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Optimal Selection of the Mother Wavelet in WPT Analysis and Its Influence in Cracked Railway Axles Detection

Marta Zamorano *, María Jesús Gómez and Cristina Castejón

MAQLAB Group, Mechanical Engineering Department, Universidad Carlos III de Madrid, Av. de la Universidad 30, 28911 Leganés, Spain; mjggarci@ing.uc3m.es (M.J.G.); castejon@ing.uc3m.es (C.C.)
* Correspondence: mazamora@ing.uc3m.es

Abstract: The detection of cracked railway axles by processing vibratory signals measured during operation is the focus of this study. The rotodynamic theory is applied to this specific purpose but, in practice and for real systems, there is no consensus on applying the results obtained from theory. Finding reliable patterns that change during operation would have advantages over other currently applied methods, such as non-destructive testing (NDT) techniques, because data between inspections would be obtained during operation. Vibratory signal processing techniques in the time-frequency domain, such as wavelet packet transform (WPT), have proved to be reliable to obtain patterns. The aim of this work is to develop a methodology to select the optimal function associated with the WPT, the mother wavelet (MW), and to find diagnostic patterns for cracked railway axle detection. In previous related works, the Daubechies 6 MW was commonly used for all speed/load conditions and defects. In this work, it was found that the Symlet 9 MW works better, so a comparative study was carried out with both functions, and it was observed that the success rates obtained with Daubechies 6 are improved when using Symlet 9. Specifically, defects above 16.6% of the shaft diameter were reliably detected, with no false alarms. To validate the proposed methodology, experimental vibratory signals of a healthy scaled railway axle were obtained and then the same axle was tested with a transverse crack located close to a section change (where this type of defect typically appears) for nine different crack depths.

Keywords: mother wavelet; wavelet packet transform; vibration analysis; condition monitoring; railway axles; crack detection; crack diagnosis

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1. Introduction

Rotating shafts are safety-critical components in machines, and, specifically, railway axles are such in the rolling stock. This is why detecting any problem or defect in time is of vital importance. One of the most challenging defects that may appear is cracks, whose propagation can be rapid due to the phenomenon of fatigue [1]. For the detection of these cracks, the most widespread approach is to carry out traditional non-destructive testing (NDT) techniques, such as ultrasonic testing and magnetic particle testing. However, these techniques involve some gaps [2] together with design methods, such as damage tolerance [3]. For this purpose, axles are periodically uninstalled and checked at suitable service interruptions planned for maintenance operations, which implies high costs. Thus, the condition of the axle is not known between inspections.

To solve this issue, currently, a second and new approach is being developed, which is focused on condition-based maintenance, implementing tools and methods in real time or on-demand structural health monitoring (SHM) of railway axles. In the aeronautical and civil fields, this type of technique has traditionally proven to be more effective than traditional NDT in terms of maintenance costs, reducing them by 30% [4,5], and, moreover, they improve the control over the mechanical system, detecting defects much earlier.

In the specific case of railway axles, this type of maintenance technique is still under study. However, there are some studies related to SHM available in the literature. Some of the advances that can be found in the bibliography related to these components are based on the use of acoustic emission (AE) [6] and based on the use of vibration signals, which are passive NDT techniques that avoid the stopping and disassembly of the axle, knowing its condition at all times and allowing one to extend the inspection intervals.

Regarding the use of vibration signals, there are works that have used low-frequency vibrations, specifically energy changes observed in the first harmonics of the rotation speed (1×, 2×, or 3×), such as [7] and [8]. The work of [7] detected these changes using traditional signal processing techniques such as Fast Fourier Transform (FFT) for signals coming both from numerical simulations and experimental data. It has also been found that significant changes in energy occur in the harmonics and subharmonics (1/3) of natural frequencies of the axle when a transverse crack appears [8]. In other cases, general energy patterns of the vibration signal in a wide frequency range have also been used to train intelligent classification systems with good results in works, such as [9] and [10]. In the works [8–10], wavelet packet transform (WPT) was implemented using, as a mother wavelet (MW), Daubechies 6 for experimental signals. However, the use of this mother wavelet is not properly justified and is based only on experience.

