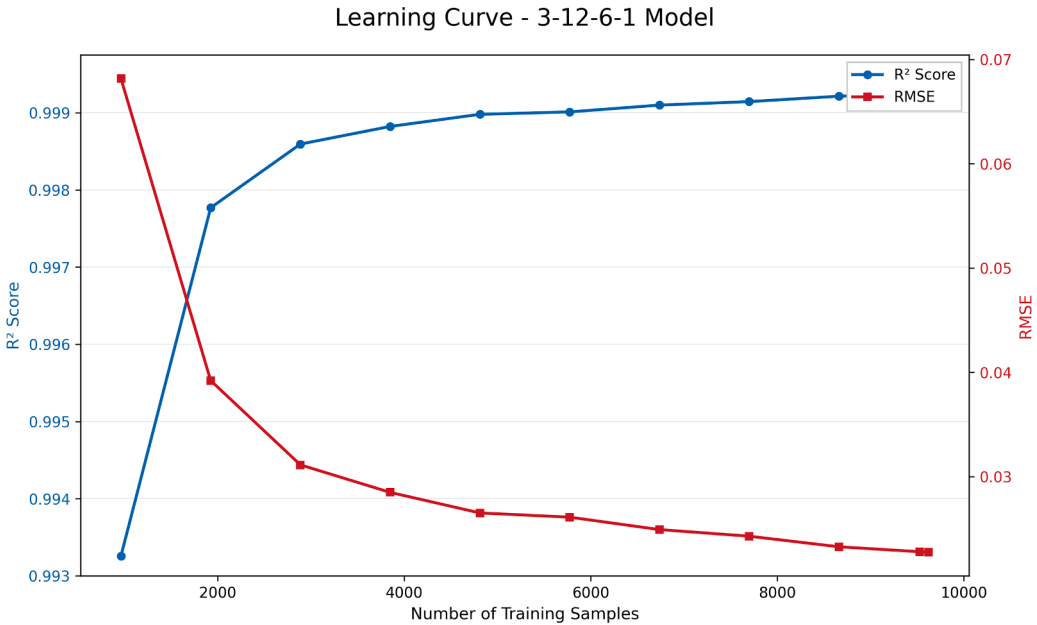


Fig. 13. Learning curve showing model performance as a function of training dataset size for the 3-12-6-1 architecture. The coefficient of determination (R^2) increases rapidly for small datasets and stabilizes above 0.99 for large sample sizes.

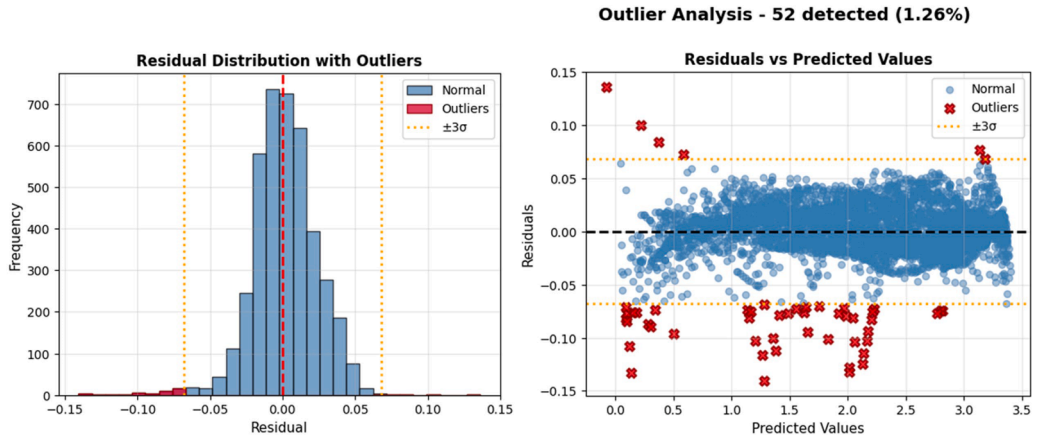


Fig. 14. Residuals distribution (3-12-6-1) (left), and residual distribution vs predicted power (3-12-6-1)(right).