However, for crack detection through wavelet techniques, the choice of the associated parameters, especially the mother wavelet, is essential to obtain good results. Works such as [11] proposed a method for crack detection in shafts based on CWT (continuous wavelet transform) and compared the prediction accuracies for different mother wavelets (MWs). It concluded that Mexican Hat and Morlet showed better results than using the Daubechies family. Works such as [12] proposed a combined wavelet analysis, using CWT, DWT (discrete wavelet transform), and WPT (wavelet packet transform). In this last analysis, mother wavelets daubechies 1 and 3 were used, and the Morlet MW was used to apply the CWT. WPT analysis has also been demonstrated to be very useful in previous related works such as [9,10,13,14], in which Daubechies 6 MW was used based on experience. Conducting a WPT analysis, differentiating patterns of the operating condition of the system can be obtained, but there is no consensus on the use of the mother wavelet, even though the Daubechies family has proved to be very useful in crack detection using vibration signal analysis [15]. The question of which mother wavelet is the optimal for each case still remains in many fields, and different approaches are proposed. In civil engineering structures, the works of [16,17] dealt with this question to analyze time-varying and nonstationary signals. Reference [18] analyzed this issue to identify surface topography irregularities. In the AE field, [19] proposed a methodology to analyze composite materials and [20] analyzed them in the railway field. In the vibrations field, [21,22] also dealt with the optimal selection of the mother wavelet. It seems clear that it is an issue of great interest and needs to be further explored, which is one of the main objectives of this paper. The choice of the mother wavelet is so crucial that it can determine if a maintenance process is optimal in terms of false alarms and computation time, so it needs to be analyzed for each particular case.

In this work, a new methodology is proposed for the detection of cracks in railway axles, considering only the energy in the first three harmonics of the rotation frequency (1×, 2×, and 3×), which are the most effective fault frequencies for crack detection according to the bibliography related to rotordynamics [23,24], considerably reducing the amount of data to be processed and, thus, the computation time of the applied algorithm with respect to previous related work [22], which analyzed the energy of the whole signal and with large frequency bands. This methodology incorporates an intelligent classification system, using an algorithm based on K-Nearest Neighbor (KNN) and improves a previous proposal in Ref. [22], since the algorithm proposed guarantees the selection of the mother wavelet that offers the clearest diagnostic pattern, which is a packet that includes the third harmonic of the rotation frequency (3×). In the specific case of the experimental data in this work, the use of the selected mother wavelet (Symlet 9) results in a diagnosis with no false alarms when the axle is in good conditions (without crack), which is the clearest advantage of the proposed method, and that also allows for detecting the crack with high probability when the size begins to be large enough to require an intervention to prevent failure, thus avoiding the detection of the crack when its size does not imply a critical behavior of the axle. Outcomes are also compared to those that would be obtained using Daubechies 6, which was the MW used in previous related works [8,10,13,14], showing less standard deviation of the data using symlet 9. This will be applied for the detection of a crack located at the section change of a scaled railway axle, where cracks commonly appear in these components [2].

In summary, this work investigates the application of SHM via vibration analysis to the case of cracked railway axles, using the WPT tool, for which a specific methodology is developed to select the optimal parameters of the mentioned tool (MW). To carry this out, a scaled railway axle (1:8) was tested with nine different crack depths, which were artificially manufactured. The influence on the results of the correct selection of the MW will also be demonstrated. To the authors' knowledge, very few studies focus on the selection of these parameters before performing an analysis.

The contents are organized as follows. Section "Experimental system" gives a description of the performed test. After that, the section "Wavelet Packet Transform" describes the post-processing tool. The methodology of the mother wavelet selection and results are shown in section "methodology and results" and, after that, the "comparative study" section shows the comparison of results using different mother wavelets.

2. Experimental System

Vibratory signals are obtained from a Machinery Fault Simulator, by SpectraQuest Inc. Its main components are a motor of 750 W, which is connected to the shaft by flexible coupling. The shaft is supported by two bearings ER10, by Rexnord, and it is a railway axle model at a 1:8 scale, making the diameter 20.77 mm. First, the undamaged shaft is tested and then a crack is mechanized with a saw cut (see Figures 1 and 2) and its depth is increased after each test according to Table 1. The tests are performed at 60 Hz because it has been proven to be the best rotation frequency for diagnosis in these conditions and machines [13,14].

Figure 1. Cracked axle used to obtain vibratory signals.

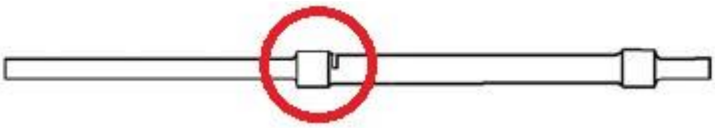

Figure 2. Location of the crack.

Table 1. Defect level of the axle (crack depth).

Defect Level	Defect Depth to Diameter (%)	Defect Depth (mm)
0	0	0
1	4.15	0.86
2	8.30	1.72
3	12.50	2.59
4	16.65	3.46
5	22.15	4.6
6	25.00	5.17
7	33.25	6.91
8	41.65	8.65
9	50.20	10.39

The vibratory signals are digitalized on a computer with an acquisition system. First, a piezoelectric accelerometer (Brüel & Kjaer 4383) is placed on top of the bearing housing closest to the crack and the motor. This is connected to a signal conditioner (Nexus Brüel & Kjaer 2693) and, later to a data acquisition card (Keithley KUSB-3100), which, in turn, is connected to the computer via USB. To obtain the signals and to set the acquisition parameters, an interface developed in Matlab® by the authors was used. The sampling frequency is 6000 Hz, and for each signal, 16,384 points are measured. This makes the duration of each signal T of 2.73 s. For each test, a total of 1000 measurements were taken.

3. Wavelet Transform

Wavelet transform (WT) is a tool that offers information both in the time and frequency domains. The signal under study is compared to a function called wavelet $\Psi s,\ \tau(t)$, described by Equation (1), by calculating the correlation coefficients, which depend on the translation (τ) and scale (s) of the mother wavelet function (Ψ) [25]. In this way, the *CWT* (continuous wavelet transform) is applied according to Equation (2).

$$\Psi s, \tau(t) = \frac{1}{\sqrt{s}} \Psi \left(\frac{1-\tau}{s} \right) \tag{1}$$

$$CWT(s,\tau) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} x(t) \psi^* \left(\frac{t-\tau}{s} \right) dt \tag{2}$$

To apply the WT in a discrete way, the DWT (discrete wavelet transform) is implemented via filters [25]. The study signal is decomposed by using a low-pass filter to obtain the wavelet approximation information (A) and a high-pass filter to obtain the wavelet detailed information (D) (Figure 3).

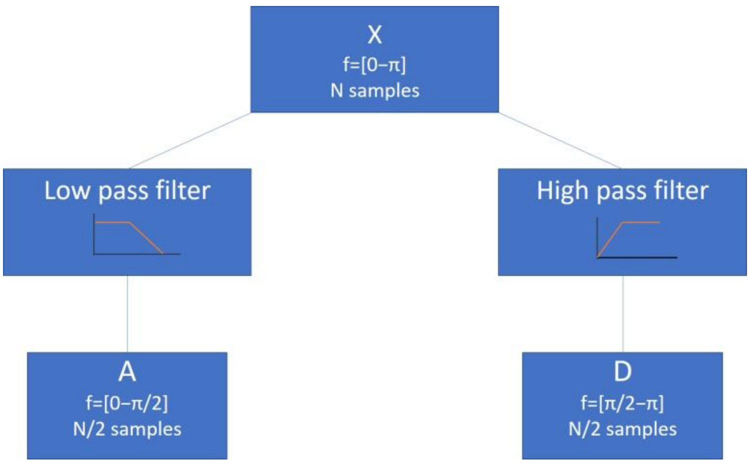

Figure 3. DWT procedure.