6.3. Tolerance of the predictive model to perturbations

As part of advanced data processing, a procedure was implemented for outlier detection based on residuals from a MLP model with the (3-12-6-1) architecture. This regression model was trained on the training set and prediction errors were calculated. Using the statistical criterion of three standard deviations (3σ), observations whose residuals exceeded this threshold were identified as outliers.

This analysis revealed the presence of 52 potential outliers among the 4124 observations in the test set, corresponding to approximately 1.26% of that subset.

The residuals distribution histogram, Fig. 14 (left), shows that most residuals are tightly concentrated around zero, forming a nearly Gaussian, symmetric distribution that rapidly decreases toward both sides.

In the analysis of the residuals versus predicted values plot, Fig. 14 (right) shows how errors behave as a function of predicted values. In the central region, which comprises the most of observations, the residuals are distributed symmetrically around zero, indicating a good fit of the model for the main range of predictions. This characteristic indicates that the neural network successfully captures the fundamental structure of the relationship between the design parameters and the power generated by the thermoacoustic engine.

Minor deviations can be observed near the prediction boundaries, where a limited number of cases show slight underestimations at lower predicted values and modest overestimations at the upper end of the scale. These edge variations are likely associated with nonlinear effects emerging at the operational limits of the system or with the presence of physical noise and measurement uncertainty inherent to the experimental data.

In general, the residual distribution demonstrates that the neural network maintains strong predictive precision, minimal bias, and consistent generalization capability, capturing the fundamental relationship between the thermoacoustic parameters and the corresponding power output.

Removing the outliers doesn't affect the model's performance. Since they represent only 1.24% of the data, their contribution to the training process is minimal, and the optimization is dominated by the remaining 98.76% of samples. In addition, the training setup limits the influence of occasional noisy points. As a result, the metrics obtained with and without outliers are almost identical, confirming that the model's predictive accuracy is not sensitive to their presence.

The negligible impact of these outliers on the overall model performance may be explained by two factors. The first, and most straightforward, is that their proportion is extremely small compared to the total dataset, making their statistical influence insignificant during training. Secondly, the occurrence of these points is likely related to how far certain parameter combinations deviated from the internal Runge–Kutta adjustment executed by DeltaEC. The software treats the thermoacoustic device as a one-dimensional sequence of connected segments and integrates the full set of governing equations in each of them using a fourth-order Runge–Kutta scheme combined with a guess–target strategy. This procedure requires exact compatibility between approximation parameters (G) and target conditions (T) for the solver to converge. When the parameter space locally induces incompatible acoustic impedances or phasing conditions, the algorithm may temporarily struggle to satisfy the target constraints, producing isolated residual deviations that manifest as outliers. Since these situations represent narrow numerical inconsistencies rather than systematic physical trends—and given that DeltaEC ensures convergence tolerances below 10^{-8} in all accepted solutions—such points do not compromise the reliability of the dataset used for training. A more detailed investigation of the origin of these deviations, and their relationship to the internal solver dynamics, lies beyond the scope of the present work and is left for future studies.

Although the present application does not involve sensitive personal or societal data, reliability and interpretability remain important

considerations when surrogate models are intended to support engineering design decisions. In the proposed framework, the surrogate is trained exclusively on physically converged DeltaEC simulations within a bounded parametric domain, which helps preserve consistency with the underlying thermoacoustic behavior. For larger or more complex thermoacoustic systems involving additional geometric or operational variables, the same methodology could be extended through higher-dimensional datasets and more advanced surrogate architectures. In such cases, communicating prediction uncertainty and defining valid operating ranges would become increasingly important to ensure robust integration into design and optimization workflows.

By integrating parametric simulation, systematic data analysis, and experimental validation, this work contributes a structured and reproducible framework for the design optimization of thermoacoustic engines based on machine learning. The study demonstrates that physics-guided, data-driven surrogate models based on multilayer neural networks can accurately reproduce DeltaEC predictions within the explored operating domain while substantially accelerating thermoacoustic design-space exploration. These results support the viability of ANN-based surrogate methodologies as practical tools for future thermoacoustic optimization and engineering-oriented design workflows.