Wavelet packet transform (WPT) applies the DWT recursively up to a decomposition level k set by the user (Figure 4). In Equation (3), $W\ (k, j)$ represents the coefficients of the signal of each packet, with j being the position of the packet within the decomposition level.

$$W\ (k, j) = \{w_1(k, j), \ldots,\ w_N(k, j)\} \tag{3}$$

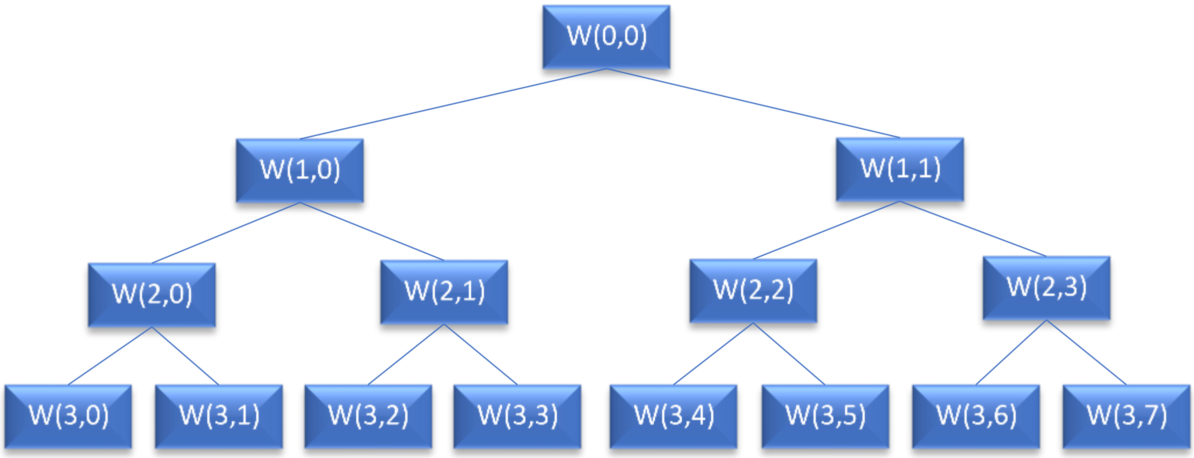

Figure 4. WPT decomposition process for a decomposition level of 3.

Given the large amount of information that is obtained, it is common to calculate the energy of the packets $E(k, j)$, described by Equation (4), to obtain clearer and more useful information to interpret.

$$E(k, j) = \sum_i \{w_i(k, j)\}^2 \tag{4}$$

The studied frequency range (3000 Hz) is divided into packets, 2 k. The methodology for the mother wavelet selection is applied with a decomposition level of 9 (since it is the one that guarantees a sufficiently narrow frequency band to ensure that the frequencies analyzed (fault frequencies) are found in a packet and not in the previous/next one [14]), obtaining 512 packets, but only those that include the first three harmonics of the rotation frequency will be analyzed. Each packet represents a frequency band of 5.86 Hz (3000/512).

4. Methodology and Results

The accelerometer measures in a discrete way, so only discrete tools can be used, such as DWT and WPT. Due to the properties of the different MW families, some families cannot be applied to discrete tools, such as Mexican Hat or Meyer, which were discarded.

Moreover, some families have different mother wavelets with different order N [26]. This parameter can be further analyzed, since it is known that the higher the order, the more computation time, so good results are sought in the shortest time possible. In this way, families with different order N were selected.

With all this, Daubechies (db), Symlets (sym), and Coiflets (coif) were chosen. Moreover, they are the most used in the bibliography. A more detailed explanation of this selection can be seen in Ref. [22].

The steps that were followed to find the optimal mother wavelet are as follows:

1. Once the vibratory signals are measured (for undamaged and defective shaft), the WPT is applied for the different mother wavelets with a decomposition level of 9, and the energy of the packets is calculated with Equation (4). The three packets corresponding to the theoretical fault frequencies (the first three harmonics of the rotating speed) will be considered.
2. The optimal order of the mother wavelet for each family is determined by a parameter called DEV. This parameter was used in previous work [22], which proved to be very useful to find the greatest difference in energy between undamaged and defective

conditions. This parameter is calculated for each defect level and for each mother wavelet. It measures the difference between the average energy values of the defective shaft and the undamaged shaft energy values. The higher this value, the greater the difference in energy between undamaged and defective states. In Equation (5), n is the number of packets, i is the packet number, P is the mean energy of packet i for a defective shaft, and O is the mean energy of packet i for an undamaged axle.

$$\mathrm{DEV} = \sqrt{\frac{\sum_{i=1}^{n} (P_i - O_i)^2}{n}} \quad (5)$$

3. A sensitivity study was carried out to find the minimum order N of the mother wavelet for which the DEV value is maximum. This was carried out for defect level 1, since it is the smallest, to guarantee the greatest differentiation with the undamaged condition. In Figure 5, the DEV evolution is shown for different orders of the mother wavelet families and for defect level 1. The number of vanishing moments is the same as the order (N) for Daubechies and Symlets families. For the Coiflets family, the number of vanishing moments is double the order (N).
4. In the case of the Coiflets family, which has a maximum order of 5, it corresponds to its maximum DEV. For the Daubechies, it was observed that the closest order with the maximum DEV was 10 and, for the Symlets family, the lower order with maximum DEV value was 11.

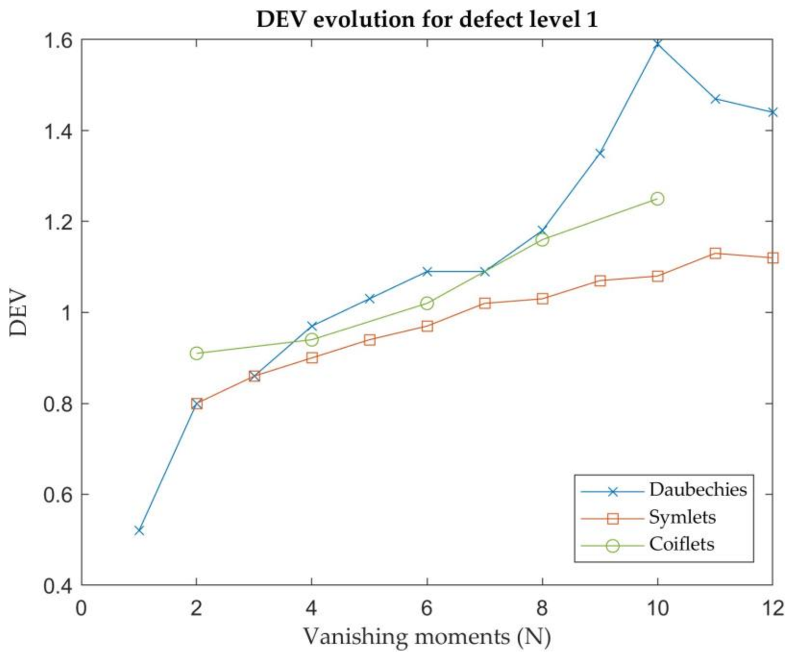

Figure 5. DEV evolution for defect level 1 and different mother wavelets.

5. In addition to these three mother wavelets (coif5, sym11, and db10), we was also decided to preselect those with lower order that have a DEV variation of less than 6% for all defect levels greater than 6 (this means a crack with a depth greater than 25% of the diameter). This DEV variation DEVvariation (%) measures the difference between the DEV values of a mother wavelet DEV(N) with the mother wavelet of maximum DEV of each family DEVmax, according to Equation (6). For example, the DEV value for each defect level using sym9 is compared with the DEV values obtained with sym11.

$$\mathrm{DEV}_{\text{variation}}\,(\%) = \frac{\mathrm{DEV}_{\max} - \mathrm{DEV(N)}}{\mathrm{DEV}_{\max}} \cdot 100 \quad (6)$$

6. In Table 2, it can be seen that sym8 does not meet the criterion (<6%), but sym9 and sym10 do. No more MW from the Coiflets and Daubechies were selected, as they do not meet the previous criterion.