Furthermore, the computational cost of surrogate models based on artificial neural networks (ANNs), whether implemented in Python or SPSS, is significantly lower than that of the DeltaEC model. Given the highly coupled and multi-parameter nature of TA-SLiCE systems, DeltaEC-based approaches for the analysis and design of thermoacoustic motors typically require iterative procedures to ensure convergence and numerical stability, resulting in a computational cost of several minutes, particularly in exhaustive parametric studies. In contrast, the artificial neural network based model configurations described in Table 4, both those obtained using Python and those developed using SPSS, are capable of providing accurate predictions in tenths of a second, enabling rapid exploration of the design space.

7. Conclusions

Thermoacoustic engines present an inherent modelling challenge because several boundary conditions are not externally imposed but instead arise from the coupled acoustic–thermodynamic dynamics of the system. Consequently, perturbing a single design variable rarely produces an isolated effect, as other parameters adjust to preserve global self-consistency. This complexity motivates the methodological contribution of this work, which quantifies the influence of key geometric and operational variables on the acoustic power delivered to the extraction branch while maintaining the remaining system behavior fixed.

Artificial neural networks were developed to predict the thermoacoustic power output from three physical parameters identified as the most influential: frequency (f), inertance diameter (DL), and compliance diameter (DC). These covariates were selected based on a preliminary statistical analysis that revealed significant positive correlations with power ($r = 0.334, 0.280, \text{ and } 0.257$, respectively; $p < 0.01$), supported by thermoacoustic theory and by DeltaEC sensitivity trends. Together, they capture the dominant physical mechanisms governing thermoacoustic performance at low drive ratios, making them suitable predictors for nonlinear surrogate modelling.

Across all experimental configurations, both SPSS-based and Python-based multilayer perceptron models achieved very high coefficients of determination, indicating that neural-network approaches are highly effective for thermoacoustic power prediction. Despite methodological differences, both modelling frameworks reproduced DeltaEC reference outputs with high fidelity, particularly when trained on sufficiently large datasets.

The comparative analysis shows a clear trend: predictive accuracy improves substantially as the training dataset increases. For medium (1300) and large (13,000) sample sizes, deviations from the experimental reference remained below 6% for SPSS and even lower for

Python, confirming strong generalization capacity. When data availability is limited (130 samples), SPSS produced more stable predictions, whereas Python achieved superior accuracy for larger datasets. The higher variability observed in the smallest dataset reflects the sensitivity of both tools to underrepresented sampling regimes.

It is important to clarify the scope of generalization of the surrogate model developed in this work. The trained ANN is designed to interpolate within the parametric space defined by the selected input variables (f , DC , and DL), while all other geometric and thermophysical parameters remain fixed. Within this domain, the model demonstrates strong predictive capability, accurately reproducing both simulated and experimental results across a wide range of operating conditions.

However, the current formulation does not directly generalize to fundamentally different TA-SLiCE geometries or configurations that fall outside the training space. Extending the model to achieve cross-geometry generalization would require the inclusion of additional input descriptors, such as resonator length, regenerator characteristics, and heat exchanger properties, as well as the generation of a significantly expanded dataset. This represents a natural extension of the present work and a promising direction for future research toward fully generalizable thermoacoustic surrogate models.

Finally, the neural networks developed in this work constitute a practical and effective substitute modelling tool for the thermoacoustic engine under study. Their application enables the intrinsic non-linearity of the system to be captured, significantly reducing simulation or calculation times compared to complex numerical or acoustic models, enabling rapid and efficient exploration of a wide design space. Once trained, the network facilitates sensitivity analysis and optimization, as it can quickly assess how variations in parameters affect the acoustic power of the device. Furthermore, the models obtained have a generalization capacity that allows them to predict the performance of new configurations, varying the dimensions of the design parameters within the range.

From a practical perspective, the proposed ANN framework is primarily intended as a fast surrogate tool for offline design exploration and parametric optimization within the validated operating range of the TA-SLiCE configuration considered in this work. Although the present formulation is not designed for real-time control applications, since the main input variables correspond to quasi-static geometric and operating parameters rather than dynamically actuated control variables, the methodology could be integrated in future optimization workflows or design-space exploration strategies aimed at accelerating thermoacoustic system development.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work the authors used ChatGPT to improve language translation. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

Funding

This research is part of the project PID2022-138700OB-I00 funded by MICIU/AEI /10.13039/501100011033 and FEDER, UE.