Table 2. Variation in the DEV for Symlets family.

Defect Level	Sym 10	Sym 9	Sym 8
6	4.05	5.00	8.06
7	3.90	4.95	7.64
8	3.32	4.19	7.17
9	3.33	4.49	7.21

7. With the preselected MW (sym9, sym10, sym11, coif5, and db10), a reliability analysis was carried out. The energy results of the packets that include the harmonics for each MW were entered into a classification system in which the undamaged (0) and defect (1) classes were differentiated. Here, the highest success rate of the model used is sought. For this, the classification learner application of Matlab®® is used, which allows for comparing the success rate of numerous models (Support Vector Machines (SVMs), K-Nearest Neighbor (KNN), decision trees, discriminant analysis, Naive Bayes, and ensemble classifiers). For this case, the best classifier is KNN cosine, which minimizes false alarms (this means it classifies the undamaged condition with a high success rate) and is also very reliable in the classification of the defective condition. This algorithm essentially classifies values by looking for the 'most similar' data points (by closeness) learned in the training stage and estimating new points based on that classification [27]. The KNN cosine algorithm is supervised, which means that the training dataset was labelled with the expected class. It is also instance based, which means that the algorithm does not explicitly learn a model (such as in logistic regression or decision trees). Instead, it memorizes the training instances that are used as a 'knowledge base' for the prediction phase. The KNN cosine algorithm calculates the cosine distance metric between the item to be classified and the rest of the items in the training dataset.
8. In Table 3, the success rate of the classification system using a KNN-cosine algorithm for each preselected MW is shown.

Table 3. Success rate of the classification system (KNN-cosine) for each mother wavelet.

Mother Wavelet	Success Rate (%)
Sym 9	94.4
Sym 10	93.6
Sym 11	93.9
Coif 5	93.6
Db 10	91.1

9. With these results, the selected mother wavelet is symlet 9 as the different conditions are better classified.
10. The results of the classification system are shown in a parallel coordinate plot (PCP) (Figure 6), which is an easy way to find the pattern that best differentiates the undamaged (0—blue) and the defective condition (1—orange) in order to facilitate the diagnostic task. In this figure, the results using sym9 and KNN-cosine are shown. The first three harmonics are on the X-axis, and the multiples of the standard deviation of the results are on the Y-axis, where '$0'$' represents the mean of the results. Dashed lines represent data not correctly classified.

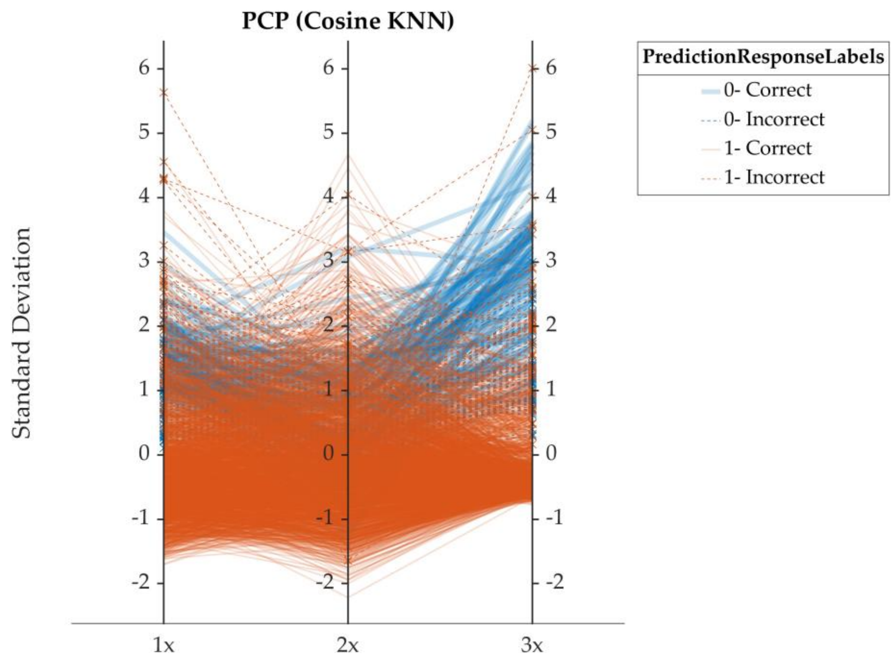

Figure 6. PCP sym9 and KNN-cosine.

In this case, the energy of the third harmonic is the best pattern for the detection of the crack, since it is the only pattern in which a differentiation between the two classes (undamaged and defective) is observed. This is in line with what was concluded in previous related works [13,14], which used the Daubechies 6 function to perform the wavelet analysis. However, in the next section, the importance of the selection of the mother wavelet will be shown and discussed in the results, thus validating and demonstrating the importance and non-conformity of choosing any parameters and promoting work prior to post-processing to optimize and guarantee good and reliable results.

5. Comparative Study

In this section, the validation of the methodology and the verification of the importance of the optimal MW selection are performed. Results using the selected MW (Symlet 9) and the results using Daubechies 6, as used in previous related works, will be compared. First, the comparison of the mean and standard variation in the packet that includes the 3× frequency will be shown, which is the only packet in which differences between undamaged and defective conditions are observed according to the PCP.

In this way, in Figure 7, the energy values (mean and standard deviation) of the 3× packet at 60 Hz are represented for both MWs, with a level of decomposition of 9.

It can be seen that the packet that includes the 3× frequency decreases its energy with the presence of a crack for both mother wavelets. This is the same as concluded in [13], but, as can be seen, the results change significantly using one mother wavelet or another. It is observed, in Figure 7, that the trend changes from level 5 if Daubechies 6 is used (begins to have an increasing trend) and the trend remains stable if Symlet 9 is used. This is a very relevant conclusion, since the larger the crack, the more important it is to detect it reliably and immediately to avoid catastrophic failure. A change in trend in energy between undamaged and defective conditions can allow for establishing an energy threshold that leads to the detection of the crack in time.

It is also seen that the standard deviation of the data is much smaller for the results obtained with Symlet 9, proving once again that the results are more stable and reliable using the optimal MW.

Finally, a probability of detection (POD) curve with a 95% lower confidence limit is shown for both mother wavelets (Figure 8).

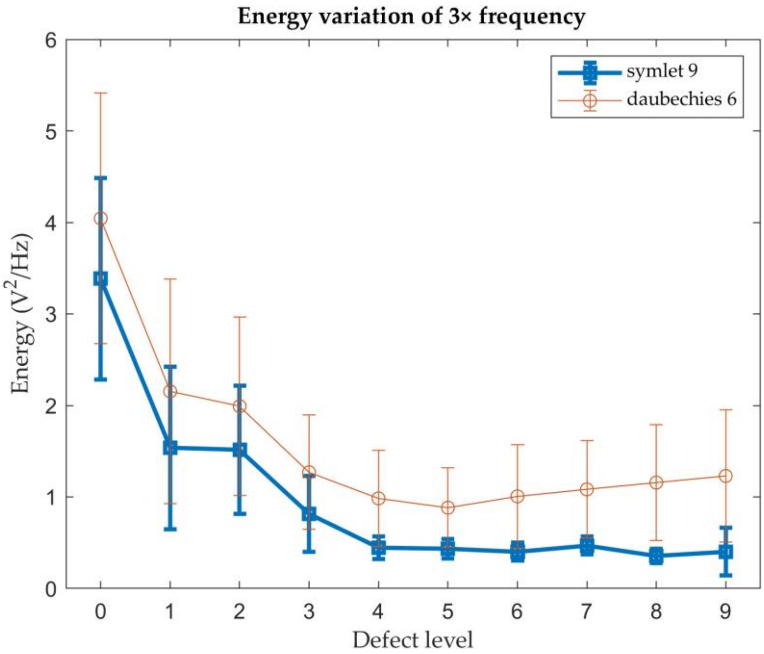

Figure 7. Energy variation of 3× frequency using Symlet 9 and Daubechies 6.