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Ángela Alonso-Solórzano: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Ana I. Velasco-Fernández:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Carmen Iniesta:** Writing – review & editing, Writing – original

draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **José Luis Olazagoitia:** Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Funding acquisition, Formal analysis, Conceptualization. **Jordi Vinolas:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Investigation, Funding acquisition.

Declaration of competing interest

The authors declare no conflict of interest.

Data availability

Data will be made available on request.

References

- [1] Swift GW. *Thermoacoustics: a unifying perspective for some engines and refrigerators*. 2nd ed. Cham, Switzerland: ASA Press/Springer.; 2017.
- [2] Backhaus S, Swift GW. A thermoacoustic-stirling heat engine: detailed study. *J Acoust Soc Am* 2000;107:3148–66. <https://doi.org/10.1121/1.429343>.
- [3] Kang H, Cheng P, Yu Z, Zheng H. A two-stage traveling-wave thermoacoustic electric generator with loudspeakers as alternators. *Appl Energy* 2015;137:9–17. <https://doi.org/10.1016/j.apenergy.2014.09.090>.
- [4] Napolitano M, Dragonetti R, Romano R. A method to optimize the regenerator parameters of a thermoacoustic engine. *Energy procedia*. Elsevier Ltd; 2017. p. 525–32. <https://doi.org/10.1016/j.egypro.2017.08.244>.
- [5] Timmer MAG, de Blok K, van der Meer TH. Review on the conversion of thermoacoustic power into electricity. *J Acoust Soc Am* 2018;143:841–57. <https://doi.org/10.1121/1.5023395>.
- [6] Ceperley PH. A pistonless Stirling engine—the traveling wave heat engine. *J Acoust Soc Am* 1979;66:1508–13. <https://doi.org/10.1121/1.383505>.
- [7] Backhaus S, Swift G. A thermoacoustic Stirling heat engine. *Nature* 1999;399:335–8. <https://doi.org/10.1038/20624>.
- [8] Zare S, Tavakolpour-Saleh AR, Aghahosseini A, Mirshekari R. Thermoacoustic Stirling engines: a review. *Int J Green Energy* 2023;20:89–111. <https://doi.org/10.1080/15435075.2021.2021414>.
- [9] De Waele ATAM. Basic treatment of onset conditions and transient effects in thermoacoustic Stirling engines. *J Sound Vib* 2009;325:974–88. <https://doi.org/10.1016/j.jsv.2009.03.043>.
- [10] Iniesta C, Olazagoitia JL, Vinolas J, Gros J. Assessing the performance of design variations of a thermoacoustic stirling engine combining laboratory tests and model results. *Machines* 2022;10:958. <https://doi.org/10.3390/machines10100958>.
- [11] Ward B, Clark J, Swift G. Design environment for low-amplitude thermoacoustic energy conversion deltaec version 6.4b2.7. users guide. Los Alamos (NM): Los Alamos National Laboratory; 2017.
- [12] Huntingford F, Kisha W. Algorithmic optimisation of the electrical power output of a low-cost, multicore thermoacoustic engine with varying resonator pressure. *Sustain Energy Technol Assess* 2022;49:101776. <https://doi.org/10.1016/j.seta.2021.101776>.
- [13] Ramadan IA, Bailliet H, Poignand G, Gardner D. Design, manufacturing and testing of a compact thermoacoustic refrigerator. *Appl Therm Eng* 2021;189:116705. <https://doi.org/10.1016/j.applthermaleng.2021.116705>.
- [14] Ward WC, Swift GW. Design environment for low-amplitude thermoacoustic engines. *J Acoust Soc Am* 1994;95:3671–2. <https://doi.org/10.1121/1.409938>.
- [15] Rott N. Damped and thermally driven acoustic oscillations in wide and narrow tubes. *J Appl Math Phys* 1969;20:230–43.
- [16] Murti P, Setiawan I, Rosafira JZ, Widyapara A, Astuti WD, Biwa T. Analysis of a standing wave thermoacoustic engine with multiple unit stages. *Int J Renew Energy Develop* 2024;13:708–15. <https://doi.org/10.61435/ijred.2024.60098>.
- [17] Tijani MEH, Lycklama à Nijeholt JA, Spoelstra S. Amplifying the power density in thermoacoustic systems using a spring component. *J Acoust Soc Am* 2024;156:202–13. <https://doi.org/10.1121/10.0026478>.
- [18] Kisha W, Riley P, McKechnie J, Hann D. Asymmetrically heated multi-stage travelling-wave thermoacoustic electricity generator. *Energy* 2021;235:121312. <https://doi.org/10.1016/j.energy.2021.121312>.
- [19] Iniesta C, Olazagoitia JL, Vinolas J, Gros J. Energy and efficiency evaluation of feedback branch design in thermoacoustic stirling-like engines. *Energies (Basel)* 2019;12:3867. <https://doi.org/10.3390/en12203867>.
- [20] Iniesta C, Olazagoitia JL, Gros J, Vinolas J, Aranceta J. Introduction to thermoacoustic Stirling engines: first steps in foundations and praxis. 1st editor. Navarra (Spain): Thomson Reuters; 2020. <https://www.thomsonreuters.es/es/tienda/duo-papel-ebook/introduction-to-thermoacoustic-stirling-engines-first-steps-in-foundations-and-praxisduo/p/10016569>.
- [21] Mumith JA, Karayiannis T, Makatsoris C. Design and optimization of a thermoacoustic heat engine using reinforcement learning. *Int J Low-Carbon Technol* 2016;11:431–9. <https://doi.org/10.1093/ijlct/ctv023>.

- [22] Tartibu LK, Sun B, Kaunda MAE. Multi-objective optimization of the stack of a thermoacoustic engine using GAMS. *Appl Soft Comput J* 2015;28:30–43. <https://doi.org/10.1016/j.asoc.2014.11.055>.
- [23] Iniasta C, Olazagoitia JL, Vinolas J, Aranceta J. Review of travelling-wave thermoacoustic electric-generator technology. *Proc Instit Mech Eng Part A: J Power Energy* 2018;232:940–57. <https://doi.org/10.1177/0957650918760627>.
- [24] Ngcukayitobi M, Tartibu LK, Bannwart FC. Performance analysis of A travelling-wave thermo-Acoustic engine using artificial neural network. In: ASME International Mechanical Engineering Congress and Exposition. 11. Proceedings (IMECE); 2021. <https://doi.org/10.1115/IMECE2021-70529>.
- [25] Ngcukayitobi M, Tartibu LK, Bannwart F. Enhancing thermo-acoustic waste heat recovery through machine learning: a comparative analysis of artificial neural network–Particle swarm optimization. *Adapt Neuro Fuzzy Infer Syst Artif Neural Netw Models AI (Switzerland)* 2024;5:237–58. <https://doi.org/10.3390/ai5010013>.
- [26] Zare S, Pourfayaz F, Tavakolpour-Saleh AR, Lashaki RA. A design method based on neural network to predict thermoacoustic Stirling engine parameters: experimental and theoretical assessment. *Energy* 2024;309:133113. <https://doi.org/10.1016/j.energy.2024.133113>.
- [27] Alamir MA. An artificial neural network model for predicting the performance of thermoacoustic refrigerators. *Int J Heat Mass Transf* 2021;164:120551. <https://doi.org/10.1016/j.ijheatmasstransfer.2020.120551>.
- [28] Machesa M, Tartibu L, Okwu M. Prediction of the oscillatory heat transfer coefficient in thermoacoustic refrigerators. *Sustainability (Switzerland)* 2021;13:9509. <https://doi.org/10.3390/su13179509>.
- [29] Yang F, Cho H, Zhang H, Zhang J, Wu Y. Artificial neural network (ANN) based prediction and optimization of an organic rankine cycle (ORC) for diesel engine waste heat recovery. *Energy Convers Manag* 2018;164:15–26. <https://doi.org/10.1016/j.enconman.2018.02.062>.
- [30] Deka MJ, Kalita P, Das D, Kamble AD, Jyoti Bora B, Sharma P, Jyoti Medhi B. An approach towards building robust neural networks models using multilayer perceptron through experimentation on different photovoltaic thermal systems. *Energy Convers Manag* 2023;292:117395. <https://doi.org/10.1016/j.enconman.2023.117395>.
- [31] El-Kenawy ESM, Ibrahim A, Mirjalili S, Zhang YD, Elnazer S, Zaki RM. Optimized ensemble algorithm for predicting metamaterial antenna parameters. *Comp Mater Continua* 2022;71:4989–5003. <https://doi.org/10.32604/cmc.2022.023884>.
- [32] Alhussan AA, Abdelhamid AA, El-Kenawy ESM, Ibrahim A, Eid MM, Khafaga DS, Ahmed AE. A binary waterwheel plant optimization algorithm for feature selection. *IEEE Access* 2023;11:94227–51. <https://doi.org/10.1109/ACCESS.2023.3312022>.