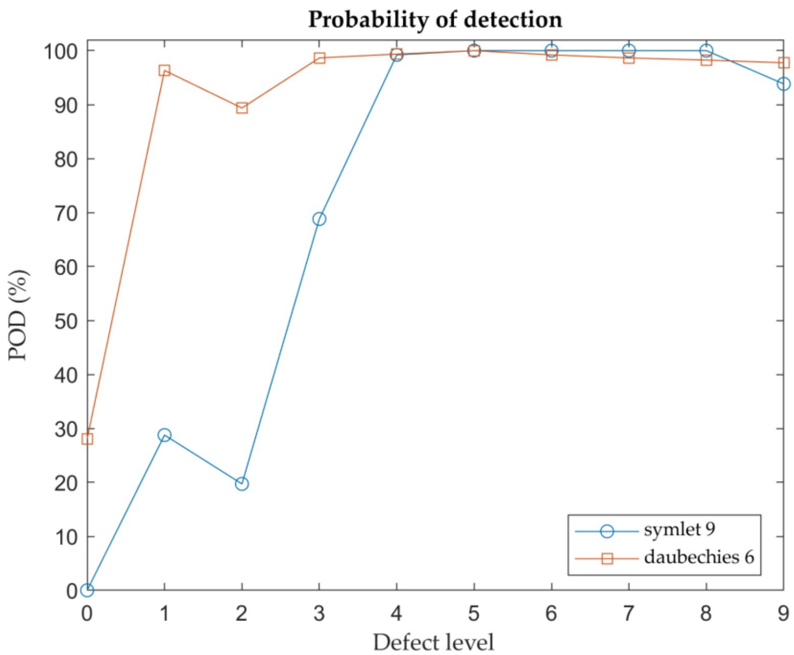

Figure 8. POD curves for symlet 9 and daubechies 6.

It can be seen that, when using Symlet 9, in this case, the false alarm rate is 0% and the defect will be detectable with high reliability from defect level 4 (16.65%). When using Daubechies 6, although the probability of defect detection is high for any level, the false alarms are almost 30%, which is not recommended for diagnostic tasks. Therefore, it is observed that with Symlet 9, the undamaged axle data are better distinguished from the defective shaft data under these conditions. These results are promising, since it is not important to detect the smallest crack sizes, because they are not considered with such a critical size that can cause serious problems. In fact, it is good that they are not detected with high probability, because this methodology detects the crack but not the size. Detecting small cracks could shorten the life of the axle and waste time performing maintenance operations that are not necessary. For all this, Symlet 9 is a much better option to carry out the analysis.

6. Conclusions

This paper investigates the application of SHM through vibration analysis for the detection of cracks in railway axles. For this, an analysis using the WPT tool was performed to analyze the energy of the theoretical fault frequencies, which are the first three harmonics of the rotation frequency. The use of this tool requires a previous selection of the parameters, such as mother wavelet, for which a specific methodology was developed incorporating intelligent classification systems that help to determine the MW with the least number of false alarms for diagnosis (in this case, the KNN-cosine model).

With all this, MW Symlet 9 was selected for the post-processing of the vibratory signals. It was also concluded that the packet, including the $3\times$ frequency, is a good pattern for crack detection. Moreover, to strengthen this selection, a comparative study was performed with the Daubechies 6 MW, which was used in previous works based on experience.

Looking at the results displayed with the POD curve, Symlet 9 shows no false alarms, and the crack is accurately detected from sizes above 16.6% of the diameter of the shaft, which is interesting, since it would detect cracks when they begin to be serious and critical and intervention would be needed to avoid a failure, whereas the POD curve for the results of Daubechies 6 shows almost 30% of false alarms, which is not recommendable for a reliable and efficient diagnostic task. In this way, previous works are improved.

It is concluded that the proposed methodology works and is useful for performing reliable, fast, and efficient maintenance for railway axles and vehicles, improving and reinforcing traditional techniques based on periodic inspection intervals.

In order to implement the methodology to real systems, the validation on a bogie test bench under realistic conditions must be conducted and, ideally and finally, on a real train in operation.

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