- [33] Rahman AA, Zhang X. Prediction of acoustic wave parameters of thermoacoustic prime mover through artificial neural network technique: practical approach for thermoacoustics. *Therm Sci Eng Progr* 2018;8:257–68. <https://doi.org/10.1016/j.tsep.2018.09.002>.
- [34] Frank M, Drikakis D, Charissis V. Machine-learning methods for computational science and engineering. *Computation* 2020;8:15. <https://doi.org/10.3390/computation8010015>.
- [35] M. Louis Croci Supervisor, M.P. Juniper Advisor, S. Cant, P. College, Data assimilation into physics-based thermoacoustic models using bayesian neural network ensembles, 2022.
- [36] Hamood A, Jaworski AJ, Mao X. Development and assessment of two-stage thermoacoustic electricity generator. *Energies (Basel)* 2019;12:1790. <https://doi.org/10.3390/en12091790>.
- [37] Abdoulla-Latiwish KOA, Mao X, Jaworski AJ. Thermoacoustic micro-electricity generator for rural dwellings in developing countries driven by waste heat from cooking activities. *Energy* 2017;134:1107–20. <https://doi.org/10.1016/j.energy.2017.05.029>.
- [38] Hamood A, Jaworski AJ, Mao X, Simpson K. Design and construction of a two-stage thermoacoustic electricity generator with push-pull linear alternator. *Energy* 2018;144:61–72. <https://doi.org/10.1016/j.energy.2017.11.148>.
- [39] Bi T, Wu Z, Zhang L, Yu G, Luo E, Dai W. Development of a 5kW traveling-wave thermoacoustic electric generator. *Appl Energy* 2017;185:1355–61. <https://doi.org/10.1016/j.apenergy.2015.12.034>.
- [40] Biwa T, Watanabe T, Penelet G. Flywheel-based traveling-wave thermoacoustic engine. *Appl Phys Lett* 2020;117:243902. <https://doi.org/10.1063/5.0022315>.
- [41] Iniasta C, Olazagoitia JL, Gros J, Vinolas J, Aranceta J. Introduction to thermoacoustic Stirling engines: first steps and praxis. In: E3S Web of Conferences. 313; 2021, 08005. <https://doi.org/10.1051/e3sconf/202131308005>.
- [42] Yu Z, Jaworski AJ, Backhaus S. Travelling-wave thermoacoustic electricity generator using an ultra-compliant alternator for utilization of low-grade thermal energy. *Appl Energy* 2012;99:135–45. <https://doi.org/10.1016/j.apenergy.2012.04.046>.
- [43] Iniasta C, Olazagoitia JL, Vinolas J, Gros J. New method to analyse and optimise thermoacoustic power generators for the recovery of residual energy. *Alex Eng J* 2020;59:3907–17. <https://doi.org/10.1016/j.aej.2020.06.046>.
- [44] Moreno-Ramírez C, Iniasta C, González A, Olazagoitia JL. Development and characterization of a low-cost sensors system for an acoustic test bench. *Sensors* 2020;20:6663. <https://doi.org/10.3390/s20226663>.